Multiplying (i) by 0.2 and (ii) by 0.3 and adding them, we get
$0.8x + 2.1x = 3.4 + 2.4$

$\Rightarrow 2.9x = 5.8$

$\Rightarrow x = \frac{5.8}{2.9} = 2$

Now, substituting x = 2 in (i), we have

$0.4 \times 2 + 0.3y = 1.7$

$\Rightarrow 0.3y = 1.7 - 0.8$

$\Rightarrow y = \frac{0.9}{0.3} = 3$

Hence, x = 2 and y = 3.

14. Solve for x and y:
$0.3x + 0.5y = 0.5,\ 0.5x + 0.7y = 0.74$
Sol:
The given system of equations is
$0.3x + 0.5y = 0.5$ ....(i)
$0.5x + 0.7y = 0.74$ ....(ii)
Multiplying (i) by 5 and (ii) by 3 and subtracting (ii) from (i), we get
$2.5y - 2.1y = 2.5 - 2.2$

$\Rightarrow 0.4y = 0.28$

$\Rightarrow y = \frac{0.28}{0.4} = 0.7$

Now, substituting y = 0.7 in (i), we have

$0.3x + 0.5 \times 0.7 = 0.5$

$\Rightarrow 0.3x = 0.50 - 0.35 = 0.15$

$\Rightarrow x = \frac{0.15}{0.3} = 0.5$

Hence, x = 0.5 and y = 0.7.

15. Solve for x and y:
$7(y + 3) - 2(x + 2) = 14,\ 4(y - 2) + 3(x - 3) = 2$
Sol:
The given equations are:
$7(y + 3) - 2(x + 2) = 14$

$\Rightarrow 7y + 21 - 2x - 4 = 14$

$\Rightarrow -2x + 7y = -3$ ....(i)

and $4(y - 2) + 3(x - 3) = 2$

$\Rightarrow 4y - 8 + 3x - 9 = 2$

$\Rightarrow 3x + 4y = 19$ ……….(ii)

On multiplying (i) by 4 and (ii) by 7, we get:

$-8x + 28y = -12$ ……(iii)

$21x + 28y = 133$ ……(iv)

On subtracting (iii) from (iv), we get:

$29x = 145$

$\Rightarrow x = 5$

On substituting x = 5 in (i), we get:

$-10 + 7y = -3$

$\Rightarrow 7y = (-3 + 10) = 7$

$\Rightarrow y = 1$

Hence, the solution is x = 5 and y = 1.

16. Solve for x and y:

$6x + 5y = 7x + 3y + 1 = 2(x + 6y – 1)$

Sol:

The given equations are:

$6x + 5y = 7x + 3y + 1 = 2(x + 6y – 1)$

$\Rightarrow 6x + 5y = 2(x + 6y – 1)$

$\Rightarrow 6x + 5y = 2x + 12y – 2$

$\Rightarrow 6x – 2x + 5y – 12y = -2$

$\Rightarrow 4x – 7y = -2$ …….(i)

and $7x + 3y + 1 = 2(x + 6y – 1)$

$\Rightarrow 7x + 3y + 1 = 2x + 12y – 2$

$\Rightarrow 7x – 2x + 3y – 12y = -2 – 1$

$\Rightarrow 5x – 9y = -3$ …….(ii)

On multiplying (i) by 9 and (ii) by 7, we get:

$36x - 63y = -18$ ……(iii)

$35x - 63y = -21$ ……(iv)

On subtracting (iv) from (iii), we get:

$x = (-18 + 21) = 3$

On substituting x = 3 in (i), we get:

$12 - 7y = -2$

$\Rightarrow 7y = (2 + 12) = 14$

$\Rightarrow y = 2$

Hence, the solution is x = 3 and y = 2.

17. Solve for x and y:

$\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x+y-12}{11}$

Sol:

The given equations are:

$\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x+y-12}{11}$

i.e., $\frac{x+y-8}{2} = \frac{3x+y-12}{11}$

By cross multiplication, we get:

$11x + 11y – 88 = 6x + 2y – 24$

$\Rightarrow 11x – 6x + 11y – 2y = -24 + 88$

$\Rightarrow 5x + 9y = 64$ …….(i)

and $\frac{x+2y-14}{3} = \frac{3x+y-12}{11}$

$\Rightarrow 11x + 22y – 154 = 9x + 3y – 36$

$\Rightarrow 11x – 9x + 22y – 3y = -36 + 154$

$\Rightarrow 2x + 19y = 118$ …….(ii)

On multiplying (i) by 19 and (ii) by 9, we get:

$95x + 171y = 1216$ ……(iii)

$18x + 171y = 1062$ ……(iv)

On subtracting (iv) from (iii), we get:

$77x = 154$

$\Rightarrow x = 2$

On substituting x = 2 in (i), we get:

$10 + 9y = 64$

$\Rightarrow 9y = (64 – 10) = 54$

$\Rightarrow y = 6$

Hence, the solution is x = 2 and y = 6.

18. Solve for x and y:

$\frac{5}{x} + 6y = 13, \frac{3}{x} + 4y = 7$

Sol:

The given equations are:

$\frac{5}{x} + 6y = 13$ ……..(i)

$\frac{3}{x} + 4y = 7$ ……..(ii)

Putting $\frac{1}{x} = u$, we get:

5u + 6y = 13 …….(iii)

3u + 4y = 7 ……(iv)

On multiplying (iii) by 4 and (iv) by 6, we get:

20u + 24y = 52 ……..(v)

18u + 24y = 42 ……..(vi)

On subtracting (vi) from (v), we get:

$2u = 10 \Rightarrow u = 5$

$\Rightarrow \frac{1}{x} = 5 \Rightarrow x = \frac{1}{5}$

On substituting $x = \frac{1}{5}$ in (i), we get:

$\frac{5}{1/3} + 6y = 13$

25 + 6y = 13

6y = (13 – 25) = -12

y = -2

Hence, the required solution is $x = \frac{1}{5}$ and y = -2.

19. Solve for x and y:

$x + \frac{6}{y} = 6,\ 3x - \frac{8}{y} = 5$

Sol:

The given equations are:

$x + \frac{6}{y} = 6$ ……..(i)

$3x - \frac{8}{y} = 5$ ……..(ii)

Putting $\frac{1}{y} = v$, we get:

x + 6v = 6 …….(iii)

3x – 8v = 5 ……(iv)

On multiplying (iii) by 4 and (iv) by 3, we get:

4x + 24v = 24 ……..(v)

9x – 24v = 15 ……..(vi)

On adding (v) from (vi), we get:

$13x = 39 \Rightarrow x = 3$

On substituting x = 3 in (i), we get:

$3 + \frac{6}{y} = 6$

$\Rightarrow \frac{6}{y} = (6 - 3) = 3 \Rightarrow 3y = 6 \Rightarrow y = 2$

Hence, the required solution is x = 3 and y = 2.

20. Solve for x and y:

$2x - \frac{3}{y} = 9, 3x + \frac{7}{y} = 2$

Sol:

The given equations are:

$2x - \frac{3}{y} = 9$ ……..(i)

$3x + \frac{7}{y} = 2$ ……..(ii)

Putting $\frac{1}{y} = v$, we get:

$2x - 3v = 6$ …….(iii)

$3x + 7v = 2$ ……(iv)

On multiplying (iii) by 7 and (iv) by 3, we get:

$14x - 21v = 63$ ……..(v)

$9x + 21v = 6$ ……..(vi)

On adding (v) from (vi), we get:

$23x = 69 \Rightarrow x = 3$

On substituting x = 3 in (i), we get:

$2 \times 3 - \frac{3}{y} = 9$

$\Rightarrow 6 - \frac{3}{y} = 9 \Rightarrow \frac{3}{y} = -3 \Rightarrow y = -1$

Hence, the required solution is x = 3 and y = -1.

21. Solve for x and y:

$\frac{3}{x} - \frac{1}{y} + 9 = 0, \frac{2}{x} + \frac{3}{y} = 5$

Sol:

The given equations are:

$\frac{3}{x} - \frac{1}{y} + 9 = 0,$

$\Rightarrow \frac{3}{x} - \frac{1}{y} = -9$ ……..(i)

$\Rightarrow \frac{2}{x} - \frac{3}{y} = 5$……..(ii)

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get:

$3u – v = -9$ …….(iii)

$2u + 3v = 5$ ……(iv)

On multiplying (iii) by 3, we get:

$9u – 3v = -27$ ……..(v)

On adding (iv) and (v), we get:

$11u = -22 \Rightarrow u = -2$

$\Rightarrow \frac{1}{x} = -2 \Rightarrow x = \frac{-1}{2}$

On substituting $x = \frac{-1}{2}$ in (i), we get:

$\frac{3}{-1/2} - \frac{1}{y} = -9$

$\Rightarrow -6 - \frac{1}{y} = -9 \Rightarrow \frac{1}{y} = (-6 + 9) = 3$

$\Rightarrow y = \frac{1}{3}$

Hence, the required solution is $x = \frac{-1}{2}$ and $y = \frac{1}{3}$.

