NCERT Solutions for Class-XII Maths

Chapter-9.6

NCERT Mathematics Class 12

1. $\frac{dy}{dx}+2y=\sin x$

1. The given differential equation is $\frac{dy}{dx}+2y=\sin x$.

This is in the form of $\frac{dy}{dx}+py=Q\left(\text{where } p=2 \text{ and } Q=\sin x\right)$

Now, $\text{I.F}=e^{\int p.dx}=e^{\int 2.dx}=e^{2x}$

The solution of the given differential equation is given by the relation,

$$y(\text{I.F.})=\int(Q\times \text{I.F.})dx+C$$

$$\Rightarrow ye^{2x}=\int \sin x.e^{2x}dx+C \qquad ...(1)$$

Let $I=\int \sin x\cdot e^{2x}$

$$\Rightarrow I=\sin x\cdot\int e^{2x}dx-\int\left(\frac{d}{dx}(\sin x)\cdot\int e^{2x}\right)dx$$

$$\Rightarrow I=\sin x\cdot\frac{e^{2x}}{2}-\int\left(\cos x.\frac{e^{2x}}{2}\right)dx$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x.\int e^{2x}-\int\left(\frac{d}{dx}(\cos x)\cdot\int e^{2x}dx\right)dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x.\frac{e^{2x}}{2}-\int\left[(-\sin x).\frac{e^{2x}}{2}\right]dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\sin}{2}-\frac{e^{2x}\cos x}{4}-\frac{1}{4}\int\left(\sin x.e^{2x}\right)dx$$

$$\Rightarrow I=\frac{e^{2x}}{4}(2\sin x-\cos x)-\frac{1}{4}I$$

$$\Rightarrow \frac{5}{4}I=\frac{e^{2x}}{4}(2\sin x-\cos x)$$

$$\Rightarrow I=\frac{e^{2x}}{5}(2\sin x-\cos x)$$

Therefore, equation (1) becomes:

$$ye^{2x} = \frac{e^{2x}}{5}(2\sin x - \cos x) + C$$

$$\Rightarrow y = \frac{1}{5}(2\sin x - \cos x) + Ce^{-2x}$$

This is the required general solution of the given differential equation.

2. $\frac{dy}{dx} + 3y = e^{-2x}$

2. It is given that $\frac{dy}{dx} + 3y = e^{-2x}$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = 3 and Q = e^{-2x})

Now, I.F. = $e^{\int p dx} = e^{\int 3dx} = e^{3x}$

Thus, the solution of the given differential equation is given by the relation:

$y(\text{I.F.}) = \int (Q \times \text{I. F.})dx + C$

$\Rightarrow ye^{3x} = \int (e^{-2x} \times e^{2x})dx + C$

$\Rightarrow ye^{3x} = \int e^{x}dx + C$

$\Rightarrow ye^{3x} = e^{x} + C$

$y \Rightarrow \; = e^{-2x} + Ce^{-3x}$

Therefore, the required general solution of the given differential equation is $y = e^{-2x} + Ce^{-3x}$.

3. $\frac{dy}{dx} + \frac{y}{x} = x^2$

3. The given differential equation is:

$$\frac{dy}{dx} + py = Q\left(\text{where } p = \frac{1}{x} \text{ and } Q = x^2\right)$$

Now, $\text{I.F} = e^{\int p\,dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

The solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int (Q \times \text{I.F.})dx + C$$

$$\Rightarrow y(x) = \int (x^2 \cdot x)dx + C$$

$$\Rightarrow xy = \int x^3 dx + C$$

$$\Rightarrow xy = \frac{x^4}{4} + C$$

This is the required general solution of the given differential equation.

