Chapter 21. Solids [Surface Area and Volume of 3-D Solids]

Exercise 21(A)

Solution 1:

Let the length, breadth and height of rectangular solid are $5x, 4x, 2x$.

Total surface area $= 1216\ \text{cm}^2$

$$2(5x \cdot 4x + 4x \cdot 2x + 2x \cdot 5x) = 1216$$
$$20x^2 + 8x^2 + 10x^2 = 608$$
$$38x^2 = 608$$
$$x^2 = \frac{608}{38} = 16$$
$$x = 4$$

Therefore, the length, breadth and height of rectangular solid are $5 \times 4 = 20\ \text{cm}$, $4 \times 4 = 16\,\text{cm}$, $2 \times 4 = 8\,\text{cm}$.

Solution 2:

Let a be the one edge of a cube.

Volume $= a^3$

$$729 = a^3$$
$$9^3 = a^3$$
$$9 = a$$
$$a = 9\,\text{cm}$$

Total surface area $= 6a^2 = 6 \times 9^2 = 486\,\text{cm}^2$

Solution 3:

Volume of cinema hall $= 100 \times 60 \times 15 = 90000\,\text{m}^3$

$150\ \text{m}^3$ requires $=$ 1 person

$90000\,\text{m}^3$ requires $= \frac{1}{150} \times 90000 =$ 600 persons

Therefore, 600 persons can sit in the hall.

Solution 4:

Let h be height of the room.

1 person requires 16 m^3

75 person requires $75 \times 16\ m^3 = 1200\ m^3$

Volume of room is 1200 m^3

$$1200 = 25 \times 9.6 \times h$$

$$h = \frac{1200}{25 \times 9.6}$$

$$h = 5\ m$$

Solution 5:

Volume of melted single cube = $3^3 + 4^3 + 5^3\ cm^3$

$= 27 + 64 + 125 cm^3$

$= 216\ cm^3$

Let a be the edge of the new cube.

Volume $= 216 cm^3$

$a^3 = 216$

$a^3 = 6^3$

$a = 6\ cm$

Therefore, 6 cm is the edge of cube.

Solution 6:

Volume of melted single cube $x^3 + 8^3 + 10^3\ cm^3$

$= x^3 + 512 + 1000 cm^3$

$= x^3 + 1512\ cm^3$

Given that 12 cm is edge of the single cube.

$$12^3 = x^3 + 1512\ cm^3$$

$$x^3 = 12^3 - 1512$$

$$x^3 = 1728 - 1512$$

$$x^3 = 216$$

$$x^3 = 6^3$$

$$x = 6\ cm$$

Solution 7:

Let the side of a cube be 'a' units.

Total surface area of one cube $= 6a^2$

Total surface area of 3 cubes $= 3 \times 6a^2 = 18a^2$

After joining 3 cubes in a row, length of Cuboid =3a

Breadth and height of cuboid = a

Total surface area of cuboid $= 2(3a^2 + a^2 + 3a^2) = 14a^2$

Ratio of total surface area of cuboid to the total surface area of 3 cubes $= \frac{14a^2}{18a^2} = \frac{7}{9}$

Solution 8:

Let the length and breadth of the room is 5x and 3x respectively.

Given that the four walls of a room at 75 paise per square met Rs. 240.

Thus,

$240 = \text{Area} \times 0.75$

$\text{Area} = \frac{240}{0.75}$

$\text{Area} = \frac{24000}{75}$

$\text{Area} = 320\text{m}$

$\text{Area} = 2 \times \text{Height}(\text{Length} + \text{Breadth})$

$320 = 2 \times 5(5x + 3x)$

$320 = 10 \times 8x$

$32 = 8x$

$x = 4$

Length $= 5x$

$= 5(4)\text{m}$

$= 20\text{m}$

Breadth $= 3x$

$= 3(4)\text{m}$

$= 12\text{m}$

Solution 9:

The area of the playground is 3650 m^2 and the gravels are 1.2 cm deep. Therefore the total volume to be covered will be:

3650 x 0.012 =43.8 m^3.

