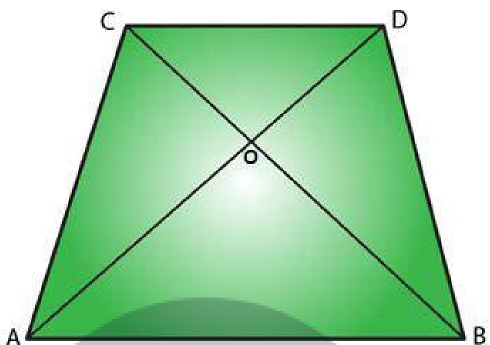
EXERCISE 17.1

Given below is a parallelogram ABCD. Complete each statement along with the definition or property used.
(i) AD =

(ii) ∠DCB =
(iii) OC =

(iv) ∠DAB + ∠CDA =



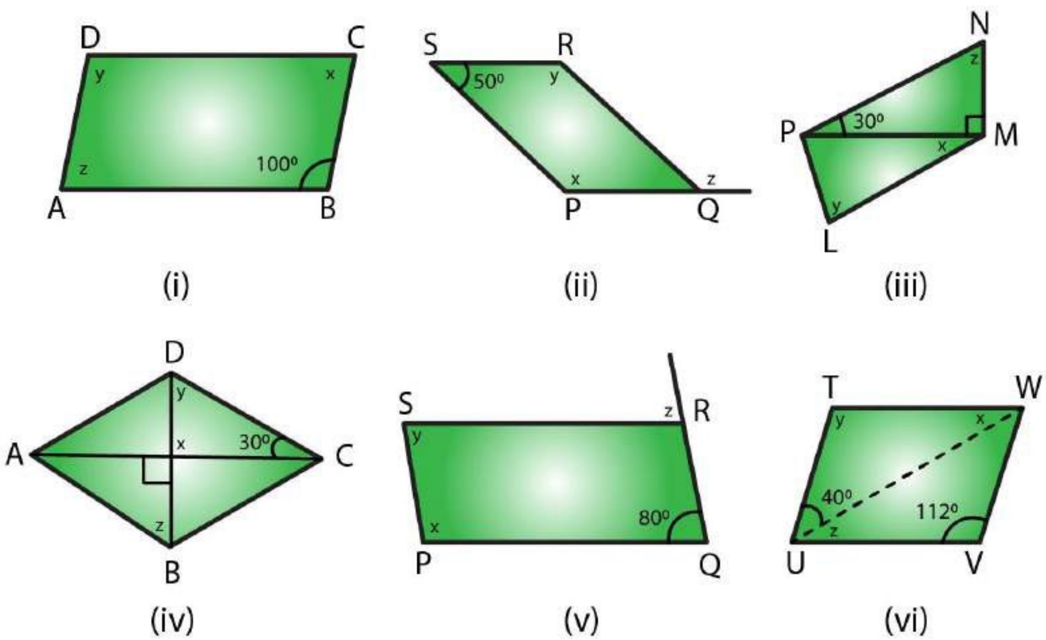
Solution:
(i) AD = BC. Because, diagonals bisect each other in a parallelogram.

(ii) ∠DCB = ∠BAD. Because, alternate interior angles are equal.

(iii) OC = OA. Because, diagonals bisect each other in a parallelogram.

(iv) ∠DAB+ ∠CDA = 180°. Because sum of adjacent angles in a parallelogram is 180°.

2. The following figures are parallelograms. Find the degree values of the unknowns x, y, z.



Solution:

(i) $\angle ABC = \angle y = 100^o$ (opposite angles are equal in a parallelogram)

$\angle x + \angle y = 180^o$ (sum of adjacent angles is $= 180^o$ in a parallelogram)

$\angle x + 100^o = 180^o$

$\angle x = 180^o - 100^o$

$= 80^o$

$\therefore \angle x = 80^o$ $\angle y = 100^o \angle z = 80^o$ (opposite angles are equal in a parallelogram)

(ii) $\angle RSP + \angle y = 180^o$ (sum of adjacent angles is $= 180^o$ in a parallelogram)

$\angle y + 50^o = 180^o$

$\angle y = 180^o - 50^o$

$= 130^o$

$\therefore \angle x = \angle y = 130^o$ (opposite angles are equal in a parallelogram)

$\angle RSP = \angle RQP = 50^o$ (opposite angles are equal in a parallelogram)

$\angle RQP + \angle z = 180^o$ (linear pair)

$50^o + \angle z = 180^o$

$\angle z = 180^o – 50^o$

$= 130^o$

$\therefore \angle x = 130^o$ $\angle y = 130^o$ $\angle z = 130^o$

(iii) In ΔPMN

$\angle NPM + \angle NMP + \angle MNP = 180°$ [Sum of all the angles of a triangle is 180°]