4. $\frac{dy}{dx} + \sec xy = \tan\left(0 \le x < \frac{\pi}{2}\right)$

4. It is given that $\frac{dy}{dx} + (\sec x)y = \tan x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = secx and Q = tanx)

Now, I.F. = $e^{\int p dx} = e^{\int \sec x dx} = e^{\log(\sec x + \tan x)} = \sec x + \tan x$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int (Q \times I.F.)dx + C$

$\Rightarrow y(\sec x + \tan x) = \int \tan x(\sec x + \tan x)dx + C$

$\Rightarrow y(\sec x + \tan x) = \int \sec x \tan x dx + \int \tan^2 x dx + C$

$\Rightarrow y(\sec x + \tan x) = \sec x + \int (\sec^2 x - 1)dx + C$

$\Rightarrow$ y(secx + tanx) = secx + tanx – x+ C

Therefore, the required general solution of the given differential equation is

y(secx + tanx) = secx + tanx – x + C.

5. $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$

5. Let $I = \int_0^{\frac{\pi}{2}} \cos 2x \, dx$

$\int \cos 2x \, dx = \left(\frac{\sin 2x}{2}\right) = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\frac{\pi}{2}\right) - F(0)$

$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right]$

$= \frac{1}{2}[\sin \pi - \sin 0]$

$= \frac{1}{2}[0 - 0] = 0$

6. $x\frac{dy}{dx} + 2y = x^2 \log x$

6. It is given that $x\frac{dy}{dx} + 2y = x^2 \log x$

$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = x\log x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $\frac{2}{x}$ and Q =xlogx)

Now, I.F. = $e^{\int p dx} = e^{\int \frac{2}{x} dx} = e^{2(\log x)} = e^{\log x^2} = x^2$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int (Q \times I.F.)dx + C$

$\Rightarrow y.x^2 = \int (x\log x.x^2)dx + C$

$\Rightarrow x^2y = \int(x^3 logx)dx + C$

$\Rightarrow x^2y = logx.\int x^3dx - \int\left[\frac{d}{dx}(logx).\int x^3dx\right]dx + C$

$\Rightarrow x^2y = logx.\frac{x^4}{4} - \int\left(\frac{1}{x}.\frac{x^4}{4}\right)dx + C$

$\Rightarrow x^2y = \frac{x^4logx}{4} - \frac{1}{4}\int x^3dx + C$

$\Rightarrow x^2y = \frac{x^4logx}{4} - \frac{1}{4}.\frac{x^4}{4} + C$

$\Rightarrow x^2y = \frac{1}{16}x^4(4logx - 1) + C$

$\Rightarrow y = \frac{1}{16}x^2(4logx - 1) + Cx^{-2}$

Therefore, the required general solution of the given differential equation

$y = \frac{1}{16}x^2(4logx - 1) + Cx^{-2}$

7. $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

7. The given differential equation is:

$x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x^2}$

This equation is the form of a linear differential equation as:

$\frac{dy}{dx} + py = Q\left(\text{where } p = \frac{1}{x\log x} \text{ and } Q = \frac{2}{x^2}\right)$

Now, $\text{I.F} = e^{\int p\,dx} = e^{\int \frac{1}{x\log}dx} = e^{\log(\log x)} = \log x$

The general solution of the given differential equation is given by the relation,

$y(\text{I.F.}) = \int(Q \times \text{I.F.})dx + C$

$\Rightarrow y\log x = \int\left(\frac{2}{x^2}\log x\right)dx + C \qquad ...(1)$

Now, $\int\left(\frac{2}{x^2}\log x\right)dx = 2\int\left(\log x.\frac{1}{x^2}\right)dx$

$= 2\left[\log x \cdot \int\frac{1}{x^2}dx - \int\left\{\frac{d}{dx}(\log x)\cdot\int\frac{1}{x^2}dx\right\}dx\right]$

$= 2\left[\log x\left(-\frac{1}{x}\right) - \int\left(\frac{1}{x}\cdot\left(-\frac{1}{x}\right)\right)dx\right]$

$= 2\left[-\frac{\log x}{x} + \int\frac{1}{x^2}dx\right]$

$= 2\left[-\frac{\log x}{x} - \frac{1}{x}\right]$

$= -\frac{2}{x}(1 + \log x)$

Substituting the value of $\int\left(\frac{2}{x^2}\log x\right)$ in equation (1), we get:

$y \log x = -\frac{2}{x}(1 + \log x) + C$

This is the required general solution of the given differential equation.

