NCERT Solutions for Class-XII Maths

Chapter-4.2

NCERT Math Class 12

Using the property of determinants and without expanding in Exercises 1 to 7, prove that:

1. $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$

1. LHS= $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = \begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix}$ [Applying $C_1 \to C_1 + C_2$]

$= 0 =$ RHS $\quad [\because C_1 = C_3]$

2. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

2. Applying Operation $C_1 \to C_1 + C_2$ (i.e. Replacing 1st column by addition of 1st and 2nd column)

$\therefore$ LHS = $\begin{vmatrix} a-c & b-c & c-a \\ b-a & c-a & a-b \\ c-b & a-b & b-c \end{vmatrix}$

$C_1 \to C_1 + C_3$ (i.e. Replacing 1st column by addition of 1st and 3rd column)

$\therefore$ LHS = $\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$

If any one of the rows or columns of a determinant is 0 then the value of that determinant is 0.

$\therefore$ LHS = 0 = RHS

$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

3. $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$

3. $\text{LHS} = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix}$ [Applying $C_3 \rightarrow C_3 - C_1$]

$= 9\begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix}$ [Taking common 9 from C_3]

= 0 = RHS [$\because C_2 = C_3$]

4. $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

4. $\therefore \text{LHS} = \begin{vmatrix} 1 & bc & ab+ac \\ 1 & ca & bc+ab \\ 1 & ab & ac+bc \end{vmatrix}$

$C_3 \rightarrow C_2 + C_3$ (i.e. replace 3rd column by addition of 2nd and 3rd column)

$\therefore \text{LHS} = \begin{vmatrix} 1 & bc & ab+bc+ac \\ 1 & ca & ab+bc+ac \\ 1 & ab & ab+bc+ac \end{vmatrix}$

Taking ab + bc + ac outside determinant

$\therefore \text{LHS} = \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$

$C_1 \rightarrow C_1 - C_3$ (i.e. replace 1st column by subtraction of 1st and 3rd column)

$\therefore \text{LHS} = \begin{vmatrix} 0 & bc & 1 \\ 0 & ca & 1 \\ 0 & ab & 1 \end{vmatrix}$

If any one of the rows or columns of a determinant is 0 then the value of that determinant is 0.

∴ LHS = 0 = RHS

$\therefore \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

5. $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

5. $\text{LHS} = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$

$= \begin{vmatrix} 2c & 2r & 2z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$ [Applying $R_1 + R_1 + R_2 - R_3$]

$= 2\begin{vmatrix} c & r & z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$ [Taking 2 as common from R_1]

$= 2\begin{vmatrix} c & r & z \\ a & p & x \\ a+b & p+q & x+y \end{vmatrix}$ [Applying $R_2 + R_2 - R_1$]

$= 2\begin{vmatrix} c & r & z \\ a & p & x \\ b & q & y \end{vmatrix}$ [Applying $R_3 \rightarrow R_3 - R_2$]

$= -2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$ [Applying $R_1 + R_2$]

$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \text{RHS}$ [Applying $R_2 \leftrightarrow R_3$]

By using properties of determinants, in Exercises 8 to 14, show that:

6. $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

6. $R_1 \rightarrow cR_1$ (i.e. replace 1st row by multiplying it with c)
As we are multiplying we should also divide c so that the original given determinant is not changed

$\therefore \text{LHS} = \frac{1}{c}\begin{vmatrix} 0 & ac & -bc \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$

$R_1 \rightarrow R_1 - bR_2$ (i.e. replace 1st row by subtraction of 1st row and b times 2nd row)

$$\therefore \text{LHS} = \frac{1}{c}\begin{vmatrix} ab & ac & 0 \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

Taking a outside the determinant from 1st row

$$\therefore \text{LHS} = \frac{a}{c}\begin{vmatrix} b & c & 0 \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

If any two rows or columns of a determinant are identical then the value of that determinant is 0 because we get a row or column with all elements 0 when we when we subtract those particular rows/columns here the transformation is R1 → R1 - R3

∴LHS = 0 = RHS

$$\therefore \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

7. $$\begin{vmatrix} -a^2 & ab & ac \\ bc & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