$30° + 90° + \angle z = 180°$
$\angle z = 180° - 120°$
$= 60°$
$\angle y = \angle z = 60°$ [opposite angles are equal in a parallelogram]
$\angle z = 180° - 120°$ [sum of the adjacent angles is equal to 180° in a parallelogram]
$\angle z = 60°$
$\angle z + \angle LMN = 180°$ [sum of the adjacent angles is equal to 180° in a parallelogram]
$60° + 90° + \angle x = 180°$
$\angle x = 180° - 150°$
$\angle x = 30°$
$\therefore \angle x = 30°\ \angle y = 60°\ \angle z = 60°$

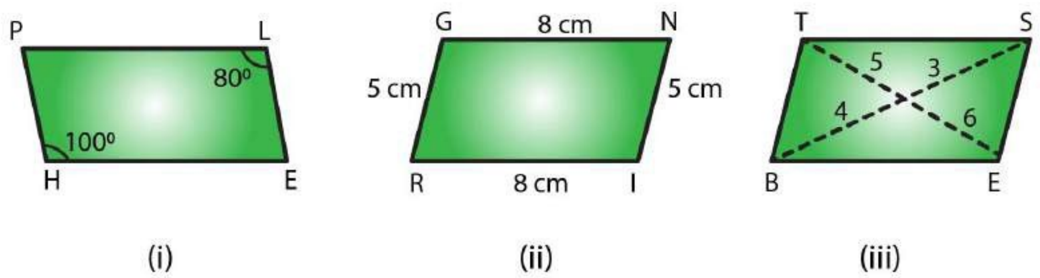
(iv) $\angle x = 90°$ [vertically opposite angles are equal]
In ΔDOC
$\angle x + \angle y + 30° = 180°$ [Sum of all the angles of a triangle is 180°]
$90° + 30° + \angle y = 180°$
$\angle y = 180° - 120°$
$\angle y = 60°$
$\angle y = \angle z = 60°$ [alternate interior angles are equal]
$\therefore \angle x = 90°\ \angle y = 60°\ \angle z = 60°$

(v) $\angle x + \angle POR = 180°$ [sum of the adjacent angles is equal to 180° in a parallelogram]
$\angle x + 80° = 180°$
$\angle x = 180° - 80°$
$\angle x = 100°$
$\angle y = 80°$ [opposite angles are equal in a parallelogram]
$\angle SRQ = \angle x = 100°$
$\angle SRQ + \angle z = 180°$ [Linear pair]
$100° + \angle z = 180°$
$\angle z = 180° - 100°$
$\angle z = 80°$
$\therefore \angle x = 100°\ \angle y = 80°\ \angle z = 80°$

(vi) $\angle y = 112°$ [In a parallelogram opposite angles are equal]
$\angle y + \angle VUT = 180°$ [In a parallelogram sum of the adjacent angles is equal to 180°]
$\angle z + 40° + 112° = 180°$
$\angle z = 180° - 152°$
$\angle z = 28°$
$\angle z = \angle x = 28°$ [alternate interior angles are equal]

∴ $\angle x = 28^\circ$ $\angle y = 112^\circ$ $\angle z = 28^\circ$

3. Can the following figures be parallelograms? Justify your answer.

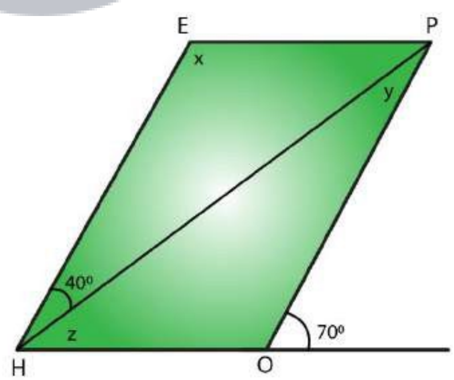


Solution:
(i) No, opposite angles are equal in a parallelogram.

(ii) Yes, opposite sides are equal and parallel in a parallelogram.

(iii) No, diagonals bisect each other in a parallelogram.