22. Solve for x and y:

$\frac{9}{x} - \frac{4}{y} = 8, \frac{13}{x} + \frac{7}{y} = 101$

Sol:

The given equations are:

$\frac{9}{x} - \frac{4}{y} = 8$ ……..(i)

$\frac{13}{x} + \frac{7}{y} = 101$ ……..(ii)

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get:

$9u - 4v = 8$ …….(iii)

$13u + 7v = 101$ ……(iv)

On multiplying (iii) by 7 and (iv) by 4, we get:

$63u - 28v = 56$ ……..(v)

$52u + 28v = 404$ ……..(vi)

On adding (v) from (vi), we get:

$115u = 460 \Rightarrow u = 4$

$\Rightarrow \frac{1}{x} = 4 \Rightarrow x = \frac{1}{4}$

On substituting $x = \frac{1}{4}$ in (i), we get:

$\frac{9}{1/4} - \frac{4}{y} = 8$

$\Rightarrow 36 - \frac{4}{y} = 8 \Rightarrow \frac{4}{y} = (36 – 8) = 28$

$y = \frac{4}{28} = \frac{1}{7}$

Hence, the required solution is $x = \frac{1}{4}$ and $y = \frac{1}{7}$.

23. Solve for x and y:

$\frac{5}{x} - \frac{3}{y} = 1, \frac{3}{2x} + \frac{2}{3y} = 5$

Sol:

The given equations are:

$\frac{5}{x} - \frac{3}{y} = 1$ ……..(i)

$\frac{3}{2x} + \frac{2}{3y} = 5$ ……..(ii)

Putting $\frac{1}{x}$ = u and $\frac{1}{y}$ = v, we get:

5u - 3v = 1 …….(iii)

$\Rightarrow \frac{3}{2}$ u + $\frac{2}{3}$ v = 5

$\Rightarrow \frac{9u+4v}{6} = 5$

$\Rightarrow$9u + 4v = 30 …….(iv)

On multiplying (iii) by 4 and (iv) by 3, we get:

20u - 12v = 4 ……..(v)

27u + 12v = 90 ……..(vi)

On adding (iv) and (v), we get:

47u = 94 $\Rightarrow$ u = 2

$\Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$

On substituting x = $\frac{1}{2}$ in (i), we get:

$\frac{5}{1/2} - \frac{3}{y} = 1$

$\Rightarrow 10 - \frac{3}{y} = 1 \Rightarrow \frac{3}{y} = (10 – 1) = 9$

$y = \frac{3}{9} = \frac{1}{3}$

Hence, the required solution is x = $\frac{1}{2}$ and y = $\frac{1}{3}$.

24. Solve for x and y:

$\frac{3}{x} + \frac{2}{y} = 12, \frac{2}{x} + \frac{3}{y} = 13$

Sol:

The given equations are:

$\frac{3}{x} + \frac{2}{y} = 12$ ……..(i)

$\frac{2}{x} + \frac{3}{y} = 13$ ……..(ii)

Multiplying (i) by 3 and (ii) by 2 and subtracting (ii) from (i), we get:

$\frac{9}{x} - \frac{4}{x} = 36 – 26$

$\Rightarrow \frac{5}{x} = 10$

$\Rightarrow x = \frac{5}{10} = \frac{1}{2}$

Now, substituting x = $\frac{1}{2}$ in (i), we have

$6 + \frac{2}{y} = 12$

$\Rightarrow \frac{2}{y} = 6$

$\Rightarrow y = \frac{1}{3}$

Hence, x = $\frac{1}{2}$ and y = $\frac{1}{3}$.

25. Solve for x and y:
4x + 6y = 3xy, 8x + 9y = 5xy
Sol:
The given equations are:
4x + 6y = 3xy …..(i)
8x + 9y = 5xy ……(ii)
From equation (i), we have:
$\frac{4x + 6y}{xy} = 3$
$\Rightarrow \frac{4}{y} + \frac{6}{x} = 3$ ……(iii)
For equation (ii), we have:
$\frac{8x + 9y}{xy} = 5$
$\Rightarrow \frac{8}{y} + \frac{9}{x} = 5$ ……(iv)
On substituting $\frac{1}{y}$ = v and $\frac{1}{x}$ = u, we get:
4v + 6u = 3 ……(v)
8v + 9u = 5 …….(vi)
On multiplying (v) by 9 and (vi) by 6, we get:
36v + 54u = 27 ….(vii)
48v + 54u = 30 ….(viii)
On subtracting (vii) from (viii), we get:
$12v = 3 \Rightarrow v = \frac{3}{12} = \frac{1}{4}$
$\Rightarrow \frac{1}{y} = \frac{1}{4} \Rightarrow y = 4$
On substituting y =4 in (iii), we get:
$\frac{4}{4} + \frac{6}{x} = 3$
$\Rightarrow 1 + \frac{6}{x} = 3 \Rightarrow \frac{6}{x} = (3 - 1) = 2$
$\Rightarrow 2x = 6 \Rightarrow x = \frac{6}{2} = 3$
Hence, the required solution is x = 3 and y = 4.

26. Solve for x and y:
x + y = 5xy, 3x + 2y = 13xy
Sol:
The given equations are:
x + y = 5xy …..(i)
3x + 2y = 13xy ……(ii)

From equation (i), we have:

$\frac{x+y}{xy} = 5$

$\Rightarrow \frac{1}{y} + \frac{1}{x} = 5$ ……(iii)

For equation (ii), we have:

$\frac{3x+2y}{xy} = 13$

$\Rightarrow \frac{3}{y} + \frac{2}{x} = 13$ ……(iv)

On substituting $\frac{1}{y} = v$ and $\frac{1}{x} = u$, we get:

$v + u = 5$ ……(v)

$3v + 2u = 13$ …….(vi)

On multiplying (v) by 2, we get:

$2v + 2u = 10$ ….(vii)

On subtracting (vii) from (vi), we get:

$v = 3$

$\Rightarrow \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$

On substituting $y = \frac{1}{3}$ in (iii), we get:

$\frac{1}{1/3} + \frac{1}{x} = 5$

$\Rightarrow 3 + \frac{1}{x} = 5 \Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$

Hence, the required solution is $x = \frac{1}{2}$ and $y = \frac{1}{3}$ or x= 0 and y = 0.

27. Solve for x and y:

$\frac{5}{x+y} - \frac{2}{x-y} = -1$, $\frac{15}{x+y} - \frac{7}{x-y} = 10$

Sol:

The given equations are

$\frac{5}{x+y} - \frac{2}{x-y} = -1$ ……(i)

$\frac{15}{x+y} - \frac{7}{x-y} = 10$ ……(ii)

Substituting $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$ in (i) and (ii), we get

$5u - 2v = -1$ ……..(iii)

$15u + 7v = 10$ …….(iv)

Multiplying (iii) by 3 and subtracting it from (iv), we get

$7v + 6v = 10 + 3$

$\Rightarrow 13v = 13$

$\Rightarrow v = 1$

$\Rightarrow x - y = 1 \qquad \left(\because \frac{1}{x-y} = v\right) \qquad$ …..(v)

Now, substituting v = 1 in (iii), we get

$5u - 2 = -1$

$\Rightarrow 5u = 1$

$\Rightarrow u = \frac{1}{5}$

$x + y = 5$ …….(vi)

Adding (v) and (vi), we get

$2x = 6 \Rightarrow x = 3$

Substituting x = 3 in (vi), we have

$3 + y = 5 \Rightarrow y = 5 - 3 = 2$

Hence, x = 3 and y = 2.

28. Solve for x and y:

$\frac{3}{x+y} + \frac{2}{x-y} = 2, \frac{3}{x+y} + \frac{2}{x-y} = 2$

Sol:

The given equations are

$\frac{3}{x+y} + \frac{2}{x-y} = 2$ ……(i)

$\frac{9}{x+y} - \frac{4}{x-y} = 1$ ……(ii)

Substituting $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$, we get:

$3u + 2v = 2$ ……..(iii)

$9u - 4v = 1$ …….(iv)

On multiplying (iii) by 2, we get:

$6u + 4v = 4$ …..(v)

On adding (iv) and (v), we get:

$15u = 5$

$\Rightarrow u = \frac{5}{15} = \frac{1}{3}$

$\Rightarrow \frac{1}{x+y} = \frac{1}{3} \Rightarrow x + y = 3$ …….(vi)

On substituting $u = \frac{1}{3}$ in (iii), we get

$1 + 2v = 2$

$\Rightarrow 2v = 1$

$\Rightarrow v = \frac{1}{2}$

$\Rightarrow \frac{1}{x-y} = \frac{1}{2} \Rightarrow x - y = 2$ …….(vii)

On adding (vi) and (vii), we get

$2x = 5$

$\Rightarrow x = \frac{5}{2}$

On substituting $x = \frac{5}{2}$ in (vi), we have

$\frac{5}{2} + y = 3$

$\Rightarrow y = \left(3 - \frac{5}{2}\right) = \frac{1}{2}$

Hence, the required solution is $x = \frac{5}{2}$ and $y = \frac{1}{2}$.

29. Solve for x and y:

$\frac{5}{x+1} + \frac{2}{y-1} = \frac{1}{2}, \frac{10}{x+1} - \frac{2}{y-1} = \frac{5}{2}$, where $x \neq 1, y \neq 1$.

