EXERCISE 16.3

1. Evaluate each of the following:
(i) $^8P_3$
(ii) $^{10}P_4$
(iii) $^6P_6$
(iv) P (6, 4)
Solution:
(i) $^8P_3$
We know that, $^8P_3$ can be written as P (8, 3)
By using the formula,
P (n, r) = n!/(n - r)!
P (8, 3) = 8!/(8 - 3)!
= 8!/5!
= (8 × 7 × 6 × 5!)/5!
= 8 × 7 × 6
= 336
∴ $^8P_3$ = 336

(ii) $^{10}P_4$
We know that, $^{10}P_4$ can be written as P (10, 4)
By using the formula,
P (n, r) = n!/(n - r)!
P (10, 4) = 10!/(10 - 4)!
= 10!/6!
= (10 × 9 × 8 × 7 × 6!)/6!
= 10 × 9 × 8 × 7
= 5040
∴ $^{10}P_4$ = 5040

(iii) $^6P_6$
We know that, $^6P_6$ can be written as P (6, 6)
By using the formula,
P (n, r) = n!/(n - r)!
P (6, 6) = 6!/(6 - 6)!
= 6!/0!
= (6 × 5 × 4 × 3 × 2 × 1)/1 [Since, 0! = 1]
= 6 × 5 × 4 × 3 × 2 × 1
= 720

∴ $^6P_6 = 720$

(iv) P (6, 4)
By using the formula,
P (n, r) = n!/(n - r)!
P (6, 4) = 6!/(6 - 4)!
= 6!/2!
= (6 × 5 × 4 × 3 × 2!)/2!
= 6 × 5 × 4 × 3
= 360
∴ P (6, 4) = 360

2. If P (5, r) = P (6, r – 1), find r.
Solution:
Given:
P (5, r) = P (6, r – 1)
By using the formula,
P (n, r) = n!/(n - r)!
P (5, r) = 5!/(5 - r)!
P (6, r-1) = 6!/(6 – (r-1))!
= 6!/(6 – r + 1)!
= 6!/(7 - r)!
So, from the question,
P (5, r) = P (6, r – 1)
Substituting the obtained values in above expression we get,
5!/(5 - r)! = 6!/(7 - r)!
Upon evaluating,
(7 - r)! / (5 – r)! = 6!/5!
[(7 - r) (7 – r - 1) (7 – r - 2)!] / (5 - r)! = (6 × 5!)/5!
[(7 - r) (6 - r) (5 - r)!] / (5 - r)! = 6
(7 - r) (6 - r) = 6
$42 – 6r – 7r + r^2 = 6$
$42 – 6 – 13r + r^2 = 0$
$r^2 – 13r + 36 = 0$
$r^2 – 9r – 4r + 36 = 0$
r(r – 9) – 4(r – 9) = 0
(r – 9) (r – 4) = 0
r = 9 or 4
For, P (n, r): r ≤ n

∴ r = 4 [for, P (5, r)]

3. If 5 P(4, n) = 6 P(5, n – 1), find n.
Solution:
Given:
5 P(4, n) = 6 P(5, n – 1)
By using the formula,
P (n, r) = n!/(n - r)!
P (4, n) = 4!/(4 - n)!
P (5, n-1) = 5!/(5 – (n-1))!
= 5!/(5 – n + 1)!
= 5!/(6 - n)!
So, from the question,
5 P(4, n) = 6 P(5, n – 1)
Substituting the obtained values in above expression we get,
5 × 4!/(4 - n)! = 6 × 5!/(6 - n)!
Upon evaluating,
(6 - n)! / (4 - n)! = 6/5 × 5!/4!
[(6 - n) (6 – n - 1) (6 – n - 2)!] / (4 - n)! = (6 × 5 × 4!) / (5 × 4!)
[(6 - n) (5 - n) (4 - n)!] / (4 - n)! = 6
(6 – n) (5 – n) = 6
$30 – 6n – 5n + n^2 = 6$
$30 – 6 – 11n + n^2 = 0$
$n^2 – 11n + 24 = 0$
$n^2 – 8n – 3n + 24 = 0$
n(n – 8) – 3(n – 8) = 0
(n – 8) (n – 3) = 0
n = 8 or 3
For, P (n, r): r ≤ n
∴ n = 3 [for, P (4, n)]