4. In the adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the geometrical truths you use to find them.

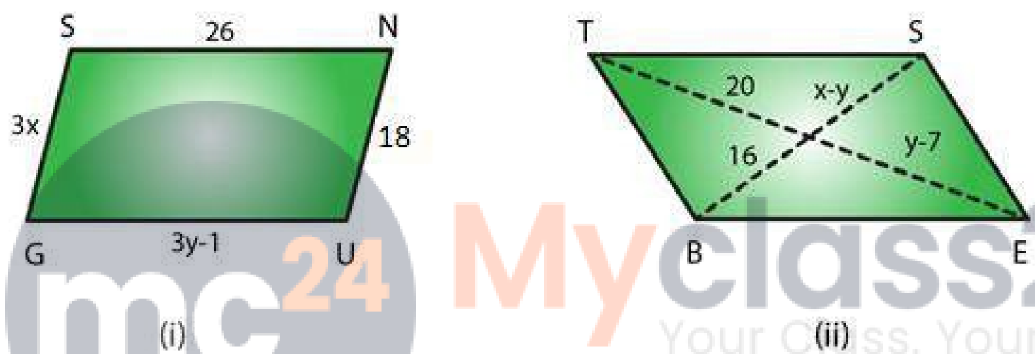


Solution:
We know that
$\angle POH + 70^\circ = 180^\circ$ [Linear pair]
$\angle POH = 180^\circ - 70^\circ$
$\angle POH = 110^\circ$

∠POH = ∠x = 110° [opposite angles are equal in a parallelogram]
∠x + ∠z + 40° = 180° [sum of the adjacent angles is equal to 180° in a parallelogram]
110° + ∠z + 40° = 180°
∠z = 180° - 150°
∠z = 30°

∠z +∠y = 70°
∠y + 30° = 70°
∠y = 70°- 30°
∠y = 40°

5. In the following figures *GUNS* and *RUNS* are parallelograms. Find *x* and *y*.



Solution:
(i) 3y – 1 = 26 [opposite sides are of equal length in a parallelogram]
3y = 26 + 1
y = 27/3
y = 9

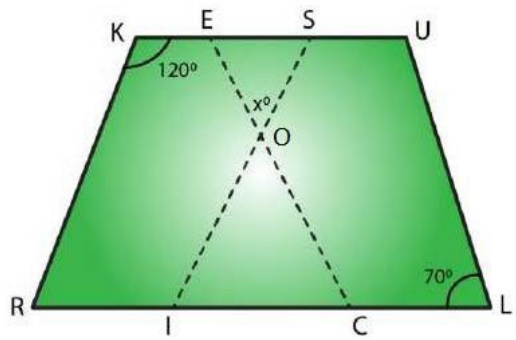
3x = 18 [opposite sides are of equal length in a parallelogram]
x = 18/3
x = 6
∴ x = 6 and y = 9

(ii) y – 7 = 20 [diagonals bisect each other in a parallelogram]
y = 20 + 7
y = 27

x - y = 16 [diagonals bisect each other in a parallelogram]
x -27 = 16

x = 16 + 27
= 43
∴ x = 43 and y = 27

6. In the following figure RISK and CLUE are parallelograms. Find the measure of x.



Solution:
In parallelogram RISK
∠RKS + ∠KSI = 180° [sum of the adjacent angles is equal to 180° in a parallelogram]
120° + ∠KSI = 180°
∠KSI = 180° - 120°
∠KSI = 60°

In parallelogram CLUE
∠CEU = ∠CLU = 70° [opposite angles are equal in a parallelogram]

In ΔEOS
70° + ∠x + 60° = 180° [Sum of angles of a triangles is 180°]
∠x = 180° - 130°
∠x = 50°
∴ x = 50°

7. Two opposite angles of a parallelogram are (3*x* - 2)° and (50 - *x*)°. Find the measure of each angle of the parallelogram.
Solution:
We know that opposite angles of a parallelogram are equal.
So, (3x - 2)° = (50 - x)°
3x° - 2° = 50° - x°

$3x° + x° = 50° + 2°$
$4x° = 52°$
$x° = 52°/4$
$= 13°$

Measure of opposite angles are,
$(3x – 2)° = 3×13 - 2 = 37°$
$(50 - x)° = 50 - 13 = 37°$
We know that Sum of adjacent angles = 180°
Other two angles are 180° - 37° = 143°
∴ Measure of each angle is 37°, 143°, 37°, 143°

8. If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Solution:

Let us consider one of the adjacent angle as x°
Other adjacent angle is = $2x°/3$
We know that sum of adjacent angles = 180°
So,
$x° + 2x°/3 = 180°$
$(3x° + 2x°)/3 = 180°$
$5x°/3 = 180°$
$x° = 180°×3/5$
$= 108°$
Other angle is = 180° – 108° = 72°
∴ Angles of a parallelogram are 72°, 72°, 108°, 108°

9. The measure of one angle of a parallelogram is 70°. What are the measures of the remaining angles?

Solution:

Let us consider one of the adjacent angle as x°
Other adjacent angle = 70°
We know that sum of adjacent angles = 180°
So,
$x° + 70° = 180°$
$x° = 180° – 70°$
$= 110°$
∴ Measures of the remaining angles are 70°, 70°, 110° and 110°

10. Two adjacent angles of a parallelogram are as 1 : 2. Find the measures of all the

angles of the parallelogram.