Sol:

The given equations are

$\frac{5}{x+1} + \frac{2}{y-1} = \frac{1}{2}$ ……(i)

$\frac{10}{x+1} - \frac{2}{y-1} = \frac{5}{2}$ ……(ii)

Substituting $\frac{1}{x+1} = u$ and $\frac{1}{y-1} = v$, we get:

$5u - 2v = \frac{1}{2}$ ……..(iii)

$10u + 2v = \frac{5}{2}$ …….(iv)

On adding (iii) and (iv), we get:

$15u = 3$

$\Rightarrow u = \frac{3}{15} = \frac{1}{5}$

$\Rightarrow \frac{1}{x+1} = \frac{1}{5} \Rightarrow x + 1 = 5 \Rightarrow x = 4$

On substituting $u = \frac{1}{5}$ in (iii), we get

$5 \times \frac{1}{5} - 2v = \frac{1}{2} \Rightarrow 1 - 2v = \frac{1}{2}$

$\Rightarrow 2v = \frac{1}{2} \Rightarrow v = \frac{1}{4}$

$\Rightarrow \frac{1}{y-1} = \frac{1}{4} \Rightarrow y - 1 = 4 \Rightarrow y = 5$

Hence, the required solution is x = 4 and y = 5.

30. Solve for x and y:

$\frac{44}{x+y} + \frac{30}{x-y} = 10, \frac{55}{x+y} - \frac{40}{x-y} = 13.$

Sol:

The given equations are

$\frac{44}{x+y} + \frac{30}{x-y} = 10$ ……(i)

$\frac{55}{x+y} - \frac{40}{x-y} = 13$ ……(ii)

Putting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v, we get:
44u + 30v = 10 ……..(iii)
55u + 40v = 13 …….(iv)
On multiplying (iii) by 4 and (iv) by 3, we get:
176u + 120v = 40 …..(v)
165u + 120v = 39 …..(vi)
On subtracting (vi) and (v), we get:
11u = 1
$\Rightarrow u = \frac{1}{11}$
$\Rightarrow \frac{1}{x+y} = \frac{1}{11} \Rightarrow x + y = 11$ …….(vii)
On substituting u = $\frac{1}{11}$ in (iii), we get:
4 + 30v = 10
$\Rightarrow 30v = 6$
$\Rightarrow v = \frac{6}{30} = \frac{1}{5}$
$\Rightarrow \frac{1}{x-y} = \frac{1}{5} \Rightarrow x - y = 5$ …….(viii)
On adding (vii) and (viii), we get
2x = 16
$\Rightarrow x = 8$
On substituting x = 8 in (vii), we get:
8 + y = 11
$\Rightarrow y = 11 - 8 = 3$
Hence, the required solution is x = 8 and y = 3.

31. Solve for x and y:
$\frac{10}{x+y} + \frac{2}{x-y} = 4$, $\frac{15}{x+y} - \frac{9}{x-y} = -2$, where $x \neq y$, $x \neq -y$.
Sol:
The given equations are
$\frac{10}{x+y} + \frac{2}{x-y} = 4$ ……(i)
$\frac{15}{x+y} - \frac{9}{x-y} = -2$ ……(ii)
Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v in (i) and (ii), we get:
10u + 2v = 4 ……..(iii)
15u - 9v = -2 …….(iv)
Multiplying (iii) by 9 and (iv) by 2 and adding, we get:
90u + 30u = 36 – 4

$\Rightarrow 120u = 32$

$\Rightarrow u = \frac{32}{120} = \frac{4}{15}$

$\Rightarrow x + y = \frac{15}{4}$ $\left(\because \frac{1}{x+y} = u\right)$ …..(v)

On substituting $u = \frac{4}{15}$ in (iii), we get:

$10 \times \frac{4}{15} + 2v = 4$

$\frac{8}{3} + 2v = 4$

$\Rightarrow 2v = 4 - \frac{8}{3} = \frac{4}{3}$

$\Rightarrow v = \frac{2}{3}$

$\Rightarrow x - y = \frac{3}{2}$ $\left(\because \frac{1}{x-y} = v\right)$ …..(vi)

Adding (v) and (vi), we get

$2x = \frac{15}{4} + \frac{3}{2} \Rightarrow 2x = \frac{21}{4} \Rightarrow x = \frac{21}{8}$

Substituting $x = \frac{21}{8}$ in (v), we have

$\frac{21}{8} + y = \frac{15}{4} \Rightarrow y = \frac{15}{4} - \frac{21}{8} = \frac{9}{8}$

Hence, $x = \frac{21}{8}$ and $y = \frac{9}{8}$.

32. Solve for x and y:

$71x + 37y = 253$, $37x + 71y = 287$

Sol:

The given equations are:

$71x + 37y = 253$ …..(i)

$37x + 71y = 287$ ……(ii)

On adding (i) and (ii), we get:

$108x + 108y = 540$

$\Rightarrow 108(x + y) = 540$

$\Rightarrow (x + y) = 5$ ……(iii)

On subtracting (ii) from (i), we get:

$34x - 34y = -34$

$\Rightarrow 34(x - y) = -34$

$\Rightarrow (x - y) = -1$ ……(iv)

On adding (iii) from (i), we get:

$2x = 5 - 1 = 4$

$\Rightarrow x = 2$

On subtracting (iv) from (iii), we get:

$2y = 5 + 1 = 6$

$\Rightarrow y = 3$

Hence, the required solution is x = 2 and y = 3.

33. Solve for x and y:

$217x + 131y = 913$, $131x + 217y = 827$

Sol:

The given equations are:

$217x + 131y = 913$ …..(i)

$131x + 217y = 827$ ……(ii)

On adding (i) and (ii), we get:

$348x + 348y = 1740$

$\Rightarrow 348(x + y) = 1740$

$\Rightarrow x + y = 5$ ……(iii)

On subtracting (ii) from (i), we get:

$86x - 86y = 86$

$\Rightarrow 86(x - y) = 86$

$\Rightarrow x - y = 1$ ……(iv)

On adding (iii) from (i), we get:

$2x = 6$

$\Rightarrow x = 3$

On substituting x = 3 in (iii), we get:

$3 + y = 5$

$\Rightarrow y = 5 - 3 = 2$

Hence, the required solution is x = 3 and y = 2.

34. Solve for x and y:

$23x - 29y = 98$, $29x - 23y = 110$

Sol:

The given equations are:

$23x - 29y = 98$ …..(i)

$29x - 23y = 110$ ……(ii)

Adding (i) and (ii), we get:

$52x - 52y = 208$

$\Rightarrow x - y = 4$ ……(iii)

Subtracting (i) from (ii), we get:

$6x + 6y = 12$

$\Rightarrow x + y = 2$ ……(iv)

Now, adding equation (iii) and (iv), we get:

$2x = 6$

$\Rightarrow x = 3$

On substituting x = 3 in (iv), we have:

$3 + y = 2$

$\Rightarrow y = 2 – 3 = -1$

Hence, x = 3 and y = -1.

35. Solve for x and y:

$\frac{5}{x} + \frac{2}{y} = 6, \frac{-5}{x} + \frac{4}{y} = -3$

Sol:

The given equations can be written as

$\frac{5}{x} + \frac{2}{y} = 6$ …….(i)

$\frac{-5}{x} + \frac{4}{y} = -3$ ……(ii)

Adding (i) and (ii), we get

$\frac{6}{y} = 3 \Rightarrow y = 2$

Substituting y = 2 in (i), we have

$\frac{5}{x} + \frac{2}{2} = 6 \Rightarrow x = 1$

Hence, x = 1 and y = 2.

36. Solve for x and y:

$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}, \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$

Sol:

The given equations are

$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$ ……(i)

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$

$\frac{1}{3x+y} - \frac{1}{3x-y} = -\frac{1}{4}$ (Multiplying by 2) ……(ii)

Substituting $\frac{1}{3x+y}$ = u and $\frac{1}{3x-y}$ = v in (i) and (ii), we get:

$u + v = \frac{3}{4}$ ……..(iii)

$u – v = -\frac{1}{4}$ …….(iv)

Adding (iii) and (iv), we get:

$2u = \frac{1}{2}$

$\Rightarrow$ u = $\frac{1}{4}$

$\Rightarrow$3x + y = 4 $\left(\because \frac{1}{3x+y} = u\right)$ .....(v)

Now, substituting u = $\frac{1}{4}$ in (iii), we get:

$\frac{1}{4}$ + v = $\frac{3}{4}$

v = $\frac{3}{4}$ - $\frac{1}{4}$

$\Rightarrow$v = $\frac{1}{2}$

$\Rightarrow$ 3x – y = 2 $\left(\because \frac{1}{3x-y} = v\right)$ .....(vi)

Adding (v) and (vi), we get

6x = 6 $\Rightarrow$x = 1

Substituting x = 1 in (v), we have

3 + y = 4 $\Rightarrow$y = 1

Hence, x = 1 and y = 1.

37. Solve for x and y:

$\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = -\frac{3}{2}, \frac{1}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$ where x + 2y ≠ 0 and 3x – 2y ≠ 0.