7. $$\text{LHS} = \begin{vmatrix} -a^2 & ab & ac \\ bc & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

$$= abc\begin{vmatrix} -a & a & a \\ b & -b & b \\ c & c & -c \end{vmatrix}$$ [Taking a, b, c as common from C_1, C_2, C_3]

$$= a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$ [Taking a, b, c as common from R_1, R_2, R_3]

$$= a^2b^2c^2\begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$ [Applying $C_1 \to C_1 + C_2$]

$$= a^2b^2c^2\{2(1+1)\}$$ [Expanding along C_1]

$$= 4a^2b^2c^2 = \text{RHS}$$

8. (i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

8. R1 → R1 - R2 (i.e. Replacing 1st row by subtraction of 1st and 2nd row)

R2 → R2 - R3 (i.e. Replacing 2nd row by subtraction of 2nd and 3rd row)

$\therefore \text{LHS} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$

Since we know $a^2 - b^2 = (a + b)(a - b)$

Therefore taking (a - b) and (b - c) outside the determinant from 1st and 2nd row respectively

$\therefore \text{LHS} = (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$

R1 → R1 - R2 (i.e. Replacing 1st row by subtraction of 1st and 2nd row)

$\therefore \text{LHS} = (a-b)(b-c)\begin{vmatrix} 0 & 0 & a-c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$

Expanding the determinant along 1st column

$\therefore \text{LHS} = (a-b)(b-c)\begin{vmatrix} 0 & a-c \\ 1 & b+c \end{vmatrix}$

∴LHS = (a - b)(b - c)(0 - (a - c))

∴LHS = (a - b)(b - c)(c - a) = RHS

$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

Solution: C1 → C1 - C2 (i.e. Replacing 1st column by subtraction of 1st and 2nd column)

C2 → C2 - C3 (i.e. Replacing 2nd column by subtraction of 2nd and 3rd column)

$$\therefore \text{LHS} = \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

We have $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ and $b^3 - c^3 = (b - c)(b^2 + bc + c^2)$

Therefore taking (a - b) and (b - c) outside the determinant from 1st and 2nd column respectively

$$\therefore \text{LHS} = (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+ab+b^2 & b^2+bc+c^2 & c^3 \end{vmatrix}$$

C1 → C1 - C2 (i.e. Replacing 1st column by subtraction of 1st and 2nd column)

$$\therefore \text{LHS} = (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (a^2-c^2)+b(a-c) & b^2+bc+c^2 & c^3 \end{vmatrix}$$

As $a^2 - c^2) = (a + c)(a - c)$ therefore taking (a - c) outside the determinant from 1st column we get

$$\therefore \text{LHS} = (a-b)(b-c)(a-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (a+c)+b & b^2+bc+c^2 & c^3 \end{vmatrix}$$

Expanding the determinant along 1st row

∴LHS = (a - b)(b - c)(a - c)(- (a + b + c))

Adjusting the minus sign with (a - c)

∴LHS = (a - b)(b - c)(c - a)(a + b + c) = RHS

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

9. $$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

9. $$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = \begin{vmatrix} x^2 & x^2 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix}$$ [Applying $R_1 \to xR_1$, $R_2 \to yR_2$, $R_3 \to zR_3$]

$$= xyz\begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$ [Taking xyz as common from C_3]

$$= xyz\begin{vmatrix} x^2 - y^2 & x^3 - y^3 & 0 \\ y^2 - z^2 & y^3 - z^3 & 0 \\ z^2 & z^3 & 1 \end{vmatrix}$$ [Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$]

$$= xyz(x-y)(y-z)\begin{vmatrix} x+y & x^2+xy+y^2 & 0 \\ y+z & y^2+yz+z^2 & 0 \\ z^2 & z^3 & 1 \end{vmatrix}$$ [Taking x – y as common from R_1 and y – z from R_2]

$= xyz(x - y)(y - z)\{(x + y)(y^2 + y^2 + z^2) - (y + z)(x^2 + xy + y^2)\}$ [Expanding along C_3]

$= xyz(x - y)(y - z)\{xy^2 + xyz + xz^2 + y^3 + y^2z + yz^2 - (x^2y + xy^2 + y^2 + x^2z + xyz + y^2z\}\}$

$= xyz(x - y)(y - z)\{xz^2 + yz^2 - x^2y - x^2z\} = xyz(x - y)(y - z)\{xz^2 - x^2z + yz^2 - x^2y\}$