Solution:

Let us consider one of the adjacent angle as x°

Other adjacent angle = 2x°

We know that sum of adjacent angles = 180°

So,

$x^o + 2x^o = 180^o$

$3x^o = 180^o$

$x^o = 180^o/3$

$= 60^o$

So other angle is $2x = 2\times60 = 120^o$

∴ Measures of the remaining angles are 60°, 60°, 120° and 120°

11. In a parallelogram ABCD, ∠D= 135°, determine the measure of ∠A and ∠B.

Solution:

Given, one of the adjacent angle ∠D = 135°

Let other adjacent angle ∠A be = x°

We know that sum of adjacent angles = 180°

$x^o + 135^o = 180^o$

$x^o = 180^o – 135^o$

$= 45^o$

$\angle A = x^o = 45^o$

We know that measure of opposite angles are equal in a parallelogram.

So, $\angle A = \angle C = 45^o$

And $\angle D = \angle B = 135^o$

12. ABCD is a parallelogram in which ∠A = 70°. Compute ∠B, ∠C and ∠D.

Solution:

Given, one of the adjacent angle $\angle A = 70^o$

Other adjacent angle $\angle B$ be $= x^o$

We know that sum of adjacent angles = 180°

$x^o + 70^o = 180^o$

$x^o = 180^o – 70^o$

$= 110^o$

$\angle B = x^o = 110^o$

We know that measure of opposite angles are equal in a parallelogram.

So, $\angle A = \angle C = 70^o$

And $\angle D = \angle B = 110^o$

13. The sum of two opposite angles of a parallelogram is 130°. Find all the angles of the parallelogram.

Solution:

Consider ABCD as a parallelogram

$\angle A + \angle C = 130^o$

Here $\angle A$ and $\angle C$ are opposite angles

So $\angle C = 130/2 = 65^0$

We know that sum of adjacent angles is 180^0

$\angle B + \angle D = 180^0$

$65^0 + \angle D = 180^o$

$\angle D = 180^0 - 65^0 = 115^0$

$\angle D = \angle B = 115^o$ (opposite angles)

Therefore, $\angle A = 65^0$, $\angle B = 115^o$, $\angle C = 65^0$, $\angle D = 115^0$.

14. All the angles of a quadrilateral are equal to each other. Find the measure of each. Is the quadrilateral a parallelogram? What special type of parallelogram is it?

Solution:

Let us consider each angle of a parallelogram as x^o

We know that sum of angles = 360^o

$x^o + x^o + x^o + x^o = 360^o$

$4 x^o = 360^o$

$x^o = 360^o/4$

$= 90^o$

∴ Measure of each angle is 90^o

Yes, this quadrilateral is a parallelogram.

Since each angle of a parallelogram is equal to 90°, so it is a rectangle.

15. Two adjacent sides of a parallelogram are 4 cm and 3 cm respectively. Find its perimeter.

Solution:

We know that opposite sides of a parallelogram are parallel and equal.

So, Perimeter = Sum of all sides (there are 4 sides)

Perimeter = 4 + 3 + 4 + 3

= 14 cm

∴ Perimeter is 14cm.

16. The perimeter of a parallelogram is 150 cm. One of its sides is greater than the other by 25 cm. Find the length of the sides of the parallelogram.

Solution:

Given, Perimeter of the parallelogram = 150 cm
Let us consider one of the sides as = ‘x’ cm
Other side as = (x + 25) cm
We know that opposite sides of a parallelogram are parallel and equal.
So, Perimeter = Sum of all sides
x + x + 25 + x + x + 25 = 150
4x + 50 = 150
4x = 150 – 50
x = 100/4
= 25
∴ Sides of the parallelogram are (x) = 25 cm and (x+25) = 50 cm.

17. The shorter side of a parallelogram is 4.8 cm and the longer side is half as much again as the shorter side. Find the perimeter of the parallelogram.

Solution:

Given, Shorter side of the parallelogram = 4.8 cm
Longer side of the parallelogram = 4.8 + 4.8/2
= 4.8 + 2.4
= 7.2cm
We know that opposite sides of a parallelogram are parallel and equal.
So, Perimeter = Sum of all sides
Perimeter of the parallelogram = 4.8 + 7.2 + 4.8 + 7.2
= 24cm
∴ Perimeter of the parallelogram is 24 cm.