Sol:

The given equations are

$\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = -\frac{3}{2}$ ......(i)

$\frac{1}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$ ......(ii)

Putting $\frac{1}{x+2y}$ = u and $\frac{1}{3x-2y}$ = v, we get:

$\frac{1}{2}$u + $\frac{5}{3}$v = - $\frac{3}{2}$ ....(iii)

$\frac{5}{4}$u – $\frac{3}{5}$v = $\frac{61}{60}$ ....(iv)

On multiplying (iii) by 6 and (iv) by 20, we get:

3u + 10v = -9 .....(v)

25u – 12v = $\frac{61}{3}$ .....(vi)

On multiplying (v) by 6 and (vi) by 5, we get

18u + 60v = -54 ....(vii)

125u – 60v = $\frac{305}{3}$ ....(viii)

On adding(vii) and (viii), we get:

143u = $\frac{305}{3}$ – 54 = $\frac{305-162}{3}$ = $\frac{143}{3}$

$\Rightarrow$u = $\frac{1}{3}$ = $\frac{1}{x+2y}$

$\Rightarrow x + 2y = 3$ …….(ix)

On substituting $u = \frac{1}{3}$ in (v), we get:

$1 + 10v = -9$

$\Rightarrow 10v = -10$

$\Rightarrow v = -1$

$\Rightarrow \frac{1}{3x-2y} = -1 \Rightarrow 3x - 2y = -1$ …….(x)

On adding (ix) and (x), we get:

$4x = 2$

$\Rightarrow x = \frac{1}{2}$

On substituting $x = \frac{1}{2}$ in (x), we get:

$\frac{3}{2} - 2y = -1$

$2y = \left(\frac{3}{2} + 1\right) = \frac{5}{2}$

$y = \frac{5}{4}$

Hence, the required solution is $x = \frac{1}{2}$ and $y = \frac{5}{4}$.

38. Solve for x and y:

$\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}, \frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$

Sol:

The given equations are

$\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}$ ……(i)

$\frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$ ……(ii)

Substituting $\frac{1}{3x+2y} = u$ and $\frac{1}{3x-2y} = v$, in (i) and (ii), we get:

$2u + 3v = \frac{17}{5}$ ……..(iii)

$5u + v = 2$ …….(iv)

Multiplying (iv) by 3 and subtracting from (iii), we get:

$2u - 15u = \frac{17}{5} - 6$

$\Rightarrow -13u = \frac{-13}{5} \Rightarrow u = \frac{1}{5}$

$\Rightarrow 3x + 2y = 5$ $\left(\because \frac{1}{3x+2y} = u\right)$ …..(v)

Now, substituting $u = \frac{1}{5}$ in (iv), we get

$1 + v = 2 \Rightarrow v = 1$

$\Rightarrow 3x - 2y = 1$ $\left(\because \frac{1}{3x-2y} = v\right)$ …….(vi)

Adding(v) and (vi), we get:

$\Rightarrow 6x = 6 \Rightarrow x = 1$

Substituting x = 1 in (v), we get:

$3 + 2y = 5 \Rightarrow y = 1$

Hence, x = 1 and y =1.

39. Solve for x and y:

$\frac{3}{x} + \frac{6}{y} = 7, \frac{9}{x} + \frac{3}{y} = 11$

Sol:

The given equations can be written as

$\frac{3}{x} + \frac{6}{y} = 7$ ....(i)

$\frac{9}{x} + \frac{3}{y} = 11$ ......(ii)

Multiplying (i) by 3 and subtracting (ii) from it, we get

$\frac{18}{y} - \frac{3}{y} = 21 - 11$

$\Rightarrow \frac{15}{y} = 10$

$\Rightarrow y = \frac{15}{10} = \frac{3}{2}$

Substituting $y = \frac{3}{2}$ in (i), we have

$\frac{3}{x} + \frac{6 \times 2}{3} = 7$

$\Rightarrow \frac{3}{x} = 7 - 4 = 3$

Hence, x = 1 and $y = \frac{3}{2}$.

40. Solve for x and y:

$x + y = a + b, ax - by = a^2 - b^2$

Sol:

The given equations are

$x + y = a + b$ ....(i)

$ax - by = a^2 - b^2$ ....(ii)

Multiplying (i) by b and adding it with (ii), we get

$bx + ax = ab + b^2 + a^2 - b^2$

$\Rightarrow x = \frac{ab + a^2}{a+b} = a$

Substituting x = a in (i), we have

$a + y = a + b$

$\Rightarrow y = b$

Hence, x = a and y = b.

41. Solve for x and y:

$\frac{x}{a} + \frac{y}{b} = 2$, $ax - by = (a^2 - b^2)$

Sol:

The given equations are:

$\frac{x}{a} + \frac{y}{b} = 2$

$\Rightarrow \frac{bx+ay}{ab} = 2$ [Taking LCM]

$\Rightarrow bx + ay = 2ab$ …….(i)

Again, $ax - by = (a^2 - b^2)$ …..(ii)

On multiplying (i) by b and (ii) by a, we get:

$b^2x + bay = 2ab^2$ ……..(iii)

$a^2x - bay = a(a^2 - b^2)$ …….(iv)

On adding (iii) from (iv), we get:

$(b^2 + a^2)x = 2a^2b + a(a^2 - b^2)$

$\Rightarrow (b^2 + a^2)x = 2ab^2 + a^3 - ab^2$

$\Rightarrow (b^2 + a^2)x = ab^2 + a^3$

$\Rightarrow (b^2 + a^2)x = a(b^2 + a^2)$

$\Rightarrow x = \frac{a(b^2+a^2)}{(b^2+a^2)} = a$

On substituting x = a in (i), we get:

$ba + ay = 2ab$

$\Rightarrow ay = ab$

$\Rightarrow y = b$

Hence, the solution is x = a and y = b.

42. Solve for x and y:

$px + qy = p - q$,

$qx - py = p + q$

Sol:

The given equations are

$px + qy = p - q$ ………(i)

$qx - py = p + q$ ………(ii)

Multiplying (i) by p and (ii) by q and adding them , we get

$p^2x + q^2x = p^2 - pq + pq + q^2$

$x = \frac{p^2+q^2}{p^2+q^2} = 1$

Substituting x = 1 in (i), we have

$p + qy = p - q$

$\Rightarrow qy = -p$

$\Rightarrow y = -1$
Hence, x = 1 and y = -1.

43. Solve for x and y:
$\frac{x}{a} - \frac{y}{b} = 0$, $ax + by = a^2 + b^2$
Sol:
The given equations can be written as
$\frac{x}{a} - \frac{y}{b} = 0$ ....(i)
$ax + by = a^2 + b^2$ ......(ii)
From (i),
$y = \frac{bx}{a}$
Substituting $y = \frac{bx}{a}$ in (ii), we get
$ax + \frac{b \times bx}{a} = a^2 + b^2$
$\Rightarrow x = \frac{(a^2 + b^2) \times a}{a^2 + b^2} = a$
Now, substitute x = a in (ii) to get
$a^2 + by = a^2 + b^2$
$\Rightarrow by = b^2$
$\Rightarrow y = b$
Hence, x = a and y = b.

44. Solve for x and y:
6(ax + by) = 3a + 2b,
6(bx – ay) = 3b – 2a
Sol:
The given equations are
6(ax + by) = 3a + 2b
$\Rightarrow 6ax + 6by = 3a + 2b$ ....(i)
and 6(bx – ay) = 3b – 2a
$\Rightarrow 6bx - 6ay = 3b - 2a$ ....(ii)
On multiplying (i) by a and (ii) by b, we get
$6a^2x + 6aby = 3a^2 + 2ab$ ....(iii)
$6b^2x - 6aby = 3b^2 - 2ab$ ....(iv)
On adding (iii) and (iv), we get
$6(a^2 + b^2)x = 3(a^2 + b^2)$
$x = \frac{3(a^2 + b^2)}{6(a^2 + b^2)} = \frac{1}{2}$

On substituting $x = \frac{1}{2}$ in (i), we get:

$6a \times \frac{1}{2} + 6by = 3a + 2b$

$6by = 2b$

$y = \frac{2b}{6b} = \frac{1}{3}$

Hence, the required solution is $x = \frac{1}{2}$ and $y = \frac{1}{3}$.

45. Solve for x and y:

$ax - by = a^2 + b^2$, $x + y = 2a$

Sol:

The given equations are

$ax - by = a^2 + b^2$ ………(i)

$x + y = 2a$ ………(ii)

From (ii)

$y = 2a - x$

Substituting $y = 2a - x$ in (i), we get

$ax - b(2a - x) = a^2 + b^2$

$\Rightarrow ax - 2ab + bx = a^2 + b^2$

$\Rightarrow x = \frac{a^2 + b^2 + 2ab}{a+b} = \frac{(a+b)^2}{a+b} = a + b$

Now, substitute $x = a + b$ in (ii) to get

$a + b + y = 2a$

$\Rightarrow y = a - b$

Hence, $x = a + b$ and $y = a - b$.

46. Solve for x and y:

$\frac{bx}{a} - \frac{ay}{b} + a + b = 0$, $bx - ay + 2ab = 0$

Sol:

The given equations are:

$\frac{bx}{a} - \frac{ay}{b} + a + b = 0$

By taking LCM, we get:

$b^2x - a^2y = -a^2b - b^2a$ …..(i)

and $bx - ay + 2ab = 0$

$bx - ay = -2ab$ …….(ii)

On multiplying (ii) by a, we get:

$abx - a^2y = -2a^2b$ ……(iii)

On subtracting (i) from (iii), we get:

$abx - b^2x = 2a^2b + a^2b + b^2a = -a^2b + b^2a$

$\Rightarrow x(ab - b^2) = -ab(a - b)$

$\Rightarrow x(a - b)b = -ab(a - b)$

$\therefore x = \frac{-ab(a-b)}{(a-b)b} = -a$

On substituting x = -a in (i), we get:

$b^2(-a) - a^2y = -a^2b - b^2a$

$\Rightarrow -b^2a - a^2y = -a^2b - b^2a$

$\Rightarrow -a^2y = -a^2b$

$\Rightarrow y = b$

Hence, the solution is x = -a and y = b.