4. If P(n, 5) = 20 P(n, 3), find n.
Solution:
Given:
P(n, 5) = 20 P(n, 3)
By using the formula,
P (n, r) = n!/(n - r)!
P (n, 5) = n!/(n - 5)!
P (n, 3) = n!/(n – 3)!

So, from the question,
$P(n, 5) = 20\ P(n, 3)$
Substituting the obtained values in above expression we get,
$n!/(n - 5)! = 20 \times n!/(n - 3)!$
Upon evaluating,
$n!\ (n - 3)! / n!\ (n - 5)! = 20$
$[(n - 3)\ (n - 3 - 1)\ (n - 3 - 2)!] / (n - 5)! = 20$
$[(n - 3)\ (n - 4)\ (n - 5)!] / (n - 5)! = 20$
$(n - 3)\ (n - 4) = 20$
$n^2 - 3n - 4n + 12 = 20$
$n^2 - 7n + 12 - 20 = 0$
$n^2 - 7n - 8 = 0$
$n^2 - 8n + n - 8 = 0$
$n(n - 8) - 1(n - 8) = 0$
$(n - 8)\ (n - 1) = 0$
$n = 8$ or 1
For, $P(n, r)$: $n \geq r$
$\therefore n = 8$ [for, $P(n, 5)$]

5. If $^nP_4 = 360$, find the value of n.
Solution:
Given:
$^nP_4 = 360$
$^nP_4$ can be written as $P(n, 4)$
By using the formula,
$P(n, r) = n!/(n - r)!$
$P(n, 4) = n!/(n - 4)!$
So, from the question,
$^nP_4 = P(n, 4) = 360$
Substituting the obtained values in above expression we get,
$n!/(n - 4)! = 360$
$[n(n - 1)\ (n - 2)\ (n - 3)\ (n - 4)!] / (n - 4)! = 360$
$n\ (n - 1)\ (n - 2)\ (n - 3) = 360$
$n\ (n - 1)\ (n - 2)\ (n - 3) = 6\times5\times4\times3$
On comparing,
The value of n is 6.

6. If $P(9, r) = 3024$, find r.
Solution:

Given:
P (9, r) = 3024
By using the formula,
P (n, r) = n!/(n - r)!
P (9, r) = 9!/(9 - r)!
So, from the question,
P (9, r) = 3024
Substituting the obtained values in above expression we get,
9!/(9 - r)! = 3024
1/(9 - r)! = 3024/9!
= 3024/(9×8×7×6×5×4×3×2×1)
= 3024/(3024×5×4×3×2×1)
= 1/5!
(9 – r)! = 5!
9 – r = 5
-r = 5 – 9
-r = -4
∴ The value of r is 4.

7. If P (11, r) = P (12, r – 1), find r.
Solution:
Given:
P (11, r) = P (12, r – 1)
By using the formula,
P (n, r) = n!/(n - r)!
P (11, r) = 11!/(11 - r)!
P (12, r-1) = 12!/(12 – (r-1))!
= 12!/(12 – r + 1)!
= 12!/(13 - r)!
So, from the question,
P (11, r) = P (12, r – 1)
Substituting the obtained values in above expression we get,
11!/(11 - r)! = 12!/(13 - r)!
Upon evaluating,
(13 - r)! / (11 - r)! = 12!/11!
[(13 - r) (13 – r - 1) (13 – r - 2)!] / (11 - r)! = (12×11!)/11!
[(13 - r) (12 - r) (11 -r)!] / (11 - r)! = 12
(13 – r) (12 – r) = 12
156 – 12r – 13r + r^2 = 12