$= xyz(x - y)(y - z)\{xz(z - x) + y(z^2 - x^2)\}$

$= xyz(x - y)(y - z)(z - x)\{x^2 + y(z + x)\}$

$= (x - y)(y - z)(z - x)(xy + yz + zx) =$ RHS

10. (i) $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

(ii) $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

10. R1 → R1 + R2 + R3 (i.e. replace 1st row by addition of 1st, 2nd and 3rd row)

$$\therefore \text{LHS} = \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Taking 5x + 4 outside the determinant from 1st row

$$\therefore \text{LHS} = (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

C2 → C2 - C1 (i.e. replace 2nd column by subtraction of 2nd and 1st column)

C3 → C3 - C1 (i.e. replace 3rd column by subtraction of 3rd and 1st column)

$$\therefore \text{LHS} = (5x+4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & 4-x & 0 \\ 2x & 0 & 4-x \end{vmatrix}$$

Expanding the determinant along 1st row

$$\therefore \text{LHS} = (5x+4)\begin{vmatrix} 4-x & 0 \\ 0 & 4-x \end{vmatrix}$$

$\therefore$LHS = (5x - 4) (4 - x)^2 = RHS

$$\therefore \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

(ii) $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

Solution:

R1 → R1 + R2 + R3 (i.e. replace 1st row by addition of 1st, 2nd and 3rd row)

$$\therefore \text{LHS} = \begin{vmatrix} 3y+k & 3y+k & 3y+k \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

Taking 3y + k outside the determinant from 1st row

$$\therefore \text{LHS} = (3y+k)\begin{vmatrix} 1 & 1 & 1 \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

C2 → C2 - C1 (i.e. replace 2nd column by subtraction of 2nd and 1st column)

C3 → C3 - C1 (i.e. replace 3rd column by subtraction of 3rd and 1st column)

$$\therefore \text{LHS} = (3y+k)\begin{vmatrix} 1 & 0 & 0 \\ y & k & 0 \\ y & 0 & k \end{vmatrix}$$

Expanding the determinant along 1st row

$$\therefore \text{LHS} = (3y+k)\begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix}$$

$\therefore$LHS = (3y + k) k^2 = RHS

$$\therefore \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$$

11. (i) $\begin{vmatrix} a-b-c & 2a & 2a \\ ab & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

(ii) $\begin{vmatrix} x+y+c & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

11. (i) LHS $= \begin{vmatrix} a-b-c & 2a & 2a \\ ab & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ [Applying $R_1 \to R_1 + R_2 + R_3$]

$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ [Taking a + b + c as common from R_1]

$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ a+b+c & -a-bc & ab \\ 0 & a+b+c & c-a-b \end{vmatrix}$ [By $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$]

$= (a + b + c)\ \{(a + b + c)^2 - 0\}$ [Expanding along R_1]

$= (a + b + c)^3 =$ RHS

(ii) LHS $= \begin{vmatrix} x+y+c & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$

$= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$ [Applying $C_1 \to C_1 + C_2 + C_3$]

$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$ [Taking 2(x + y + z) common from C_1]

$= 2(x+y+z)\begin{vmatrix} 0 & -(x+y+z) & 0 \\ 0 & x+y+z & -(x+y+z) \\ 1 & x & z+x+2y \end{vmatrix}$ [By $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$]

12. $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

12. R1 → R1 - xR2 (i.e. replace 1st row by subtraction of 1st row and 'x' times 2nd row)

$\therefore \text{LHS} = \begin{vmatrix} 1-x^3 & 0 & 0 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$

Taking $(1 - x^3)$ outside the determinant from 1st row

$\therefore \text{LHS} = \left(1-x^3\right) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$

Expanding the determinant along 1st row

$\therefore \text{LHS} = \left(1-x^3\right) \begin{vmatrix} 1 & x \\ x^2 & 1 \end{vmatrix}$

$\therefore \text{LHS} = (1 - x^3)^2 = \text{RHS}$

$\therefore \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

13. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

13. $\text{LHS} = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

$= \frac{1}{2} \begin{vmatrix} 1+a^2-b^2 & 2ab & -2ab \\ 2ab & 1-a^2+b^2 & 2a^2 \\ 2b & -2a & a-a^3-ab^2 \end{vmatrix}$ [Applying $C_3 \to aC_3$]