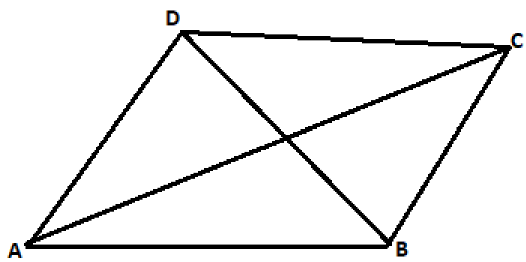
18. Two adjacent angles of a parallelogram are (3*x*-4)° and (3*x*+10)°. Find the angles of the parallelogram.

Solution:

We know that adjacent angles of a parallelogram are equal.
So, (3x - 4)° + (3x + 10)° = 180°
3x° + 3x° – 4 + 10 = 180°
6x = 180° - 6°
x = 174°/6
= 29°

Measure of adjacent angles are,
(3x – 4)° = 3×29 - 4 = 83°
(3x + 10)° = 3×29 + 10 = 97°
We know that Sum of adjacent angles = 180°
∴ Measure of each angle is 83°, 97°, 83°, 97°

19. In a parallelogram ABCD, the diagonals bisect each other at O.
If ∠ABC =30°, ∠BDC= 10° and ∠CAB =70°. Find:
∠DAB, ∠ADC, ∠BCD, ∠AOD, ∠DOC, ∠BOC, ∠AOB, ∠ACD, ∠CAB, ∠ADB, ∠ACB , ∠DBC, and ∠DBA.
Solution:
Firstly let us draw a parallelogram



Given, ∠ABC = 30°,
∠ABC = ∠ADC = 30° [We know that measure of opposite angles are equal in a parallelogram]
∠BDC = 10°
∠CAB =70°
∠BDA = ∠ADB = ∠ADC - ∠BDC = 30° - 10° = 20°
∠DAB = 180° - 30° = 150°
∠ADB = ∠DBC = 20° (alternate angles)

∠BCD = ∠DAB = 150° [we know, opposite angles are equal in a parallelogram]
∠DBA = ∠BDC = 10° [we know, Alternate interior angles are equal]

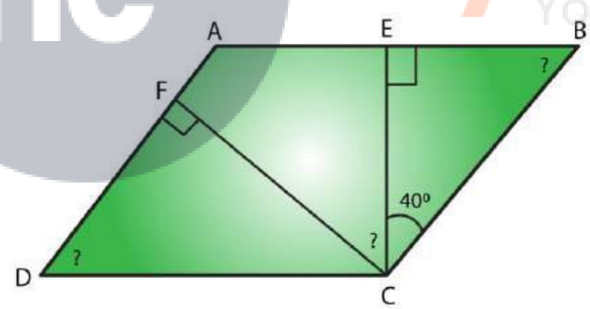
In ΔABC
∠CAB + ∠ABC + ∠BCA = 180° [since, sum of all angles of a triangle is 180°]
70° + 30° + ∠BCA = 180°
∠BCA = 180° – 100°
= 80°
∠DAB = ∠DAC + ∠CAB = 70° + 80° = 150°
∠BCD = 150° (opposite angle of the parallelogram)
∠DCA = ∠CAB = 70°

In ΔDOC
∠BDC + ∠ACD + ∠DOC = 180° [since, sum of all angles of a triangle is 180°]
10° + 70° + ∠DOC = 180°
∠DOC = 180°- 80°

∠DOC = 100°
So, ∠DOC = ∠AOB = 100° [Vertically opposite angles are equal]
∠DOC + ∠AOD = 180° [Linear pair]
100° + ∠AOD = 180°
∠AOD = 180°- 100°
∠AOD = 80°
So, ∠AOD = ∠BOC = 80° [Vertically opposite angles are equal]
∠CAB = 70°

∠ABC + ∠BCD = 180° [In a parallelogram sum of adjacent angles is 180°]
30° + ∠ACB + ∠ACD = 180°
30° + ∠ACB + 70° = 180°
∠ACB = 180° - 100°
∠ACB = 80°
∴ ∠DAB = 150°, ∠ADC = 30, ∠BCD = 150°, ∠AOD = 80°, ∠DOC = 100°, ∠BOC = 80°, ∠AOB = 100°, ∠ACD = 70°, ∠CAB = 70°, ∠ADB = 20°, ∠ACB = 80°, ∠DBC = 20°, and ∠DBA = 10°.