8. $(1 + x^2)dy + 2xy\,dx = \cot x dx\,(x \neq 0)$

8. It is given that $(1 + x^2)dy + 2xydx = cotxdx$

$\Rightarrow \frac{dy}{dx} + \frac{2xy}{(1+x^2)} = \frac{cotx}{1+x^2}$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $\frac{2x}{(1+x^2)}$ and Q = $\frac{cotx}{1+x^2}$)

Now, I.F. = $e^{\int pdx} = e^{\int \frac{2x}{(1+x^2)}dx} = e^{log(1+x^2)} = 1 + x^2$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int (Q \times I.F.)dx + C$

$\Rightarrow y.(1 + x^2) = \int \left[\frac{cotx}{1+x^2}.(1 + x^2)\right] dx + C$

$\Rightarrow y.(1 + x^2) = \int cotxdx + C$

$\Rightarrow y(1 + x^2) = log|sinx| + C$

Therefore, the required general solution of the given differential equation is

$$y(1 + x^2) = log|sinx| + C$$

9. $x\frac{dy}{dx} + y - x + xy\cot x = 0\,(x \neq 0)$

9. $x\frac{dy}{dx} + y - x + xy \cot x = 0$

$\Rightarrow x\frac{dy}{dx} + y(1 + x\cot x) = x$

$\Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$

This equation is a linear differential equation of the form:

$\frac{dy}{dx} + py = Q\left(\text{where } p = \frac{1}{x} + \cot x \text{ and } Q = 1\right)$

Now, $\text{I.F} = e^{\int p\,dx} = e^{\int\left(\frac{1}{x}+\cot x\right)dx} = e^{\log x + \log(\sin x)} = e^{\log(x \sin x)} = x \sin x$

The general solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int(Q \times \text{I.F.})dx + C$$

$$\Rightarrow y(x\sin x) = \int(1 \times x\sin x)dx + C$$

$$\Rightarrow y(x\sin x) = \int(x\sin x)dx + C$$

$$\Rightarrow y(x\sin x) = x\int \sin x\, dx - \int\left[\frac{d}{dx}(x)\cdot\int \sin x\, dx\right] + C$$

$$\Rightarrow y(x\sin x) = x(-\cos x) - \int 1\cdot(-\cos x)dx + C$$

$$\Rightarrow y(x\sin x) = -x\cos x + \sin x + C$$

$$\Rightarrow y = \frac{-x\cos x}{x\sin x} + \frac{\sin x}{x\sin x} + \frac{C}{x\sin x}$$

$$\Rightarrow y = -\cot\cdot x + \frac{1}{x} + \frac{C}{x\sin x}$$

10. $(x+y)\frac{dy}{dx} = 1$

10. It is given that $(x+y)\frac{dy}{dx} = 1$

$\Longrightarrow \frac{dy}{dx} = \frac{1}{x+y}$

$\Longrightarrow \frac{dx}{dy} = x + y$

$\Longrightarrow \frac{dx}{dy} - x = y$

This is equation in the form of $\frac{dy}{dx} + px = Q$ (where, p = -1 and Q = y)

Now, I.F. = $e^{\int p dy} = e^{\int -dy} = e^{-y}$

Thus, the solution of the given differential equation is given by the relation:

x(I.F.) = $\int(Q \times I.F.)dy + C$

$\Longrightarrow xe^{-y} = \int[y.e^{-y}]dy + C$

$\Longrightarrow xe^{-y} = y\int e^{-dy} - \int\left[\frac{d}{dy}(y)\int e^{-y}dy\right]dy + C$

$\Longrightarrow xe^{-y} = y(-e^{-y}) - \int(-e^{-y})dy + C$

$\Longrightarrow xe^{-y} = -ye^{-y} + \int e^{-y}dy + C$

$\Longrightarrow xe^{-y} = -ye^{-y} - e^{-y} + C$

$\Longrightarrow$ x = - y – 1 + Ce^{y}

$\Longrightarrow$ x + y + 1 = Ce^{y}

Therefore, the required general solution of the given differential equation is x + y + 1 = Ce^{y}.