47. Solve for x and y:

$\frac{bx}{a} + \frac{ay}{b} = a^2 + b^2$, $x + y = 2ab$

Sol:

The given equations are:

$\frac{bx}{a} + \frac{ay}{b} = a^2 + b^2$

By taking LCM, we get:

$\frac{b^2x + a^2y}{ab} = a^2 + b^2$

$\Rightarrow b^2x + a^2y = (ab)a^2 + b^2$

$\Rightarrow b^2x + a^2y = a^3b + ab^3$ …..(i)

Also, $x + y = 2ab$ …….(ii)

On multiplying (ii) by a^2, we get:

$a^2x + a^2y = 2a^3b$ ……(iii)

On subtracting (iii) from (i), we get:

$(b^2 - a^2)x = a^3b + ab^3 - 2a^3b$

$\Rightarrow (b^2 - a^2)x = -a^3b + ab^3$

$\Rightarrow (b^2 - a^2)x = ab(b^2 - a^2)$

$\Rightarrow (b^2 - a^2)x = ab(b^2 - a^2)$

$\therefore x = \frac{ab(b^2 - a^2)}{(b^2 - a^2)} = ab$

On substituting x = ab in (i), we get:

$b^2(ab) + a^2y = a^3b + ab^3$

$\Rightarrow a^2y = a^3b$

$\Rightarrow \frac{a^3b}{a^2} = ab$

Hence, the solution is x = ab and y = ab.

48. Solve for x and y:
$x + y = a + b$, $ax - by = a^2 - b^2$
Sol:
The given equations are
$x + y = a + b$ ....(i)
$ax - by = a^2 - b^2$ ....(ii)
From (i)
$y = a + b - x$
Substituting $y = a + b - x$ in (ii), we get
$ax - b(a + b - x) = a^2 - b^2$
$\Rightarrow ax - ab - b^2 + bx = a^2 - b^2$
$\Rightarrow x = \frac{a^2 + ab}{a+b} = a$
Now, substitute $x = a$ in (i) to get
$a + y = a + b$
$\Rightarrow y = b$
Hence, $x = a$ and $y = b$.

49. Solve for x and y:
$a^2x + b^2y = c^2$, $b^2x + a^2y = d^2$
Sol:
The given equations are
$a^2x + b^2y = c^2$ ....(i)
$b^2x + a^2y = d^2$ ....(ii)
Multiplying (i) by a^2 and (ii) by b^2 and subtracting, we get
$a^4x - b^4x = a^2c^2 - b^2d^2$
$\Rightarrow x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}$
Now, multiplying (i) by b^2 and (ii) by a^2 and subtracting, we get
$b^4y - a^4y = b^2c^2 - a^2d^2$
$\Rightarrow y = \frac{b^2c^2 - a^2d^2}{b^4 - a^4}$
Hence, $x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}$ and $y = \frac{b^2c^2 - a^2d^2}{b^4 - a^4}$.

50. Solve for x and y:
$\frac{x}{a} + \frac{y}{b} = a + b$, $\frac{x}{a^2} + \frac{y}{b^2} = 2$
Sol:
The given equations are
$\frac{x}{a} + \frac{y}{b} = a + b$ ....(i)
$\frac{x}{a^2} + \frac{y}{b^2} = 2$ ......(ii)

Multiplying (i) by b and (ii) by b^2 and subtracting, we get

$\frac{bx}{a} - \frac{b^2x}{a^2} = ab + b^2 - 2b^2$

$\Rightarrow \frac{ab - b^2}{a^2} x = ab - b^2$

$\Rightarrow x = \frac{(ab - b^2)a^2}{ab - b^2} = a^2$

Now, substituting $x = a^2$ in (i) we get

$\frac{a^2}{a} + \frac{y}{b} = a + b$

$\Rightarrow \frac{y}{b} = a + b – a = b$

$\Rightarrow y = b^2$

Hence, $x = a^2$ and $y = b^2$.

Exercise – 3C

1. Solve the system of equations by using the method of cross multiplication:

x + 2y + 1 = 0,

2x – 3y – 12 = 0.

Sol:

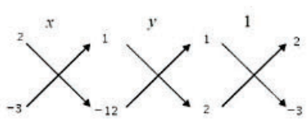
The given equations are:

x + 2y + 1 = 0 …….(i)

2x – 3y – 12 = 0 …….(ii)

Here $a_1 = 1$, $b_1 = 2$, $c_1 = 1$, $a_2 = 2$, $b_2 = -3$ and $c_2 = -12$

By cross multiplication, we have:



$\therefore \frac{x}{[2 \times (-12) - 1 \times (-3)]} = \frac{y}{[1 \times 2 - 1 \times (-12)]} = \frac{1}{[1 \times (-3) - 2 \times 2]}$

$\Rightarrow \frac{x}{(-24+3)} = \frac{y}{(2+12)} = \frac{1}{(-3-4)}$

$\Rightarrow \frac{x}{(-21)} = \frac{y}{(14)} = \frac{1}{(-7)}$

$\Rightarrow x = \frac{-21}{-7} = 3,\ y = \frac{14}{-7} = -2$

Hence, x = 3 and y = -2 is the required solution.

2. Solve the system of equations by using the method of cross multiplication:

3x - 2y + 3 = 0,

4x + 3y – 47 = 0

Sol:

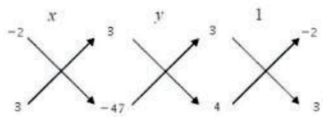
The given equations are:

$3x - 2y + 3 = 0$ ....(i)

$4x + 3y - 47 = 0$ ....(ii)

Here $a_1 = 3$, $b_1 = -2$, $c_1 = 3$, $a_2 = 4$, $b_2 = 3$ and $c_2 = -47$

By cross multiplication, we have:



$$\therefore \frac{x}{[(-2)\times(-47)-3\times3]} = \frac{y}{[3\times4-(-47)\times3]} = \frac{1}{[3\times3-(-2)\times4]}$$

$$\Rightarrow \frac{x}{(94-9)} = \frac{y}{(12+141)} = \frac{1}{(9+8)}$$

$$\Rightarrow \frac{x}{(85)} = \frac{y}{(153)} = \frac{1}{(17)}$$

$$\Rightarrow x = \frac{85}{17} = 5,\ y = \frac{153}{17} = 9$$

Hence, $x = 5$ and $y = 9$ is the required solution.

3. Solve the system of equations by using the method of cross multiplication:

$6x - 5y - 16 = 0$,

$7x - 13y + 10 = 0$

Sol:

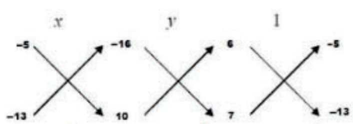
The given equations are:

$6x - 5y - 16 = 0$ ....(i)

$7x - 13y + 10 = 0$ ....(ii)

Here $a_1 = 6$, $b_1 = -5$, $c_1 = -16$, $a_2 = 7$, $b_2 = -13$ and $c_2 = 10$

By cross multiplication, we have:



$$\therefore \frac{x}{[(-5)\times10-(-16)\times(-13)]} = \frac{y}{[(-16)\times7-10\times6]} = \frac{1}{[6\times(-13)-(-5)\times7]}$$

$$\Rightarrow \frac{x}{(-50-208)} = \frac{y}{(-112-60)} = \frac{1}{(-78+35)}$$

$$\Rightarrow \frac{x}{(-258)} = \frac{y}{(-172)} = \frac{1}{(43)}$$

$$\Rightarrow x = \frac{-258}{-43} = 6,\ y = \frac{-172}{-43} = 4$$

Hence, $x = 6$ and $y = 4$ is the required solution.

4. Solve the system of equations by using the method of cross multiplication:

$3x + 2y + 25 = 0$, $2x + y + 10 = 0$

Sol:

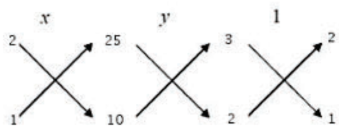
The given equations are:

$3x + 2y + 25 = 0$ ....(i)

$2x + y + 10 = 0$ ....(ii)

Here $a_1 = 3$, $b_1 = 2$, $c_1 = 25$, $a_2 = 2$, $b_2 = 1$ and $c_2 = 10$

By cross multiplication, we have:



$\therefore \frac{x}{[2\times10-25\times1]} = \frac{y}{[25\times2-10\times3]} = \frac{1}{[3\times1-2\times2]}$

$\Rightarrow \frac{x}{(20-25)} = \frac{y}{(50-30)} = \frac{1}{(3-4)}$

$\Rightarrow \frac{x}{(-5)} = \frac{y}{20} = \frac{1}{(-1)}$

$\Rightarrow x = \frac{-5}{-1} = 5$, $y = \frac{20}{(-1)} = -20$

Hence, x = 5 and y = -20 is the required solution.

5. Solve the system of equations by using the method of cross multiplication:

$2x + 5y - 1 = 0$, $2x + 3y - 3 = 0$

Sol:

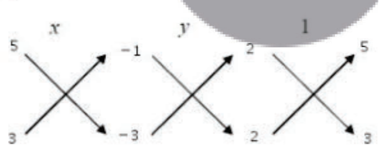
The given equations may be written as:

$2x + 5y - 1 = 0$ ....(i)

$2x + 3y - 3 = 0$ ....(ii)

Here $a_1 = 2$, $b_1 = 5$, $c_1 = -1$, $a_2 = 2$, $b_2 = 3$ and $c_2 = -3$

By cross multiplication, we have:



$\therefore \frac{x}{[5\times(-3)-3\times(-1)]} = \frac{y}{[(-1)\times2-(-3)\times2]} = \frac{1}{[2\times3-2\times5]}$

$\Rightarrow \frac{x}{(-15+3)} = \frac{y}{(-2+6)} = \frac{1}{(6-10)}$

$\Rightarrow \frac{x}{-12} = \frac{y}{4} = \frac{1}{-4}$

$\Rightarrow x = \frac{-12}{-4} = 3$, $y = \frac{4}{-4} = -1$

Hence, x = 3 and y = -1 is the required solution.

