Chapter 23. Trigonometrical Ratios of Standard Angles [Including Evaluation of an Expression Involving Trigonometric Ratios]

Exercise 23(A)

Solution 1:

(i) $\sin 30^\circ \cos 30^\circ = \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

(ii) $\tan 30^0 \tan 60^0 = \frac{1}{\sqrt{3}}\left(\sqrt{3}\right) = 1$

(iii) $\cos^2 60^0 + \sin^2 30^0 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

(iv) $\operatorname{cosec}^2 60^\circ - \tan^2 30^\circ = \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3} - \frac{1}{3} = 1$

(v) $\sin^2 30^\circ + \cos^2 30^\circ + \cot^2 45^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 1^2 = \frac{1}{4} + \frac{3}{4} + 1 = 2$

(vi)

$$\cos^2 60^\circ + \sec^2 30^\circ + \tan^2 45^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 + 1^2$$
$$= \frac{1}{4} + \frac{4}{3} + 1$$
$$= \frac{3+16+12}{12}$$
$$= \frac{31}{12}$$
$$= 2\frac{7}{12}$$

Solution 2:

(i) $\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 + 1^2 + \left(\sqrt{3}\right)^2 = \frac{1}{3} + 1 + 3 = \frac{13}{3} = 4\frac{1}{3}$

(ii) $\frac{\tan 45^0}{\operatorname{cosec} 30^0} + \frac{\sec 60^0}{\cot 45^0} - \frac{5 \sin 90^0}{2 \cos 0^0} = \frac{1}{2} + \frac{2}{1} - \frac{5}{2} = \frac{1+4-5}{2} = 0$

(iii) $3 \sin^2 30^0 + 2 \tan^2 60^0 - 5 \cos^2 45^0$

$$= 3\left(\frac{1}{2}\right)^2 + 2\left(\sqrt{3}\right)^2 - 5\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{3}{4} + 6 - \frac{5}{2} = \frac{3+24-10}{4} = 4\frac{1}{4}$$

Solution 3:

(i) LHS=sin 60° cos 30° + cos 60°. sin 30°

$$= \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}+\frac{1}{2}\frac{1}{2}=\frac{3}{4}+\frac{1}{4}=1=\text{RHS}$$

(ii) LHS=cos 30°. cos 60° - sin 30°. sin 60°

$$= \frac{\sqrt{3}}{2}\frac{1}{2}-\frac{1}{2}\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}=0=\text{RHS}$$

(iii) LHS= cosec^2 45° - cot^2 45°

$$=\left(\sqrt{2}\right)^2-1^2=2-1=1=\text{RHS}$$

(iv) LHS= cos^2 30° - sin^2 30°

$$=\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{1}{2}\right)^2=\frac{3}{4}-\frac{1}{4}=\frac{1}{2}=\cos 60^0=\text{RHS}$$

(v) LHS= $\left(\frac{\tan 60^o+1}{\tan 60^o-1}\right)^2$

$$=\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2=\frac{4+2\sqrt{3}}{4-2\sqrt{3}}=\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}=\frac{1+\cos 30^0}{1-\cos 30^0}=\text{RHS}$$

(vi) LHS= $3\ \text{cosec}^2 60^o-2\cot^2 30^o+\sec^2 45^o$

$$=3\left(\frac{2}{\sqrt{3}}\right)^2-2\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2=4-6+2=0=\text{RHS}$$

Solution 4:

(i)

$RHS =$

$$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}=\frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}=\frac{\sqrt{3}}{2}$$

$$LHS=\sin(2\times 30^\circ)=\sin 60^\circ=\frac{\sqrt{3}}{2}$$

$\therefore LHS = RHS$

(ii)

RHS,

$$\frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{1}{2}$$

LHS,

$$\cos(2\times 30^\circ) = \cos 60^\circ = \frac{1}{2}$$

LHS = *RHS*

(iii)

RHS,

$$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

LHS,

$$\tan(2\times 30^\circ) = \tan 60^\circ = \sqrt{3}$$

LHS = *RHS*

Solution 5:

Given that AB = BC = x

$$\therefore AC = \sqrt{AB^2+BC^2} = \sqrt{x^2+x^2} = x\sqrt{2}$$

(i) $\sin 45^\circ = \frac{AB}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$

(ii) $\cos 45^\circ = \frac{BC}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$

(iii) $\tan 45^\circ = \frac{AB}{BC} = \frac{x}{x} = 1$

Solution 6:

(*i*) $LHS = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$RHS = 2\sin 60^\circ \cos 60^\circ = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