Since the cost of per cubic meter is Rs. 6.40, therefore the total cost will be:
43.8 x Rs.6.40 = Rs.280.32

Solution 10:

We know that

$$1\ mm = \frac{1}{10}\ cm$$

$$8\ mm = \frac{8}{10}\ cm$$

Volume = Base area × Height

$$\Rightarrow 2880\ cm^3 = x \times x \times \frac{8}{10}$$

$$\Rightarrow 2880 \times \frac{10}{8} = x^2$$

$$\Rightarrow x^2 = 3600$$

$$\Rightarrow x = 60\ cm$$

Solution 11:

External volume of the box= $27 \times 19 \times 11\ cm^3 = 5643\ cm^3$

Since, external dimensions are 27 cm, 19 cm, 11 cm; thickness of the wood is 1.5 cm.

∴ Internal dimensions

$$= (27 - 2 \times 1.5)\ cm, (19 - 2 \times 1.5)\ cm, (11 - 2 \times 1.5)\ cm$$

$$= 24\ cm, 16\ cm, 8\ cm$$

Hence, internal volume of box= $(24 \times 16 \times 8)\ cm^3 = 3072\ cm^3$

(i)

Volume of wood in the box= $5643\ cm^3 - 3072\ cm^3 = 2571\ cm^3$

(ii)

Cost of wood= Rs 1.20×2571 =Rs 3085.2

(iii)

Vol. of 4 cm cube= $4^3 = 64\ cm^3$

Number of 4 cm cubes that could be placed into the box

$$= \frac{3072}{64} = 48$$

Solution 12:

Area of sheet= Surface area of the tank

$\Rightarrow$Length of the sheet$\times$ its width=Area of 4 walls of the tank +Area of its base

$\Rightarrow$Length of the sheet $\times$ 2.5 m= $2(20+12)\times 8 m^2 + 20\times 12 m^2$

$\Rightarrow$Length of the sheet= 300.8 m

Cost of the sheet = 300.8 $\times$Rs 12.50 = Rs 3760

Solution 13:

Let exterior height is h cm. Then interior dimensions are 78-3=75, 19-3=16 and h-3 (subtract two thicknesses of wood). Interior volume = 75 x 16 x (h-3) which must = 15 cu dm

= 15000 cm^3

(1 dm = 10cm, 1 cu dm = 10^3 cm^3).

15000 cm^3 = 75 x 16 x (h-3)

$\Rightarrow$h-3 = 15000/(75x16) = 12.5 cm $\Rightarrow$h = 15.5 cm.

Solution 14:

(i)

If the side of the cube= *a* cm

The length of its diagonal= $a\sqrt{3}$ cm

And,

$$(a\sqrt{3})^2 = 1875$$
$$a = 25 cm$$

(ii)

Total surface area of the cube= $6a^2$

$= 6(25)^2 = 3750 cm^2$

Solution 15:

Given that the volume of the iron in the tube 192 cm^3

Let the thickness of the tube $= x$ cm

$\therefore$ Side of the external square $= (5+2x)$ cm

$\because$ Ext. vol. of the tube $-$ its internal vol. = volume of iron in the tube, we have,

$$(5+2x)(5+2x)\times 8 - 5\times 5\times 8 = 192$$
$$(25+4x^2+20x)\times 8 - 200 = 192$$
$$200+32x^2+160x-200 = 192$$
$$32x^2+160x-192 = 0$$
$$x^2+5x-6 = 0$$
$$x^2+6x-x-6 = 0$$
$$x(x+6)-(x+6) = 0$$
$$(x+6)(x-1) = 0$$
$$x-1 = 0$$
$$x = 1$$

Therefore, thickness is 1 cm.

Solution 16:

Let l be the length of the edge of each cube.

The length of the resulting cuboid $= 4\times l = 4l$ cm

Let width (b) = l cm and its height (h) = l cm

$\therefore$ The total surface area of the resulting cuboid

$$= 2(l\times b + b\times h + h\times l)$$

$$648 = 2(4l\times l + l\times l + l\times 4l)$$
$$4l^2 + l^2 + 4l^2 = 324$$
$$9l^2 = 324$$
$$l^2 = 36$$
$$l = 6\text{ cm}$$

Therefore, the length of each cube is 6 cm.

$$\frac{\text{Surface area of the resulting cuboid}}{\text{Surface area of cube}} = \frac{648}{6l^2}$$
$$\frac{\text{Surface area of the resulting cuboid}}{\text{Surface area of cube}} = \frac{648}{6(6)^2}$$
$$\frac{\text{Surface area of the resulting cuboid}}{\text{Surface area of cube}} = \frac{648}{216} = \frac{3}{1} = 3:1$$