EXERCISE 5.3

1. Match the APs given in column A with suitable common differences given in column B.

Column A		Column B	
(A_1)	**2, – 2, – 6, –10,...**	**(B_1)**	**2/3**
(A_2)	**$a = –18$, $n = 10$, $a_n = 0$**	**(B_2)**	**– 5**
(A_3)	**$a = 0$, $a_{10} = 6$**	**(B_3)**	**4**
(A_4)	**$a_2 = 13$, $a_4 = 3$**	**(B_4)**	**– 4**
		(B_5)	**2**
		(B_6)	**1/2**
		(B_7)	**5**

Solution:

(A_1) AP is 2, - 2, - 6, - 10, ….
So common difference is simply
$a_2 - a_1 = - 2 - 2 = - 4 = (B_3)$

(A_2) Given
First term, a = - 18
No of terms, n = 10
Last term, $a_n = 0$
By using the nth term formula
$a_n = a + (n - 1)d$
$0 = - 18 + (10 - 1)d$
$18 = 9d$
$d = 2 = (B_5)$

(A_3) Given
First term, a = 0
Tenth term, $a_{10} = 6$
By using the nth term formula
$a_n = a + (n - 1)d$
$a_{10} = a + 9d$
$6 = 0 + 9d$
$d = 2/3 = (B_6)$

(A_4) Let the first term be a and common difference be d
Given that
$a_2 = 13$
$a_4 = 3$
$a_2 - a_4 = 10$
$a + d - (a + 3d) = 10$
$d - 3d = 10$
$- 2d = 10$
$d = - 5= (B_1)$

2. Verify that each of the following is an AP, and then write its next three terms.
(i) 0, 1/4, 1/2, 3/4,...
Solution:

Here,
$a_1 = 0$
$a_2 = \frac{1}{4}$
$a_3 = \frac{1}{2}$
$a_4 = \frac{3}{4}$
$a_2 - a_1 = \frac{1}{4} - 0 = \frac{1}{4}$
$a_3 - a_2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$
$a_4 - a_3 = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$
Since, difference of successive terms are equal,
Hence, 0, 1/4, 1/2, 3/4... is an AP with common difference ¼.
Therefore, the next three term will be,
$\frac{3}{4} + \frac{1}{4}, \frac{3}{4} + 2(\frac{1}{4}), \frac{3}{4} + 3(\frac{1}{4})$
1, 5/4 , 3/2

(ii) 5, 14/3, 13/3, 4...
Solution:

Here,
$a_1 = 5$
$a_2 = 14/3$
$a_3 = 13/3$
$a_4 = 4$
$a_2 - a_1 = 14/3 - 5 = -1/3$
$a_3 - a_2 = 13/3 - 14/3 = -1/3$
$a_4 - a_3 = 4 - 13/3 = -1/3$
Since, difference of successive terms are equal,
Hence, 5, 14/3, 13/3, 4... is an AP with common difference -1/3.
Therefore, the next three term will be,
4 + (-1/3), 4 + 2(-1/3), 4 + 3(-1/3)
11/3 , 10/3, 3

(iii) √3 , 2√3, 3√3,...
Solution:

Here,
$a_1 = \sqrt{3}$
$a_2 = 2\sqrt{3}$
$a_3 = 3\sqrt{3}$
$a_4 = 4\sqrt{3}$
$a_2 - a_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$
$a_3 - a_2 = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$
$a_4 - a_3 = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$
Since, difference of successive terms are equal,
Hence, √3 , 2√3, 3√3,... is an AP with common difference √3.
Therefore, the next three term will be,

$4\sqrt{3} + \sqrt{3}, 4\sqrt{3} + 2\sqrt{3}, 4\sqrt{3} + 3\sqrt{3}$
$5\sqrt{3}, 6\sqrt{3}, 7\sqrt{3}$

(iv) $a + b, (a + 1) + b, (a + 1) + (b + 1), \ldots$
Solution:

Here
$a_1 = a + b$
$a_2 = (a + 1) + b$
$a_3 = (a + 1) + (b + 1)$
$a_2 - a_1 = (a + 1) + b - (a + b) = 1$
$a_3 - a_2 = (a + 1) + (b + 1) - (a + 1) - b = 1$
Since, difference of successive terms are equal,
Hence, $a + b, (a + 1) + b, (a + 1) + (b + 1), \ldots$ is an AP with common difference 1.
Therefore, the next three term will be,
$(a + 1) + (b + 1) + 1, (a + 1) + (b + 1) + 1(2), (a + 1) + (b + 1) + 1(3)$
$(a + 2) + (b + 1), (a + 2) + (b + 2), (a + 3) + (b + 2)$

(v) $a, 2a + 1, 3a + 2, 4a + 3, \ldots$
Solution:

Here $a_1 = a$
$a_2 = 2a + 1$
$a_3 = 3a + 2$
$a_4 = 4a + 3$
$a_2 - a_1 = (2a + 1) – (a) = a + 1$
$a_3 - a_2 = (3a + 2) – (2a + 1) = a + 1$
$a_4 - a_3 = (4a + 3) – (3a+2) = a + 1$
Since, difference of successive terms are equal,
Hence, $a, 2a + 1, 3a + 2, 4a + 3, \ldots$ is an AP with common difference a+1.
Therefore, the next three term will be,
$4a + 3 +(a + 1), 4a + 3 + 2(a + 1), 4a + 3 + 3(a + 1)$
$5a + 4, 6a + 5, 7a + 6$

3. Write the first three terms of the APs when *a* and *d* are as given below:

(i) $a = 1/2, d = -1/6$
(ii) $a = –5, d = –3$
(iii) $a = 2, d = 1/\sqrt{2}$

Solution:

(i) $a = 1/2, d = -1/6$
We know that,
First three terms of AP are :
$a, a + d, a + 2d$
$\frac{1}{2}, \frac{1}{2} + (-1/6), \frac{1}{2} + 2(-1/6)$
$\frac{1}{2}, 1/3, 1/6$

(ii) $a = –5, d = –3$
We know that,

First three terms of AP are :
a, a + d, a + 2d
-5, - 5 + 1 (- 3), - 5 + 2 (- 3)
- 5, - 8, - 11

(iii) $a = \sqrt{2}$, $d = 1/\sqrt{2}$
We know that,
First three terms of AP are :
a, a + d, a + 2d
$\sqrt{2}$, $\sqrt{2}+1/\sqrt{2}$, $\sqrt{2}+2/\sqrt{2}$
$\sqrt{2}$, $3/\sqrt{2}$, $4/\sqrt{2}$

4. Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.
Solution:

For a, 7, b, 23, c… to be in AP
it has to satisfy the condition,
$a_5 - a_4 = a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = d$
Where d is thecommon difference
7 - a = b - 7 = 23 - b = c – 23 …(1)
Let us equate,
b - 7 = 23 – b
2b = 30
b = 15 (eqn 1)
And,
7 - a = b – 7
From eqn 1
7 - a = 15 – 7
a = - 1
And,
c - 23 = 23 – b
c - 23 = 23 – 15
c - 23 = 8
c = 31
So a = - 1
b = 15
c = 31
Then, we can say that, the sequence - 1, 7, 15, 23, 31 is an AP

5. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.
Solution:

We know that,
The first term of an AP = a
And, the common difference = d.
According to the question,
5^{th} term, $a_5 = 19$

Using the n^{th} term formula,
$a_n = a + (n - 1)d$
We get,
$a + 4d = 19$
$a = 19 - 4d$ …(1)
Also,
20^{th} term - 8^{th} term = 20
$a + 19d - (a + 7d) = 20$
$12d = 20$
$d = 4/3$
Substituting $d = 4/3$ in equation 1,
We get,
$a = 19 – 4(4/3)$
$a = 41/3$
Then, the AP becomes,
$41/3, 41/3 + 4/3 , 41/3 + 2(4/3)$
$41/3, 15, 49/3$