$LHS = RHS$

(*ii*) $LHS = 4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ)$

$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 + (1)^4\right]$

$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right] = \frac{4 \times 2}{16} + 3 \times \frac{1}{2} = 2$

$RHS = 2$

$LHS = RHS$

Solution 7:

(i)

The angle, x is acute and hence we have, 0<x<90 degrees

We know that

$\cos^2 x + \sin^2 x = 1$

$\Rightarrow 2\sin^2 x = 1 \quad [\text{since } \cos x = \sin x]$

$\Rightarrow \sin x = \frac{1}{\sqrt{2}}$

$\Rightarrow x = 45°$

(ii)

$\sec A = \operatorname{cosec} A$

$\cos A = \sin A$

$\cos^2 A = \sin^2 A$

$\cos^2 A = 1 - \cos^2 A$

$2\cos^2 A = 1$

$\cos A = \frac{1}{\sqrt{2}}$

$A = 45^0$

(iii)

$\tan\theta = \cot\theta$

$\tan\theta = \frac{1}{\tan\theta}$

$\tan^2\theta = 1$

$\tan\theta = 1$

$\tan\theta = \tan 45^0$

$\theta = 45^0$

(iv)

$\sin x = \cos y = \sin(90° - y)$

If x and y are acute angles,

$x = 90° - y$

$\Rightarrow x + y = 90°$

Hence x and y are complementary angles

Solution 8:

(i)

$$\sin x = \cos y = \sin\left(\frac{\pi}{2} - y\right)$$

if x and y are acute angles,

$$x = \frac{\pi}{2} - y$$

$$x + y = \frac{\pi}{2}$$

$\therefore$ x + y = 45° is false.

(ii)

$$\sec\theta \cdot \cot\theta = \frac{1}{\cos\theta}\frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

Secθ. Cot θ = cosec θ is true

(iii)

$$\sin^2\theta + \cos^2\theta = \sin^2\theta + 1 - \sin^2\theta = 1$$

Solution 9:

(i) For acute angles, remember what sine means: opposite over hypotenuse. If we increase the angle, then the opposite side gets larger. That means "opposite/hypotenuse" gets larger or increases.
(ii) For acute angles, remember what cosine means: base over hypotenuse. If we increase the angle, then the hypotenuse side gets larger. That means "base/hypotenuse" gets smaller or decreases.
(iii) For acute angles, remember what tangent means: opposite over base. If we decrease the angle, then the opposite side gets smaller. That means "opposite /base" gets decreases.

Solution 10:

(i) $\sin 60^0 = \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.87$

(ii) $\frac{2}{\tan 30^0} = \frac{2}{\frac{1}{\sqrt{3}}} = 2\sqrt{3} = 2 \times 1.732 = 3.46$

Solution 11:

(i) Given that A= 15^0

$$\frac{\cos 3A - 2\cos 4A}{\sin 3A + 2\sin 4A} = \frac{\cos\left(3\times 15^0\right) - 2\cos\left(4\times 15^0\right)}{\sin\left(3\times 15^0\right) + 2\sin\left(4\times 15^0\right)}$$

$$= \frac{\cos 45^0 - 2\cos 60^0}{\sin 45^0 + 2\sin 60^0}$$

$$= \frac{\frac{1}{\sqrt{2}} - 2\left(\frac{1}{2}\right)}{\frac{1}{\sqrt{2}} + 2\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{\frac{1}{\sqrt{2}} - 1}{\frac{1}{\sqrt{2}} + \sqrt{3}}$$

$$= \frac{1-\sqrt{2}}{1+\sqrt{6}}$$

$$= \frac{1}{5}\left(\sqrt{6} - 1 - 2\sqrt{3} + \sqrt{2}\right)$$

(ii) Given that B= 20^0

$$\frac{3\sin 3B + 2\cos\left(2B + 5^0\right)}{2\cos 3B - \sin\left(2B - 10^0\right)} = \frac{3\sin 3\times 20^0 + 2\cos\left(2\times 20^0 + 5^0\right)}{2\cos 3\times 20^0 - \sin\left(2\times 20^0 - 10^0\right)}$$

$$= \frac{3\sin 60^0 + 2\cos 45^0}{2\cos 60^0 - \sin 30^0}$$

$$= \frac{3\left(\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{\sqrt{2}}\right)}{2\left(\frac{1}{2}\right) - \frac{1}{2}}$$

$$= \frac{\frac{3\sqrt{3}}{2} + \sqrt{2}}{2}$$

$$= 3\sqrt{3} + 2\sqrt{2}$$