Exercise 8(A)

Solution:

From remainder theorem, we know that when a polynomial f (x) is divided by (x – a), then the remainder is f(a).

(i) Given, $f(x) = x^4 - 3x^2 + 2x + 1$ is divided by x - 1

So, remainder = $f(1) = (1)^4 - 3(1)^2 + 2(1) + 1 = 1 - 3 + 2 + 1 = 1$

(ii) Given, $f(x) = x^3 + 3x^2 - 12x + 4$ is divided by x – 2

So, remainder = $f(2) = (2)^3 + 3(2)^2 - 12(2) + 4 = 8 + 12 - 24 + 4 = 0$

(iii) Given, $f(x) = x^4 + 1$ is divided by x + 1

So, remainder = $f(-1) = (-1)^4 + 1 = 2$

(i) Solution:

(x - a) is a factor of a polynomial f(x) if the remainder, when f(x) is divided by (x - a), is 0, i.e., if f(a) = 0.

(i) $f(x) = 5x^2 + 15x - 50$

$f(2) = 5(2)^2 + 15(2) - 50 = 20 + 30 - 50 = 0$

As the remainder is zero for x = 2

Thus, we can conclude that (x – 2) is a factor of $5x^2 + 15x - 50$

(ii) $f(x) = 3x^2 - x - 2$

$f(-2/3) = 3(-2/3)^2 - (-2/3) - 2 = 4/3 + 2/3 - 2 = 2 - 2 = 0$

As the remainder is zero for x = -2/3

Thus, we can conclude that (3x + 2) is a factor of $3x^2 - x - 2$

Solution:

From remainder theorem we know that when a polynomial f (x) is divided by x - a, then the remainder is f(a).

Here, $f(x) = 2x^3 + 3x^2 - 5x - 6$

(i) $f(-1) = 2(-1)^3 + 3(-1)^2 - 5(-1) - 6 = -2 + 3 + 5 - 6 = 0$

⇒ Remainder is zero for x = -1
Therefore, (x + 1) is a factor of the polynomial f(x).

(ii) $f(1/2) = 2(1/2)^3 + 3(1/2)^2 – 5(1/2) – 6$
$= ¼ + ¾ - 5/2 – 6$
$= -5/2 – 5 = -15/2$
⇒ Remainder is not equals to zero for x = 1/2
Therefore, (2x - 1) is not a factor of the polynomial f(x).

(iii) $f(-2) = 2(-2)^3 + 3(-2)^2 - 5(-2) - 6 = -16 + 12 + 10 - 6 = 0$
⇒ Remainder is zero for x = -2
Therefore, (x + 2) is a factor of the polynomial f(x).

Solution:

(ii) Given, 2x + 1 is a factor of $f(x) = 2x^2 + ax - 3$. So, f(-1/2) = 0
$2(-1/2)^2 + a(-1/2) - 3 = 0$
½ - a/2 – 3 = 0
1 – a – 6 = 0
a = -5

(iii) Given, 3x - 4 is a factor of $g(x) = 3x^2 + 2x - k$. So, f(4/3) = 0
$3(4/3)^2 + 2(4/3) – k = 0$
16/3 + 8/3 – k = 0
24/3 = k
k = 8

2. Find the values of constants a and b when x - 2 and x + 3 both are the factors of expression $x^3 + ax^2 + bx - 12$.
Solution:

Here, $f(x) = x^3 + ax^2 + bx – 12$
Given, x – 2 and x + 3 both are the factors of f(x)
So,
f(2) and f(-3) both should be equal to zero.
$f(2) = (2)^3 + a(2)^2 + b(2) – 12$
0 = 8 + 4a + 2b – 12
0 = 4a + 2b – 4
2a + b = 2 …. (1)

Now,
$f(-3) = (-3)^3 + a(-3)^2 + b(-3) – 12$

0 = -27 + 9a – 3b – 12
9a – 3b – 39 = 0
3a – b = 13 …. (2)

Adding (1) and (2), we get,
5a = 15
Thus, a = 3
Putting the value of a in (1), we have
6 + b = 2
Thus, b = -4

3. Find the value of k, if 2x + 1 is a factor of $(3k + 2)x^3 + (k - 1)$.
Solution:

Let take $f(x) = (3k + 2)x^3 + (k - 1)$
Now, 2x + 1 = 0
x = -1/2
As, 2x + 1 is a factor of f(x) then the remainder should be 0.
$f(-1/2) = (3k + 2)(-1/2)^3 + (k - 1) = 0$

$$\Rightarrow \frac{-(3k+2)}{8} + (k-1) = 0$$

$$\Rightarrow \frac{-3k-2+8k-8}{8} = 0$$

5k = 10 = 0
k = 2

4. Find the value of a, if x - 2 is a factor of $2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$.
Solution:

Given, $f(x) = 2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$ and x – 2 is a factor of f(x).
So, x - 2 = 0; x = 2
Hence, f(2) = 0
$2(2)^5 - 6(2)^4 - 2a(2)^3 + 6a(2)^2 + 4a(2) + 8 = 0$
64 - 96 - 16a + 24a + 8a + 8 = 0
-24 + 16a = 0
16a = 24
Thus, a = 1.5

5. Find the values of m and n so that x - 1 and x + 2 both are factors of $x^3 + (3m + 1)x^2 + nx - 18$.
Solution:

Let $f(x) = x^3 + (3m + 1)x^2 + nx – 18$
Given, (x – 1) and (x + 2) are the factors of f(x).
So,
x - 1 = 0; x = 1 and x + 2 = 0; x = -2
f(1) and f(-2) both should be equal to zero.

$(1)^3 + (3m + 1)(1)^2 + n(1) – 18 = 0$
$1 + 3m + 1 + n – 18 = 0$
$3m + n – 16 = 0$ ….. (1)
And,
$(-2)^3 + (3m + 1)(-2)^2 + n(-2) – 18 = 0$
$8 + 12m + 4 - 2n – 18 = 0$
$12m – 2n – 22 = 0$
$6m – n – 11 = 0$ ….. (2)

Adding (1) and (2), we get,
$9m - 27 = 0$
Thus, $m = 3$
Putting the value of m in (1), we have
$3(3) + n - 16 =0$
$9 + n - 16 = 0$
Therefore, $n = 7$