20. Find the angles marked with a question mark shown in Figure.



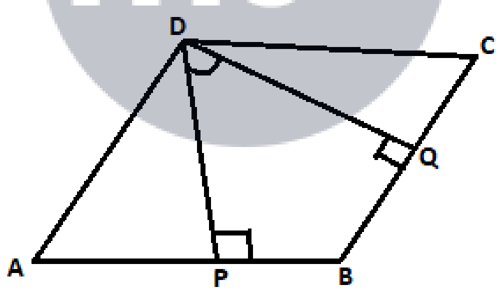
Solution:
In ΔBEC
∠BEC + ∠ECB +∠CBE = 180° [Sum of angles of a triangle is 180°]
90° + 40° + ∠CBE = 180°
∠CBE = 180°-130°
∠CBE = 50°
∠CBE = ∠ADC = 50° (Opposite angles of a parallelogram are equal)

∠B = ∠D = 50° [Opposite angles of a parallelogram are equal]

∠A + ∠B = 180° [Sum of adjacent angles of a triangle is 180°]
∠A + 50° = 180°
∠A = 180°-50°
So, ∠A = 130°

In ΔDFC
∠DFC + ∠FCD +∠CDF = 180° [Sum of angles of a triangle is 180°]
90° + ∠FCD + 50° = 180°
∠FCD = 180°-140°
∠FCD = 40°
∠A = ∠C = 130° [Opposite angles of a parallelogram are equal]
∠C = ∠FCE +∠BCE + ∠FCD
∠FCD + 40° + 40° = 130°
∠FCD = 130° - 80°
∠FCD = 50°
∴ ∠EBC = 50º, ∠ADC = 50º and ∠FCD = 50º

21. The angle between the altitudes of a parallelogram, through the same vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.
Solution:

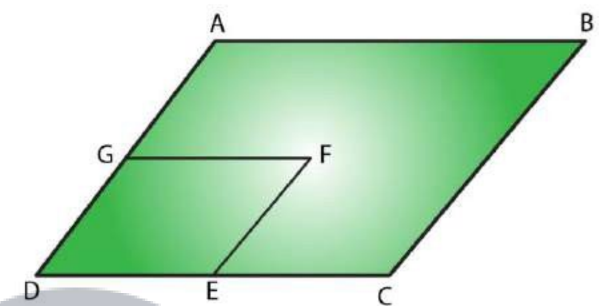


Let us consider a parallelogram, ABCD. Where, DP⊥AB and DQ ⊥BC.
Given ∠PDQ = 60°
In quadrilateral DPBQ
∠PDQ + ∠DPB + ∠B + ∠BQD = 360° [Sum of all the angles of a Quadrilateral is 360°]
60° + 90° + ∠B + 90° = 360°
∠B = 360° – 240°
∠B = 120°

∠B = ∠D = 120° [Opposite angles of parallelogram are equal]
∠B + ∠C = 180° [Sum of adjacent interior angles in a parallelogram is 180°]

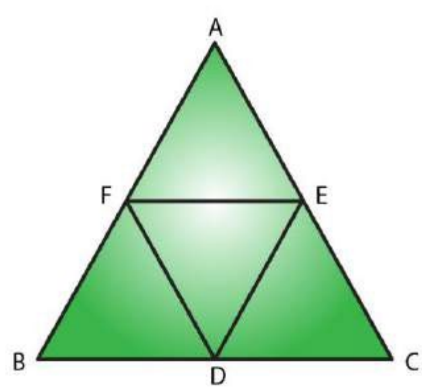
120° + ∠C = 180°
∠C = 180° – 120° = 60°
∠A = ∠C = 60° (Opposite angles of parallelogram are equal)
∴ Angles of a parallelogram are 60°, 120°, 60°, 120°

22. In Figure, ABCD and AEFG are parallelograms. If ∠C =55°, what is the measure of ∠F?



Solution:
In parallelogram ABCD
∠C = ∠A = 55° [In a parallelogram opposite angles are equal in a parallelogram]
In parallelogram AEFG
∠A = ∠F = 55° [In a parallelogram opposite angles are equal in a parallelogram]
∴ Measure of ∠F = 55°

23. In Figure, BDEF and DCEF are each a parallelogram. Is it true that BD = DC? Why or why not?

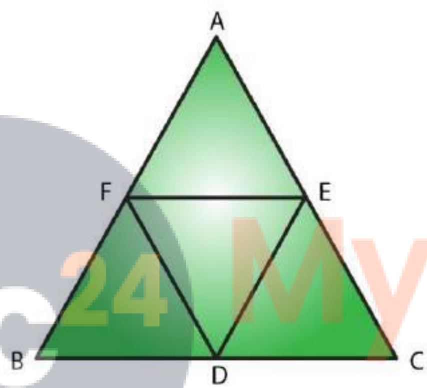


Solution:
In parallelogram BDEF
BD = EF [In a parallelogram opposite sides are equal]