11. y dx + (x – y^2)dy = 0

11. y dx + (x – y^2)dy = 0

$\Rightarrow ydx = (y^2 - x)dy = 0$

$\Rightarrow \frac{dx}{dy} = \frac{y^2 - x}{y} = y - \frac{x}{y}$

$\Rightarrow \frac{dx}{dy} + \frac{x}{y} = y$

This is a linear differential equation of the form:

$\frac{dy}{dx} + px = Q\left(\text{where } p = \frac{1}{y} \text{ and } Q = y\right)$

Now, $\text{I.F} = e^{\int p\,dy} = e^{\int \frac{1}{y}dy} = e^{\log y} = y$

The general solution of the given differential equation is given by the relation,

$x(\text{I.F.}) = \int(Q \times \text{I.F.})dy + C$

$\Rightarrow xy = \int(y \cdot y)dy + C$

$\Rightarrow xy = \int y^2 dy + C$

$\Rightarrow xy = \frac{y^3}{3} + C$

$\Rightarrow x = \frac{y^2}{3} + \frac{C}{y}$

12. $\left(x + 3y^2\right)\frac{dy}{dx} = y(y > 0)$

12. It is given that $(x + 3y^2)\frac{dy}{dx} = y$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x+3y^2}$

$\Rightarrow \frac{dx}{dy} = \frac{x+3y^2}{y} = \frac{x}{y} + 3y$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 3y$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, $p = -\frac{1}{y}$ and $Q = 3y$)

Now, I.F. $= e^{\int pdy} = e^{-\int \frac{dy}{y}} = e^{-\log y} = e^{\log\left(\frac{1}{y}\right)} = \frac{1}{y}$

Thus, the solution of the given differential equation is given by the relation:

$x(\text{I.F.}) = \int(Q \times I.F.)dy + C$

$\Rightarrow x.\frac{1}{y} = \int\left[3y.\frac{1}{y}\right]dy + C$

$\Rightarrow \frac{x}{y} = 3y + C$

$\Rightarrow x = 3y^2 + Cy$

Therefore, the required general solution of the given differential equation is $x = 3y^2 + Cy$.

13. $\frac{dy}{dx}+2y\tan x=\sin x;\ y=0$ when $x=\frac{\pi}{3}$

13. The given differential equation is $\frac{dy}{dx}+2y\tan x=\sin x$.

This is a linear equation of the form:

$\frac{dy}{dx}+py=Q\left(\text{where } p=2\tan x \text{ and } Q=\sin x\right)$

Now, $I.F=e^{\int p\,dx}=e^{\int 2\tan x dx}=e^{2\log|\sec x|}=e^{\log\left(\sec^2 x\right)}=\sec^2 x$

The general solution of the given differential equation is given by the relation,

$y\left(I.F.\right)=\int\left(Q\times I.F.\right)dx+C$

$\Rightarrow y\left(\sec^2 x\right)=\int\left(\sin x\cdot\sec^2 x\right)dx+C$

$\Rightarrow y\sec^2 x=\int\left(\sec x\cdot\tan x\right)dx+C$

$\Rightarrow y\sec^2 x=\sec x+C$ … (1)

Now, $y=0$ at $x=\frac{\pi}{3}$

Therefore,

$0\times\sec^2\frac{\pi}{3}=\sec\frac{\pi}{3}+C$

$\Rightarrow 0=2+C$

$\Rightarrow C=-2$

Substituting $C=-2$ in equation (1), we get:

$y\sec^2 x=\sec x-2$

$\Rightarrow y=\cos x-2\cos^2 x$

Hence, the required solution of the given differential equation is $y=\cos x-2\cos^2 x$.