$156 - 12 - 25r + r^2 = 0$
$r^2 - 25r + 144 = 0$
$r^2 - 16r - 9r + 144 = 0$
$r(r - 16) - 9(r - 16) = 0$
$(r - 9)(r - 16) = 0$
r = 9 or 16
For, P (n, r): $r \leq n$
∴ r = 9 [for, P (11, r)]

8. If P(n, 4) = 12. P(n, 2), find n.
Solution:
Given:
P (n, 4) = 12. P (n, 2)
By using the formula,
P (n, r) = n!/(n - r)!
P (n, 4) = n!/(n - 4)!
P (n, 2) = n!/(n - 2)!
So, from the question,
P (n, 4) = 12. P (n, 2)
Substituting the obtained values in above expression we get,
n!/(n - 4)! = 12 × n!/(n - 2)!
Upon evaluating,
n! (n - 2)! / n! (n - 4)! = 12
[(n - 2) (n – 2 -1) (n – 2 - 2)!] / (n - 4)! = 12
[(n - 2) (n - 3) (n - 4)!] / (n - 4)! = 12
(n – 2) (n – 3) = 12
$n^2 - 3n - 2n + 6 = 12$
$n^2 - 5n + 6 - 12 = 0$
$n^2 - 5n - 6 = 0$
$n^2 - 6n + n - 6 = 0$
n (n – 6) – 1(n – 6) = 0
(n – 6) (n – 1) = 0
n = 6 or 1
For, P (n, r): $n \geq r$
∴ n = 6 [for, P (n, 4)]

9. If P(n – 1, 3) : P(n, 4) = 1 : 9, find n.
Solution:
Given:

P (n – 1, 3): P (n, 4) = 1 : 9
P (n – 1, 3)/ P (n, 4) = 1 / 9
By using the formula,
P (n, r) = n!/(n - r)!
P (n – 1, 3) = (n - 1)! / (n – 1 - 3)!
= (n - 1)! / (n - 4)!
P (n, 4) = n!/(n - 4)!
So, from the question,
P (n – 1, 3)/ P (n, 4) = 1 / 9
Substituting the obtained values in above expression we get,
[(n - 1)! / (n - 4)!] / [n!/(n - 4)!] = 1/9
[(n - 1)! / (n - 4)!] × [(n - 4)! / n!] = 1/9
(n - 1)!/n! = 1/9
(n - 1)!/n (n - 1)! = 1/9
1/n = 1/9
n = 9
∴ The value of n is 9.

10. If P(2n – 1, n) : P(2n + 1, n – 1) = 22 : 7 find n.
Solution:
Given:
P(2n – 1, n) : P(2n + 1, n – 1) = 22 : 7
P(2n – 1, n) / P(2n + 1, n – 1) = 22 / 7
By using the formula,
P (n, r) = n!/(n - r)!
P (2n – 1, n) = (2n - 1)! / (2n – 1 – n)!
= (2n - 1)! / (n - 1)!
P (2n + 1, n - 1) = (2n + 1)! / (2n + 1 – n + 1)!
= (2n + 1)! / (n + 2)!
So, from the question,
P(2n – 1, n) / P(2n + 1, n – 1) = 22 / 7
Substituting the obtained values in above expression we get,
[(2n - 1)! / (n - 1)!] / [(2n + 1)! / (n + 2)!] = 22/7
[(2n - 1)! / (n - 1)!] × [(n + 2)! / (2n + 1)!] = 22/7
[(2n - 1)! / (n - 1)!] × [(n + 2) (n + 2 - 1) (n + 2 - 2) (n + 2 - 3)!] / [(2n + 1) (2n + 1 - 1) (2n + 1 - 2)] = 22/7
[(2n - 1)! / (n - 1)!] × [(n + 2) (n + 1) n(n - 1)!] / [(2n + 1) 2n (2n - 1)!] = 22/7
[(n + 2) (n + 1)] / (2n + 1)2 = 22/7
7(n + 2) (n + 1) = 22×2 (2n + 1)