$= \frac{1}{a} \begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2ab & 1-a^2+b^2 & 1+a^2+b^2 \\ 2ab & -2a & -a-a^3-ab^2 \end{vmatrix}$ [Applying $C_3 \to C_3 + C_2$]

$$= \frac{1+a^2+b^2}{a}\begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2ab & 1-a^2+b^2 & 1 \\ 2b & -2a & -a \end{vmatrix}$$ [Taking 1 + a^2 + b_2 as common C_3]

$$= \frac{1+a^2+b^2}{a}\begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2a^2b & 1-a^3+ab^2 & a \\ 2b & -2a & -a \end{vmatrix}$$ [Applying $R_2 \to aR_2$]

$$= \frac{1+a^2+b^2}{a^2}\begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2a^2b+2b & -a-a^3+ab^2 & 0 \\ 2b & -2a & -a \end{vmatrix}$$ [Applying $R_2 \to R_2 + R_3$]

$$= (1+a^2+b^2)\begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2a^2b+2b & -a-a^3+ab^2 & 0 \\ 2b & -2a & -a \end{vmatrix}$$ [Applying $R_2 \to R_2 + R_3$]

$$= (1+a^2+b^2)\begin{vmatrix} 1+a^2-b^2 & 2b & 0 \\ 2a^2b+2b & -1-a^2+b^2 & 0 \\ 2b & -2 & -1 \end{vmatrix}$$ [taking a as common from C_2 and C_3]

$= (1 + a^2 + b^2)\ (-1)\ \{(1 + a^2 - 6^2)\ (-1 - a^2 + b^2) - 2b\ (2a^2b + 2b)\}$ [Expanding along C_3]

$= -(1 + a^2 + b^2)\{-1 - a^2 + b^2 - a^2 - a^4 + a^2 b^2 + b^2 + a^2b^2 - b^4 - 4a^2b^2 - 4b^2\}$

$= (1 + a^2 + b^2)\ \{1 + a^4 + 4 + 2a^2 + 2a^2b^2 + 2b^2\}$

$= (1 + a^2 + b)\ (1 + a^2 + b)^2 = (1 - x^3)^2 =$ RHS

14. $$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

14. Taking out a, b and c from the determinant from 1st, 2nd and 3rd row respectively.

$$\therefore \text{LHS} = abc\begin{vmatrix} a+\frac{1}{a} & b & c \\ a & b+\frac{1}{b} & c \\ a & b & c+\frac{1}{c} \end{vmatrix}$$

$R2 \to R2 - R1$ (i.e. replace 2nd row by subtraction of 2nd and 1st row)

$R3 \to R3 - R1$ (i.e. replace 3rd row by subtraction of 3rd and 1st row)

$$\therefore \text{LHS} = abc\begin{vmatrix} a+\frac{1}{a} & b & c \\ -\frac{1}{a} & \frac{1}{b} & 0 \\ -\frac{1}{a} & 0 & \frac{1}{c} \end{vmatrix}$$

C1 → aC1 (i.e. replace 1st column by 'a' times 1st column)
C2 → bC2 (i.e. replace 2nd column by 'b' times 2nd column)
C3 → cC3 (i.e. replace 3rd column by 'c' times 3rd column)
As we are multiplying by a,b and c we should also divide by a,b and c to keep the original determinant value unchanged.

$$\therefore \text{LHS} = \frac{abc}{abc}\begin{vmatrix} a^2+1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

C1 → C1 + C2 + C3 (i.e. replace 1st column by addition of 1st, 2nd and 3rd column)

$$\therefore \text{LHS} = \begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding determinant along 1st column

$$\therefore \text{LHS} = \left(1+a^2+b^2+c^2\right)\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$\therefore$LHS = $(1 + a^2 + b^2 + c^2)$ = RHS

$$\therefore \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

15. Let A be a square matrix of order 3 x 3, then |KA| is equal to:
(A) k |A|
(B) k^2|A|
(C) k^3|A|
(D) 3k|A|

15. If B be a square matrix of order n × n, then $|kB| = k^{n-1}|B|$
Therefore, $|kA| = k^{3-1}|A| = k^2|A|$
Hence, the option (B) is correct.

16. Which of the following is correct
(A) Determinant is a square matrix.

(B) Determinant is a number associated to a matrix.
(C) Determinant is a number associated to a square matrix.
(D) None of these C.

16. Determinant is an operation which we perform on arranged numbers. A square matrix is set of arranged numbers. We perform some operations on a matrix and we get a value that value is called as determinant of that matrix hence determinant is a number associated to square matrix.