14. $\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{1+x^2};\ y=0$ when $x=1$

14. It is given that $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$

$\Rightarrow \frac{dy}{dx}+\frac{2xy}{(1+x^2)}=\frac{1}{(1+x^2)^2}$

This is equation in the form of $\frac{dy}{dx}+py=Q$ (where, $p=\frac{2x}{(1+x^2)}$ and $Q=\frac{1}{(1+x^2)^2}$)

Now, I.F. $=e^{\int pdx}=e^{\int\frac{2x}{(1+x^2)}dx}=e^{\log(1+x^2)}=1+x^2$

Thus, the solution of the given differential equation is given by the relation:

$y(I.F.)=\int(Q\times I.F.)dx+C$

$\Rightarrow y.(1+x^2)=\int\left[\frac{1}{(1+x^2)^2}.(1+x^2)\right]dx+C$

$\Rightarrow y.(1+x^2)=\int\frac{1}{(1+x^2)}dx+C$

$\Rightarrow y.(1+x^2) = tan^{-1} x + C$ ----------------(1)

Now, it is given that y = 0 at x = 1

$0 = \tan^{-1} 1 + C$

$\Rightarrow C = -\frac{\pi}{4}$

Now, Substituting the value of $C = -\frac{\pi}{4}$ in (1), we get,

$\Rightarrow y.(1+x^2) = tan^{-1} x - \frac{\pi}{4}$

Therefore, the required general solution of the given differential equation is

$$y.(1+x^2) = tan^{-1} x - \frac{\pi}{4}$$

15. $\frac{dy}{dx} - 3y\cot x = \sin 2x;\ y = 2 \text{ when } x = \frac{\pi}{2}$

15. The given differential equation is $\frac{dy}{dx} - 3y\cot x = \sin 2x$

This is a linear differential equation of the form:

$\frac{dy}{dx} + py = Q\left(\text{where } p = -3\cot x \text{ and } Q = \sin 2x\right)$

Now, $I.F = e^{\int p\,dx} = e^{-3\int \cot x\,dx} = e^{-3\log|\sin x|} = e^{\log\left|\frac{1}{\sin^3 x}\right|} = \frac{1}{\sin^3 x}$

The general solution of the given differential equation is given by the relation,

$y(I.F.) = \int (Q \times I.F.)dx + C$

$\Rightarrow y \cdot \frac{1}{\sin^3 x} = \int\left[\sin 2x \cdot \frac{1}{\sin^3 x}\right]dx + C$

$\Rightarrow y\,\text{cosec}^3 x = 2\int(\cot x\,\text{cosec} x)dx + C$

$\Rightarrow y\ \text{cosec}^3\ x = 2\text{cosec}\ x + C$

$\Rightarrow y = -\frac{2}{\text{cosec}^2 x} + \frac{3}{\text{cosec}^3 x}$

$\Rightarrow y = -2\sin^2 x + C\sin^3 x$ ...(1)

Now, $y = 2 \text{ at } x = \frac{\pi}{2}$

Therefore, we get:

$2 = -2 + C$

$\Rightarrow C = 4$

Substituting C = 4 in equation (1), we get:

$y = -2\sin^2 x + 4\sin^3 x$

$\Rightarrow y = 4\sin^3 x - 2\sin^2 x$

This is the required particular solution of the given differential equation.

16. Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

16. Let F(x,y) be the curve passing through origin and let (x,y) be a point on the curve.

We know the slope of the tangent to the curve at (x,y) is $\frac{dy}{dx}$.

According to the given conditions, we get,

$\frac{dy}{dx} = x + y$

$\Rightarrow \frac{dy}{dx} - y = x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = -1 and Q = x)

Now, I.F. = $e^{\int p dx} = e^{\int (-1) dx} = e^{-x}$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int (Q \times I.F.) dx + C$

$\Rightarrow ye^{-x} = \int xe^{-x} dx + C$------------(1)

Now, $\int xe^{-x} dx = x \int e^{-x} dx - \int \left[\frac{d}{dx}(x). \int e^{-x}\, dx\right] dx$

$= x(e^{-x}) - \int(-e^{-x}) dx$

$= x(e^{-x}) + (-e^{-x})$

$= -e^{-x}(x + 1)$

Thus, from equation (1), we get,

$\Rightarrow ye^{-x} = -e^{-x}(x + 1) + C$

$\Rightarrow$ y = -(x+1) + Ce^x

$\Rightarrow$ x + y + 1 = Ce^x--------------------(2)

Now, it is given that curve passes through origin.