$7(n^2 + n + 2n + 2) = 88n + 44$
$7(n^2 + 3n + 2) = 88n + 44$
$7n^2 + 21n + 14 = 88n + 44$
$7n^2 + 21n – 88n + 14 – 44 = 0$
$7n^2 – 67n – 30 = 0$
$7n^2 – 70n + 3n – 30 = 0$
$7n(n – 10) + 3(n – 10) = 0$
$(n – 10) (7n + 3) = 0$
$n = 10, -3/7$
We know that, $n \neq -3/7$
∴ The value of n is 10.

11. If P(n, 5) : P(n, 3) = 2 : 1, find n.
Solution:
Given:
P(n, 5) : P(n, 3) = 2 : 1
P(n, 5) / P(n, 3) = 2 / 1
By using the formula,
P (n, r) = n!/(n - r)!
P (n, 5) = n!/ (n - 5)!
P (n, 3) = n!/ (n - 3)!
So, from the question,
P (n, 5) / P(n, 3) = 2 / 1
Substituting the obtained values in above expression we get,
[n!/ (n - 5)!] / [n!/ (n - 3)!] = 2/1
[n!/ (n - 5)!] × [(n - 3)! / n!] = 2/1
(n - 3)! / (n - 5)! = 2/1
[(n - 3) (n – 3 - 1) (n – 3 – 2)!] / (n - 5)! = 2/1
[(n - 3) (n - 4) (n - 5)!] / (n - 5)! = 2/1
(n – 3)(n – 4) = 2
$n^2 – 3n – 4n + 12 = 2$
$n^2 – 7n + 12 – 2 = 0$
$n^2 – 7n + 10 = 0$
$n^2 – 5n – 2n + 10 = 0$
$n (n – 5) – 2(n – 5) = 0$
$(n – 5) (n – 2) = 0$
n = 5 or 2
For, P (n, r): $n \geq r$
∴ n = 5 [for, P (n, 5)]

12. Prove that:
1. P (1, 1) + 2. P (2, 2) + 3 . P (3, 3) + … + n . P(n, n) = P(n + 1, n + 1) – 1.
Solution:
By using the formula,
P (n, r) = n!/(n - r)!
P (n, n) = n!/(n - n)!
= n!/0!
= n! [Since, 0! = 1]
Consider LHS:
= 1. P(1, 1) + 2. P(2, 2) + 3. P(3, 3) + … + n . P(n, n)
= 1.1! + 2.2! + 3.3! +………+ n.n! [Since, P(n, n) = n!]

$$= \sum_{r=1}^{n} r.r!$$

$$= \sum_{r=1}^{n} r.r! + r! - r!$$

$$= \sum_{r=1}^{n} (r+1)r! - r!$$

$$= \sum_{r=1}^{n} (r+1)! - r!$$

= (2! – 1!) + (3! – 2!) + (4! – 3!) + ……… + (n! – (n – 1)!) + ((n+1)! – n!)
= 2! – 1! + 3! – 2! + 4! – 3! + ……… + n! – (n – 1)! + (n+1)! – n!
= (n + 1)! – 1!
= (n + 1)! – 1 [Since, P (n, n) = n!]
= P(n+1, n+1) – 1
= RHS
Hence Proved.

13. If P(15, r – 1) : P(16, r – 2) = 3 : 4, find r.
Solution:
Given:
P(15, r – 1) : P(16, r – 2) = 3 : 4
P(15, r – 1) / P(16, r – 2) = 3/4
By using the formula,
P (n, r) = n!/(n - r)!
P (15, r - 1) = 15! / (15 – r + 1)!

= 15! / (16 - r)!

P (16, r - 2) = 16!/(16 – r + 2)!

= 16!/(18 - r)!