6. Solve the system of equations by using the method of cross multiplication:

$2x + y - 35 = 0$,

$3x + 4y - 65 = 0$

Sol:

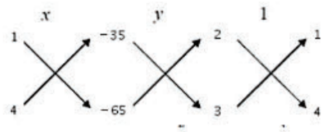
The given equations may be written as:

$2x + y - 35 = 0$ ....(i)

$3x + 4y - 65 = 0$ ....(ii)

Here $a_1 = 2$, $b_1 = 1$, $c_1 = -35$, $a_2 = 3$, $b_2 = 4$ and $c_2 = -65$

By cross multiplication, we have:



$$\therefore \frac{x}{[1\times(-65) - 4\times(-35)]} = \frac{y}{[(-35)\times 3 - (-65)\times 2]} = \frac{1}{[2\times 4 - 3\times 1]}$$

$$\Rightarrow \frac{x}{(-65+140)} = \frac{y}{(-105+130)} = \frac{1}{(8-3)}$$

$$\Rightarrow \frac{x}{75} = \frac{y}{25} = \frac{1}{5}$$

$$\Rightarrow x = \frac{75}{5} = 15,\ y = \frac{25}{5} = 5$$

Hence, x = 15 and y = 5 is the required solution.

7. Solve the system of equations by using the method of cross multiplication:

$7x - 2y - 3 = 0,$

$11x - \frac{3}{2}y - 8 = 0.$

Sol:

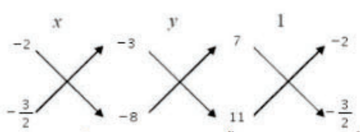
The given equations may be written as:

$7x - 2y - 3 = 0$ ....(i)

$11x - \frac{3}{2}y - 8 = 0$ ....(ii)

Here $a_1 = 7$, $b_1 = -2$, $c_1 = -3$, $a_2 = 11$, $b_2 = -\frac{3}{2}$ and $c_2 = -8$

By cross multiplication, we have:



$$\therefore \frac{x}{\left[(-2)\times(-8) - \left(-\frac{3}{2}\right)\times(-3)\right]} = \frac{y}{[(-3)\times 11 - (-8)\times 7]} = \frac{1}{\left[7\times\left(-\frac{3}{2}\right) - 11\times(-2)\right]}$$

$$\Rightarrow \frac{x}{(16-\frac{9}{2})} = \frac{y}{(-33+56)} = \frac{1}{(-\frac{21}{2}+22)}$$

$$\Rightarrow \frac{x}{\left(\frac{23}{2}\right)} = \frac{y}{23} = \frac{1}{\left(\frac{23}{2}\right)}$$

$$\Rightarrow x = \frac{\frac{23}{2}}{\frac{23}{2}} = 1,\ y = \frac{23}{\frac{23}{2}} = 2$$

Hence, x = 1 and y = 2 is the required solution.

8. Solve the system of equations by using the method of cross multiplication:

$\frac{x}{6} + \frac{y}{15} - 4 = 0,\ \frac{x}{3} - \frac{y}{12} - \frac{19}{4} = 0$

Sol:

The given equations may be written as:

$\frac{x}{6} + \frac{y}{15} - 4 = 0$ …….(i)

$\frac{x}{3}$ - $\frac{y}{12} - \frac{19}{4} = 0$ …….(ii)

Here $a_1 = \frac{1}{6}$, $b_1 = \frac{1}{15}$, $c_1 = -4$, $a_2 = \frac{1}{3}$, $b_2 = -\frac{1}{12}$ and $c_2 = -\frac{19}{4}$

By cross multiplication, we have:

$\therefore \frac{x}{\left[\frac{1}{15}\times\left(-\frac{19}{4}\right)-\left(-\frac{1}{12}\right)\times(-4)\right]} = \frac{y}{\left[(-4)\times\frac{1}{3}-\left(\frac{1}{6}\right)\times\left(-\frac{19}{4}\right)\right]} = \frac{1}{\left[\frac{1}{6}\times\left(-\frac{1}{12}\right)\times\frac{1}{3}\times\frac{1}{15}\right]}$

$\Rightarrow \frac{x}{\left(-\frac{19}{60}-\frac{1}{3}\right)} = \frac{y}{\left(-\frac{4}{3}+\frac{19}{34}\right)} = \frac{1}{\left(-\frac{1}{72}-\frac{1}{45}\right)}$

$\Rightarrow \frac{x}{\left(-\frac{39}{60}\right)} = \frac{y}{\left(-\frac{13}{24}\right)} = \frac{1}{\left(-\frac{13}{360}\right)}$

$\Rightarrow x = \left[\left(-\frac{39}{60}\right) \times \left(-\frac{360}{13}\right)\right] = 18$, $y = \left[\left(-\frac{13}{24}\right) \times \left(-\frac{360}{13}\right)\right] = 15$

Hence, x = 18 and y = 15 is the required solution.

9. Solve the system of equations by using the method of cross multiplication:

$\frac{1}{x} + \frac{1}{y} = 7$, $\frac{2}{x} + \frac{3}{y} = 17$

Sol:

Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given equations become:

u + v = 7

2u + 3v = 17

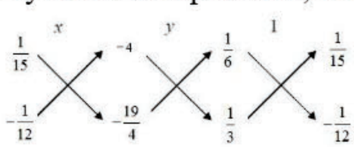
The given equations may be written as:

u + v – 7 = 0 …..(i)

2u + 3v – 17 = 0 ……(ii)

Here, $a_1 = 1$, $b_1 = 1$, $c_1 = -7$, $a_2 = 2$, $b_2 = 3$ and $c_2 = -17$

By cross multiplication, we have:



$\therefore \frac{u}{[1\times(-17) - 3\times(-7)]} = \frac{v}{[(-7)\times 2 - 1\times(-17)]} = \frac{1}{[3-2]}$

$\Rightarrow \frac{u}{(-17+21)} = \frac{v}{(-14+17)} = \frac{1}{(1)}$

$\Rightarrow \frac{u}{4} = \frac{v}{3} = \frac{1}{1}$

$\Rightarrow u = \frac{4}{1} = 4$, $v = \frac{3}{1} = 3$

$\Rightarrow \frac{1}{x} = 4$, $\frac{1}{y} = 3$

$\Rightarrow x = \frac{1}{4}$, $y = \frac{1}{3}$

Hence, $x = \frac{1}{4}$ and $y = \frac{1}{3}$ is the required solution.

10. Solve the system of equations by using the method of cross multiplication:

$\frac{5}{x+y} - \frac{2}{x-y} + 1 = 0, \frac{15}{x+y} + \frac{7}{x-y} - 10 = 0$

Sol:

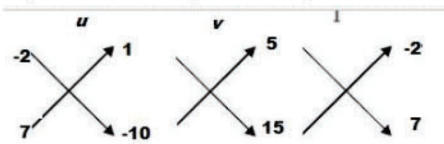
Taking $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$, the given equations become:

$5u - 2v + 1 = 0$ …..(i)

$15u + 7v - 10 = 0$ ……(ii)

Here, $a_1 = 5, b_1 = -2, c_1 = 1, a_2 = 15, b_2 = -7$ and $c_2 = -10$

By cross multiplication, we have:



$\therefore \frac{u}{[-2\times(-10) - 1\times 7]} = \frac{v}{[1\times 15 - (-10)\times 5]} = \frac{1}{[35+30]}$

$\Rightarrow \frac{u}{(20-7)} = \frac{v}{(15+50)} = \frac{1}{65}$

$\Rightarrow \frac{u}{13} = \frac{v}{65} = \frac{1}{65}$

$\Rightarrow u = \frac{13}{65} = \frac{1}{5}, v = \frac{65}{65} = 1$

$\Rightarrow \frac{1}{x+y} = \frac{1}{5}, \frac{1}{x-y} = 1$

So, $(x + y) = 5$ …….(iii)

and $(x - y) = 1$ ……(iv)

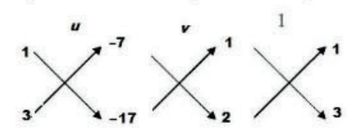
Again, the above equations (ii) and (iv) may be written as:

$x + y - 5 = 0$ …..(i)

$x - y - 1 = 0$ ……(ii)

Here, $a_1 = 1, b_1 = 1, c_1 = -5, a_2 = 1, b_2 = -1$ and $c_2 = -1$

By cross multiplication, we have:



$\therefore \frac{x}{[1\times(-1) - (-5)\times(-1)]} = \frac{y}{[(-5)\times 1 - (-1)\times 1]} = \frac{1}{[1\times(-1) - 1\times 1]}$

$\Rightarrow \frac{x}{(-1-5)} = \frac{y}{(-5+1)} = \frac{1}{(-1-1)}$

$\Rightarrow \frac{x}{-6} = \frac{v}{-4} = \frac{1}{-2}$

$\Rightarrow x = \frac{-6}{-2} = 3, y = \frac{-4}{-2} = 2$

Hence, $x = 3$ and $y = 2$ is the required solution.