In parallelogram DCEF
DC = EF [In a parallelogram opposite sides are equal]

Since, BD = EF = DC
So, BD = DC

24. In Figure, suppose it is known that DE = DF. Then, is ΔABC isosceles? Why or why not?



Solution:
In parallelogram BDEF
BD = EF and BF = DE [opposite sides are equal in a parallelogram]

In parallelogram DCEF
DC = EF and DF = CE [opposite sides are equal in a parallelogram]

In parallelogram AFDE
AF = DE and DF = AE [opposite sides are equal in a parallelogram]

So, DE = AF = BF

Similarly: DF = CE = AE

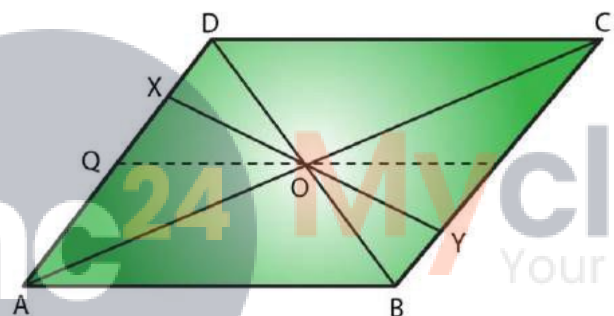
Given, DE = DF

Since, DF = DF

AF + BF = CE + AE

AB = AC
∴ ΔABC is an isosceles triangle.

25. Diagonals of parallelogram ABCD intersect at O as shown in Figure. XY contains O, and X, Y are points on opposite sides of the parallelogram. Give reasons for each of the following:
(i) OB = OD
(ii) ∠OBY = ∠ODX
(iii) ∠BOY = ∠DOX
(iv) ΔBOY = ΔDOX
Now, state if XY is bisected at O.



Solution:
(i) OB = OD
OB = OD. Since diagonals bisect each other in a parallelogram.

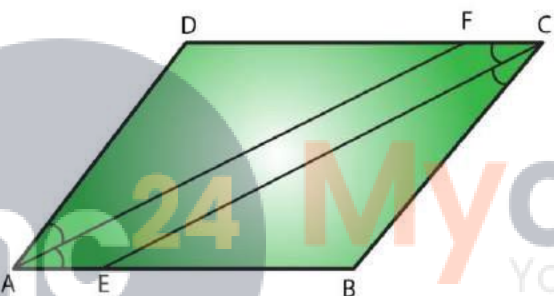
(ii) ∠OBY =∠ODX
∠OBY =∠ODX. Since alternate interior angles are equal in a parallelogram.

(iii) ∠BOY= ∠DOX
∠BOY= ∠DOX. Since vertical opposite angles are equal in a parallelogram.

(iv) ΔBOY ≅ ΔDOX
ΔBOY and ΔDOX. Since OB = OD, where diagonals bisect each other in a parallelogram.

∠OBY =∠ODX [Alternate interior angles are equal]
∠BOY= ∠DOX [Vertically opposite angles are equal]
ΔBOY ≅ΔDOX [by ASA congruence rule]
OX = OY [Corresponding parts of congruent triangles]
∴ XY is bisected at O.

26. In Fig. 17.31, ABCD is a parallelogram, CE bisects ∠C and AF bisects ∠A. In each of the following, if the statement is true, give a reason for the same:
(i) ∠A = ∠C
(ii) ∠FAB = ½ ∠A
(iii) ∠DCE = ½ ∠C
(iv) ∠CEB = ∠FAB
(v) CE ∥ AF



Solution:
(i) ∠A = ∠C
True, Since ∠A =∠C = 55° [opposite angles are equal in a parallelogram]

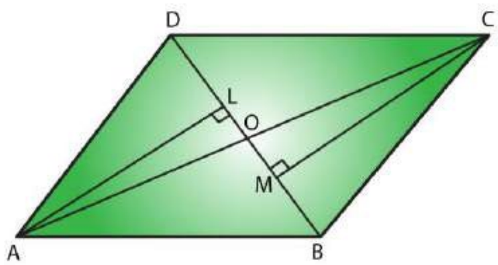
(ii) ∠FAB = ½ ∠A
True, Since AF is the angle bisector of ∠A.

(iii) ∠DCE= ½ ∠C
True, Since CE is the angle bisector of angle ∠C.

(iv) ∠CEB= ∠FAB
True,
Since ∠DCE = ∠FAB (opposite angles are equal in a parallelogram).
∠CEB = ∠DCE (alternate angles)
½ ∠C = ½ ∠A [AF and CE are angle bisectors]

(v) CE || AF
True, since one pair of opposite angles are equal, therefore quad. AEFC is a parallelogram.