Thus, equation (2) becomes:

1 = C

$\Rightarrow$ C = 1

Substituting C = 1 in equation (2), we get,

x + y – 1 = e^x

Therefore, the required general solution of the given differential equation is x + y -1 = e^x.

17. Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

17. Let F (x, y) be the curve and let (x, y) be a point on the curve. The slope of the tangent to the curve at (x, y) is $\frac{dy}{dx}$.

According to the given information:

$\frac{dy}{dx} + 5 = x + y$

$\Rightarrow \frac{dy}{dx} - y = x - 5$

This is a linear differential equation of the form:

$\frac{dy}{dx} + py = Q$ (where $p = -1$ and $Q = x - 5$)

Now, $\text{I.F} = e^{\int p\,dx} = e^{\int(-1)dx} = e^{-x}$

The general equation of the curve is given by the relation,

$y(\text{I.F.}) = \int (Q \times \text{I.F.})dx + C$

$\Rightarrow y \cdot e^{-x} = \int (x-5)e^{-x}dx + C \quad ...(1)$

Now, $\int (x-5)e^{-x}dx = (x-5)\int e^{-x}dx - \int\left[\frac{d}{dx}(x-5)\cdot\int e^{-x}dx\right]dx$

$= (x-5)(-e^{-x}) - \int(-e^{-x})dx$

$= (5-x)e^{-x} + (-e^{-x})$

$= (4-x)e^{-x}$

Therefore, equation (1) becomes:

$ye^{-x} = (4 - x)e^{-x} + C$

$\Rightarrow y = 4 - x + Ce^{x}$

$\Rightarrow x + y - 4 = Ce^{x} \quad ..(2)$

The curve passes through point (0, 2).

Therefore, equation (2) becomes:

$0 + 2 - 4 = Ce^{0}$

$\Rightarrow -2 = C$

$\Rightarrow C = -2$

Substituting $C = -2$ in equation (2), we get:

$x + y - 4 = -2e^{x}$

$\Rightarrow y = 4 - x - 2e^{x}$

This is the required equation of the curve.

18. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

(a) e^{-x}

(b) e^{-y}

(c) $\frac{1}{x}$

(d) x

18. The correct option is (C).

It is given that $x\frac{dy}{dx} - y = 2x^2$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, $p = -\frac{1}{x}$ and $Q = 2x$)

Now, I.F. $= e^{\int p dx} = e^{\int -\frac{1}{x} dx} = e^{\log(x^{-1})} = x^{-1} = \frac{1}{x}$

19. The integrating factor of the differential equation.

$\left(1-y^2\right)\frac{dx}{dy} + yx = ay\left(-1 < y < 1\right)$

(a) $\frac{1}{y^2-1}$

(b) $\frac{1}{\sqrt{y^2-1}}$

(c) $\frac{1}{1-y^2}$

(d) $\frac{1}{\sqrt{1-y^2}}$

19. The correct option is (D).

The given differential equation is:

$\left(1-y^2\right)\frac{dx}{dy} + yx = ay$

$\Rightarrow \frac{dy}{dx} + \frac{yx}{1-y^2} = \frac{ay}{1-y^2}$

This is a linear differential equation of the form:

$\frac{dx}{dy} + py = Q\left(\text{where } p = \frac{y}{1-y^2} \text{ and } Q = \frac{ay}{1-y^2}\right)$

The integrating factor (I.F) is given by the relation,

$e^{\int p\, dx}$

$\therefore \text{I.F} = e^{\int p\, dy} = e^{\int \frac{y}{1-y^2} dy} = e^{\frac{1}{2}\log\left(1-y^2\right)} = e^{\log\left[\frac{1}{\sqrt{1-y^2}}\right]} = \frac{1}{\sqrt{1-y^2}}$

Hence, the correct answer is D.