11. Solve the system of equations by using the method of cross multiplication:

$\frac{ax}{b} - \frac{by}{a} - (a + b) = 0$, $ax - by - 2ab = 0$

Sol:

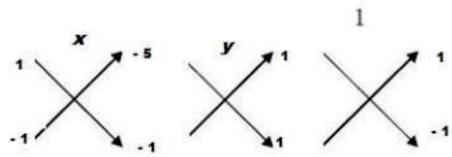
The given equations may be written as:

$\frac{ax}{b} - \frac{by}{a} - (a + b) = 0$ ……(i)

$ax - by - 2ab = 0$ ……(ii)

Here, $a_1 = \frac{a}{b}$, $b_1 = \frac{-b}{a}$, $c_1 = -(a + b)$, $a_2 = a$, $b_2 = -b$ and $c_2 = -2ab$

By cross multiplication, we have:



$$\therefore \frac{x}{\left[\left(-\frac{b}{a}\right)\times(-2ab) -(-b)\times(-(a+b))\right]} = \frac{y}{\left[-(a+b)\times a -(-2ab)\times\frac{a}{b}\right]} = \frac{1}{\left[\frac{a}{b}\times(-b) - a\times\left(-\frac{b}{a}\right)\right]}$$

$$\Rightarrow \frac{x}{(2b^2 - b(a+b))} = \frac{y}{-a(a+b)+2a^2} = \frac{1}{-a+b}$$

$$\Rightarrow \frac{x}{2b^2 - ab - b^2} = \frac{y}{-a^2 - ab + 2a^2} = \frac{1}{-a+b}$$

$$\Rightarrow \frac{x}{b^2 - ab} = \frac{y}{a^2 - ab} = \frac{1}{-(a-b)}$$

$$\Rightarrow \frac{x}{-b(a-b)} = \frac{y}{a(a-b)} = \frac{1}{-(a-b)}$$

$$\Rightarrow x = \frac{-b(a-b)}{-(a-b)} = b,\ y = \frac{a(a-b)}{-(a-b)} = -a$$

Hence, $x = b$ and $y = -a$ is the required solution.

12. Solve the system of equations by using the method of cross multiplication:

$2ax + 3by - (a + 2b) = 0$,

$3ax + 2by - (2a + b) = 0$

Sol:

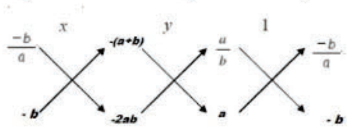
The given equations may be written as:

$2ax + 3by - (a + 2b) = 0$ ……(i)

$3ax + 2by - (2a + b) = 0$ ……(ii)

Here, $a_1 = 2a$, $b_1 = 3b$, $c_1 = -(a + 2b)$, $a_2 = 3a$, $b_2 = 2b$ and $c_2 = -(2a + b)$

By cross multiplication, we have:



$$\therefore \frac{x}{[3b\times(-(2a+b)) - 2b\times(-(a+2b))]} = \frac{y}{[-(a+2b)\times 3a - 2a\times(-(2a+b))]} = \frac{1}{[2a\times 2b - 3a\times 3b]}$$

$$\Rightarrow \frac{x}{(-6ab - 3b^2 + 2ab + 4b^2)} = \frac{y}{(-3a^2 - 6ab + 4a^2 + 2ab)} = \frac{1}{4ab - 9ab}$$

$\Rightarrow \frac{x}{b^2-4ab} = \frac{y}{a^2-4ab} = \frac{1}{-5ab}$

$\Rightarrow \frac{x}{-b(4a-b)} = \frac{y}{-a(4b-a)} = \frac{1}{-5ab}$

$\Rightarrow x = \frac{-b(4a-b)}{-5ab} = \frac{(4a-b)}{5a}, y = \frac{-a(4b-a)}{-5ab} = \frac{(4b-a)}{5b}$

Hence, $x = \frac{(4a-b)}{5a}$ and $y = \frac{(4b-a)}{5b}$ is the required solution.

13. Solve the system of equations by using the method of cross multiplication:

$\frac{a}{x} - \frac{b}{y} = 0, \frac{ab^2}{x} + \frac{a^2b}{y} = (a^2 + b^2)$, where $x \neq 0$ and $y \neq 0$.

Sol:

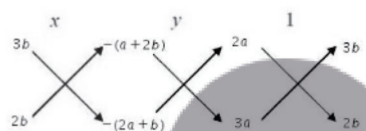
Substituting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in the given equations, we get

$au - bv + 0 = 0$ …….(i)

$ab^2u + a^2bv - (a^2 + b^2) = 0$ ……(ii)

Here, $a_1 = a$, $b_1 = -b$, $c_1 = 0$, $a_2 = ab^2$, $b_2 = a^2b$ and $c_2 = -(a^2 + b^2)$.

So, by cross-multiplication, we have



$\frac{u}{b_1c_2 - b_2c_1} = \frac{v}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{u}{(-b)[-(a^2+b^2)]-(a^2b)(0)} = \frac{v}{(0)(ab^2)-(-a^2-b^2)(a)} = \frac{1}{(a)(a^2b)-(ab^2)(-b)}$

$\Rightarrow \frac{u}{b(a^2+b^2)} = \frac{v}{a(a^2+b^2)} = \frac{1}{ab(a^2+b^2)}$

$\Rightarrow u = \frac{b(a^2+b^2)}{ab(a^2+b^2)}, v = \frac{a(a^2+b^2)}{ab(a^2+b^2)}$

$\Rightarrow u = \frac{1}{a}, v = \frac{1}{b}$

$\Rightarrow \frac{1}{x} = \frac{1}{a}, \frac{1}{y} = \frac{1}{b}$

$\Rightarrow x = a, y = b$

Hence, $x = a$ and $y = b$.

Exercise – 3D

1. Show that the following system of equations has a unique solution:

$3x + 5y = 12$,

$5x + 3y = 4$.

Also, find the solution of the given system of equations.

Sol:

The given system of equations is:

3x + 5y = 12
5x + 3y = 4
These equations are of the forms:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 3$, $b_1= 5$, $c_1 = -12$ and $a_2 = 5$, $b_2 = 3$, $c_2 = -4$
For a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., $\frac{3}{5} \neq \frac{5}{3}$
Hence, the given system of equations has a unique solution.
Again, the given equations are:
3x + 5y = 12 …..(i)
5x + 3y = 4 …..(ii)
On multiplying (i) by 3 and (ii) by 5, we get:
9x + 15y = 36 …….(iii)
25x + 15y = 20 ……(iv)
On subtracting (iii) from (iv), we get:
16x = -16
$\Rightarrow$x = -1
On substituting x = -1 in (i), we get:
3(-1) + 5y = 12
$\Rightarrow$5y = (12 + 3) = 15
$\Rightarrow$y = 3
Hence, x = -1 and y = 3 is the required solution.

2. Show that the following system of equations has a unique solution:
2x - 3y = 17,
4x + y = 13.
Also, find the solution of the given system of equations.
Sol:
The given system of equations is:
2x - 3y - 17 = 0 ….(i)
4x + y - 13 = 0 …..(ii)
The given equations are of the form
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= -3$, $c_1 = -17$ and $a_2 = 4$, $b_2 = 1$, $c_2 = -13$
Now,
$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{-3}{1} = -3$
Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, therefore the system of equations has unique solution.
Using cross multiplication method, we have

$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{x}{-3(-13)-1\times(-17)} = \frac{y}{-17\times4-(-13)\times2} = \frac{1}{2\times1-4\times(-3)}$

$\Rightarrow \frac{x}{39+17} = \frac{y}{-68+26} = \frac{1}{2+12}$

$\Rightarrow \frac{x}{56} = \frac{y}{-42} = \frac{1}{14}$

$\Rightarrow$ x = $\frac{56}{14}$, y = $\frac{-42}{14}$

$\Rightarrow$ x = 4, y = -3

Hence, x = 4 and y = -3.

3. Show that the following system of equations has a unique solution:

$\frac{x}{3} + \frac{y}{2} = 3$, x – 2y = 2.

Also, find the solution of the given system of equations.

Sol:

The given system of equations is:

$\frac{x}{3} + \frac{y}{2} = 3$

$\Rightarrow \frac{2x+3y}{6} = 3$

2x + 3y = 18

$\Rightarrow$2x + 3y – 18 = 0 ….(i)

and

x – 2y = 2

x – 2y – 2 = 0 …..(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1 = 3$, $c_1 = -18$ and $a_2 = 1$, $b_2 = -2$, $c_2 = -2$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., $\frac{2}{1} \neq \frac{3}{-2}$

Hence, the given system of equations has a unique solution.

Again, the given equations are:

2x + 3y – 18 = 0 …..(iii)

x – 2y – 2 = 0 …..(iv)

On multiplying (i) by 2 and (ii) by 3, we get:

4x + 6y – 36 = 0 …….(v)

3x - 6y – 6 = 0 ……(vi)

On adding (v) from (vi), we get:

7x = 42

$\Rightarrow$x = 6

On substituting x = 6 in (iii), we get:

$2(6) + 3y = 18$

$\Rightarrow 3y = (18 - 12) = 6$

$\Rightarrow y = 2$

Hence, $x = 6$ and $y = 2$ is the required solution.