So, from the question,

P(15, r – 1) / P(16, r – 2) = 3/4

Substituting the obtained values in above expression we get,

[15! / (16 - r)!] / [16!/(18 - r)!] = 3/4

[15! / (16 - r)!] × [(18 - r)! / 16!] = 3/4

[15! / (16 - r)!] × [(18 - r) (18 – r - 1) (18 – r - 2)!]/(16×15!) = 3/4

1/(16 - r)! × [(18 - r) (17 - r) (16 - r)!]/16 = 3/4

(18 - r) (17 - r) = 3/4 × 16

(18 – r) (17 – r) = 12

$306 – 18r – 17r + r^2 = 12$

$306 – 12 – 35r + r^2 = 0$

$r^2 – 35r + 294 = 0$

$r^2 – 21r – 14r + 294 = 0$

r(r – 21) – 14(r – 21) = 0

(r – 14) (r – 21) = 0

r = 14 or 21

For, P(n, r): r ≤ n

∴ r = 14 [for, P(15, r – 1)]

14. $^{n+5}P_{n+1} = 11(n - 1)/2\ ^{n+3}P_n$, find n.

Solution:

Given:

$^{n+5}P_{n+1} = 11(n - 1)/2\ ^{n+3}P_n$

P (n +5, n + 1) = 11(n - 1)/2 P(n + 3, n)

By using the formula,

P (n, r) = n!/(n - r)!

$$P(n+5, n+1) = \frac{(n+5)!}{(n+5-n-1)!} = \frac{(n+5)!}{4!}$$

$$P(n+3, n) = \frac{(n+3)!}{(n+3-n)!} = \frac{(n+3)!}{3!}$$

So, from the question,

P (n +5, n + 1) = 11(n - 1)/2 P (n + 3, n)

Substituting the obtained values in above expression we get,

$$\frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\frac{(n+5)!}{(n-1)(n+3)!} = \frac{11}{2} \times \frac{4!}{3!}$$

$$\frac{(n+5)(n+5-1)(n+5-2)!}{(n-1)(n+3)!} = \frac{11}{2} \times \frac{4 \times 3!}{3!}$$

$$\frac{(n+5)(n+4)(n+3)!}{(n-1)(n+3)!} = \frac{44}{2}$$

$$\frac{(n+5)(n+4)}{(n-1)} = 22$$

(n + 5) (n + 4) = 22 (n – 1)
$n^2 + 4n + 5n + 20 = 22n - 22$
$n^2 + 9n + 20 - 22n + 22 = 0$
$n^2 - 13n + 42 = 0$
$n^2 - 6n - 7n + 42 = 0$
n(n – 6) – 7(n – 6) = 0
(n – 7) (n – 6) = 0
n = 7 or 6
∴ The value of n can either be 6 or 7.

15. In how many ways can five children stand in a queue?
Solution:
Number of arrangements of 'n' things taken all at a time = P (n, n)
So by using the formula,
By using the formula,
P (n, r) = n!/(n - r)!
The total number of ways in which five children can stand in a queue = the number of arrangements of 5 things taken all at a time = P (5, 5)
So,
P (5, 5) = 5!/(5 - 5)!
= 5!/0!
= 5! [Since, 0! = 1]
= 5 × 4 × 3 × 2 × 1
= 120
Hence, Number of ways in which five children can stand in a queue are 120.

16. From among the 36 teachers in a school, one principal and one vice-principal are to be appointed. In how many ways can this be done?
Solution:

Given:
The total number of teachers in a school = 36
We know, number of arrangements of n things taken r at a time = P(n, r)
By using the formula,
P (n, r) = n!/(n - r)!
∴ The total number of ways in which this can be done = the number of arrangements of 36 things taken 2 at a time = P(36, 2)
P (36, 2) = 36!/(36 - 2)!
= 36!/34!
= (36 × 35 × 34!)/34!
= 36 × 35
= 1260
Hence, Number of ways in which one principal and one vice-principal are to be appointed out of total 36 teachers in school are 1260.