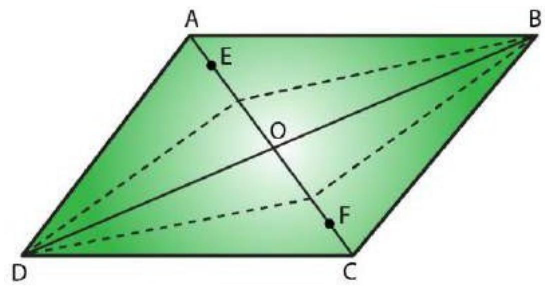
27. Diagonals of a parallelogram ABCD intersect at O. AL and CM are drawn perpendiculars to BD such that L and M lie on BD. Is AL = CM? Why or why not?
Solution:



Given, AL and CM are perpendiculars on diagonal BD.
In ΔAOL and ΔCOM
∠AOL = ∠COM (vertically opposite angle) ….. (i)
∠ALO = ∠CMO = 90° (each right angle) ……. (ii)
By using angle sum property
∠AOL + ∠ALO + ∠LAO = 180° ……… (iii)
∠COM + ∠CMO + ∠OCM = 180° ……. (iv)
From (iii) and (iv)
∠AOL + ∠ALO + ∠LAO = ∠COM + ∠CMO + ∠OCM
∠LAO = ∠OCM (from (i) and (ii))

In ΔAOL and ΔCOM
∠ALO = ∠CMO (each right angle)
AO = OC (diagonals of a parallelogram bisect each other)
∠LAO = ∠OCM (proved)
So, ΔAOL is congruent to ΔCOM
∴ AL = CM (Corresponding parts of congruent triangles)

28. Points E and F lie on diagonals AC of a parallelogram ABCD such that AE = CF. what type of quadrilateral is BFDE?
Solution:

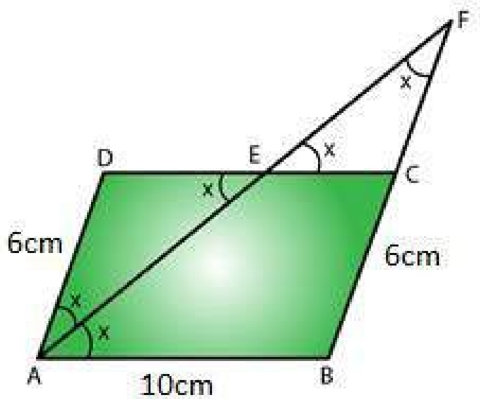


In parallelogram ABCD:
AO = OC.......................(i) (Diagonals of a parallelogram bisect each other)
AE = CF...........(ii) Given
On subtracting (ii) from (i)
AO - AE = OC - CF
EO = OF ....(iii)

In ΔDOE and ΔBOF
EO = OF (proved)
DO = OB (Diagonals of a parallelogram bisect each other)
∠DOE = ∠BOF (vertically opposite angles are equal in a parallelogram)
By the rule of SAS congruence ΔDOE ≅ ΔBOF
So, DE = BF (Corresponding parts of congruent triangles)

In ΔBOE and ΔDOF
EO = OF (proved)
DO = OB (diagonals of a parallelogram bisect each other)
∠DOF = ∠BOE (vertically opposite angles are equal in a parallelogram)
By the rule of SAS congruence ΔDOE ≅ ΔBOF
∴ DF = BE (Corresponding parts of congruent triangles)
∴ BFDE is a parallelogram, since one pair of opposite sides are equal and parallel.

29. In a parallelogram ABCD, AB = 10cm, AD = 6 cm. The bisector of ∠A meets DC in E, AE and BC produced meet at F. Find the length CF.
Solution:



In a parallelogram ABCD

Given, AB = 10 cm, AD = 6 cm

⇒ CD = AB = 10 cm and AD = BC = 6 cm [In a parallelogram opposite sides are equal]

AE is the bisector of ∠DAE = ∠BAE = x

∠BAE = ∠AED = x (alternate angles are equal)

ΔADE is an isosceles triangle. Since opposite angles in ΔADE are equal.

AD = DE = 6cm (opposite sides are equal)

CD = DE + EC

EC = CD – DE

= 10 – 6

= 4cm

∠DEA = ∠CEF = x (vertically opposite angle are equal)

∠EAD = ∠EFC = x (alternate angles are equal)

ΔEFC is an isosceles triangle. Since opposite angles in ΔEFC are equal.

CF = CE = 4cm (opposite side are equal to angles)

∴ CF = 4cm.