4. Find the value of k for which the system of equations has a unique solution:
$2x + 3y = 5$,
$kx - 6y = 8$.

Sol:

The given system of equations are
$2x + 3y – 5 = 0$
$kx - 6y - 8 = 0$
This system is of the form:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -5$ and $a_2 = k$, $b_2 = -6$, $c_2 = -8$
Now, for the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{2}{k} \neq \frac{3}{-6}$
$\Rightarrow k \neq -4$
Hence, $k \neq -4$

5. Find the value of k for which the system of equations has a unique solution:
$x – ky = 2$,
$3x + 2y + 5=0$.

Sol:

The given system of equations are
$x - ky – 2 = 0$
$3x + 2y + 5 = 0$
This system of equations is of the form:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 1$, $b_1= -k$, $c_1 = -2$ and $a_2 = 3$, $b_2 = 2$, $c_2 = 5$
Now, for the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{1}{3} \neq \frac{-k}{2}$
$\Rightarrow k \neq -\frac{2}{3}$
Hence, $k \neq -\frac{2}{3}$.

6. Find the value of k for which the system of equations has a unique solution:
$5x - 7y = 5$,
$2x + ky = 1$.
Sol:
The given system of equations are
$5x - 7y - 5 = 0$ ….(i)
$2x + ky - 1 = 0$ …(ii)
This system is of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = 5$, $b_1= -7$, $c_1 = -5$ and $a_2 = 2$, $b_2 = k$, $c_2 = -1$
Now, for the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{5}{2} \neq \frac{-7}{k}$
$\Rightarrow k \neq -\frac{14}{5}$
Hence, $k \neq -\frac{14}{5}$.

7. Find the value of k for which the system of equations has a unique solution:
$4x + ky + 8=0$,
$x + y + 1 = 0$.
Sol:
The given system of equations are
$4x + ky + 8 = 0$
$x + y + 1 = 0$
This system is of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = 4$, $b_1= k$, $c_1 = 8$ and $a_2 = 1$, $b_2 = 1$, $c_2 = 1$
For the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{4}{1} \neq \frac{k}{1}$
$\Rightarrow k \neq 4$
Hence, $k \neq 4$.

8. Find the value of k for which the system of equations has a unique solution:
$4x - 5y = k$,
$2x - 3y = 12$.
Sol:

The given system of equations are

$4x - 5y = k$

$\Rightarrow 4x - 5y - k = 0$ ....(i)

And, $2x - 3y = 12$

$\Rightarrow 2x - 3y - 12 = 0$ ...(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0,\ a_2x+b_2y+c_2 = 0$

Here, $a_1 = 4$, $b_1 = -5$, $c_1 = -k$ and $a_2 = 2$, $b_2 = -3$, $c_2 = -12$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e., $\frac{4}{2} \neq \frac{-5}{-3}$

$\Rightarrow 2 \neq \frac{5}{3} \Rightarrow 6 \neq 5$

Thus, for all real values of k, the given system of equations will have a unique solution.

9. Find the value of k for which the system of equations has a unique solution:

$kx + 3y = (k - 3)$,

$12x + ky = k$

Sol:

The given system of equations:

$kx + 3y = (k - 3)$

$\Rightarrow kx + 3y - (k - 3) = 0$ ....(i)

And, $12x + ky = k$

$\Rightarrow 12x + ky - k = 0$ ...(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0,\ a_2x+b_2y+c_2 = 0$

Here, $a_1 = k$, $b_1 = 3$, $c_1 = -(k - 3)$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e., $\frac{k}{12} \neq \frac{3}{k}$

$\Rightarrow k^2 \neq 36 \Rightarrow k \neq \pm 6$

Thus, for all real values of k, other than ± 6, the given system of equations will have a unique solution.

10. Show that the system equations

$2x - 3y = 5$,

$6x - 9y = 15$

has an infinite number of solutions

Sol:

The given system of equations:

$2x - 3y = 5$

$\Rightarrow 2x - 3y - 5 = 0$ ....(i)

$6x - 9y = 15$

$\Rightarrow 6x - 9y - 15 = 0$ ...(ii)

These equations are of the following forms:

$a_1x+b_1y+c_1 = 0,\ a_2x+b_2y+c_2 = 0$

Here, $a_1 = 2,\ b_1 = -3,\ c_1 = -5$ and $a_2 = 6,\ b_2 = -9,\ c_2 = -15$

$\therefore \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given system of equations has an infinite number of solutions.

11. Show that the system of equations

$6x + 5y = 11,$

$9x + \frac{15}{2}y = 21$

has no solution.

Sol:

The given system of equations can be written as

$6x + 5y - 11 = 0$ ....(i)

$\Rightarrow 9x + \frac{15}{2}y - 21 = 0$ ...(ii)

This system is of the form

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

Here, $a_1 = 6,\ b_1 = 5,\ c_1 = -11$ and $a_2 = 9,\ b_2 = \frac{15}{2},\ c_2 = -21$

Now,

$\frac{a_1}{a_2} = \frac{6}{9} = \frac{2}{3}$

$\frac{b_1}{b_2} = \frac{5}{\frac{15}{2}} = \frac{2}{3}$

$\frac{c_1}{c_2} = \frac{-11}{-21} = \frac{11}{21}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, therefore the given system has no solution.

12. For what value of k, the system of equations

$kx + 2y = 5,$

$3x - 4y = 10$

has (i) a unique solution, (ii) no solution?

Sol:

The given system of equations:

kx + 2y = 5

$\Rightarrow$ kx + 2y - 5 = 0 ….(i)

3x - 4y = 10

$\Rightarrow$3x - 4y - 10 = 0 …(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 2$, $c_1 = -5$ and $a_2 = 3$, $b_2 = -4$, $c_2 = -10$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{k}{3} \neq \frac{2}{-4} \Rightarrow k \neq \frac{-3}{2}$

Thus for all real values of k other than $\frac{-3}{2}$, the given system of equations will have a unique solution.

(ii) For the given system of equations to have no solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{3} = \frac{2}{-4} \neq \frac{-5}{-10}$

$\Rightarrow \frac{k}{3} = \frac{2}{-4}$ and $\frac{k}{3} \neq \frac{1}{2}$

$\Rightarrow k = \frac{-3}{2}, k \neq \frac{3}{2}$

Hence, the required value of k is $\frac{-3}{2}$.

13. For what value of k, the system of equations

x + 2y = 5,

3x + ky + 15 = 0

has (i) a unique solution, (ii) no solution?

Sol:

The given system of equations:

x + 2y = 5

$\Rightarrow$ x + 2y - 5 = 0 ….(i)

3x + ky + 15 = 0 …(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 1$, $b_1= 2$, $c_1 = -5$ and $a_2 = 3$, $b_2 = k$, $c_2 = 15$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$

Thus for all real values of k other than 6, the given system of equations will have a unique solution.

(ii) For the given system of equations to have no solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow \frac{1}{3} = \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow k = 6, k \neq -6$

Hence, the required value of k is 6.

14. For what value of k, the system of equations

$x + 2y = 3,$

$5x + ky + 7 = 0$

Have (i) a unique solution, (ii) no solution?

Also, show that there is no value of k for which the given system of equation has infinitely namely solutions

Sol:

The given system of equations:

$x + 2y = 3$

$\Rightarrow x + 2y - 3 = 0$ ….(i)

And, $5x + ky + 7 = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0,\ a_2x+b_2y+c_2 = 0$

where, $a_1 = 1, b_1= 2, c_1 = -3$ and $a_2 = 5, b_2 = k, c_2 = 7$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{1}{5} \neq \frac{2}{k} \Rightarrow k \neq 10$

Thus for all real values of k other than 10, the given system of equations will have a unique solution.

(ii) In order that the given system of equations has no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{5} \neq \frac{2}{k} \neq \frac{-3}{7}$

$\Rightarrow \frac{1}{5} \neq \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-3}{7}$

$\Rightarrow k = 10, k \neq \frac{14}{-3}$

Hence, the required value of k is 10.

There is no value of k for which the given system of equations has an infinite number of solutions.

15. Find the value of k for which the system of linear equations has an infinite number of solutions:
$2x + 3y = 7$,
$(k – 1)x + (k + 2)y = 3k$.
Sol:
The given system of equations:
$2x + 3y = 7$,
$\Rightarrow 2x + 3y - 7 = 0$ ….(i)
And, $(k – 1)x + (k + 2)y = 3k$
$\Rightarrow (k – 1)x + (k + 2)y - 3k = 0$ …(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = (k – 1)$, $b_2 = (k + 2)$, $c_2 = -3k$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{-7}{-3k}$
$\Rightarrow \frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{7}{3k}$
Now, we have the following three cases:
Case I:
$\frac{2}{(k-1)} = \frac{3}{k+2}$
$\Rightarrow 2(k + 2) = 3(k – 1) \Rightarrow 2k + 4 = 3k – 3 \Rightarrow k = 7$
Case II:
$\frac{3}{(k+2)} = \frac{7}{3k}$
$\Rightarrow 7(k + 2) = 9k \Rightarrow 7k + 14 = 9k \Rightarrow 2k = 14 \Rightarrow k = 7$
Case III:
$\frac{2}{(k-1)} = \frac{7}{3k}$
$\Rightarrow 7k – 7 = 6k \Rightarrow k = 7$
Hence, the given system of equations has an infinite number of solutions when k is equal to 7.

16. Find the value of k for which the system of linear equations has an infinite number of solutions:
$2x + (k – 2)y = k$,
$6x + (2k - 1)y = (2k + 5)$.
Sol:
The given system of equations: