$=\sin 10° \sin 30° \sin 50° \sin 70°$

$=\frac{1}{2}(2\sin 70° \sin 10°) \sin 50° \frac{1}{2}$

$=\frac{1}{4}\{\cos(70° - 10°) - \cos(70° + 10°)\} \sin 50°$

$=\frac{1}{4}\{\cos 60° \sin 50° - \cos 80° \sin 50°\}$

$=\frac{1}{4}\{\frac{1}{2}\sin 50° - \cos 80° \sin 50°\}\}$

$=\frac{1}{8}\{\sin 50° - 2\cos 80° \sin 50°\}$

$=\frac{1}{8}\{\sin 50° - (\sin(80° + 50°) - \sin(80° - 50°)\}$

$=\frac{1}{8}\{\sin 50° - \sin 130° + \sin 30°\}$

$=\frac{1}{8}\{\sin 50° - \sin 130° + \frac{1}{2}\}$

$=\frac{1}{8}\{\sin 50° - \sin(180° - 50°) + \frac{1}{2}\}$

$=\frac{1}{8}\{\sin 50° - \sin 50° + \frac{1}{2}\}$

$=\frac{1}{16}$

=R.H.S

Q. 24. Prove that

$$\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}$$

Answer : L.H.S

$$= \frac{1}{2}(2\sin 80° \sin 20°) \sin 40° \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4}\{\cos(80° - 20°) - \cos(80° + 20°)\} \sin 40°$$

$$= \frac{\sqrt{3}}{4}\{\cos 60° \sin 40° - \cos 100° \sin 40°\}$$

$$= \frac{\sqrt{3}}{4}\{\frac{1}{2}\sin 40° - \cos 100° \sin 40°\}\}$$

$$= \frac{\sqrt{3}}{8}\{\sin 40° - 2\cos 100° \sin 40°\}$$

$$= \frac{\sqrt{3}}{8}\{\sin 40° - (\sin(100° + 40°) - \sin(100° - 40°)\}$$

$$= \frac{\sqrt{3}}{8}\{\sin 40° - \sin 140° + \sin 60°\}$$

$$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin 140° + \frac{\sqrt{3}}{2}\}$$

$$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin(180° - 40°) + \frac{\sqrt{3}}{2}\}$$

$$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin 40° + \frac{\sqrt{3}}{2}\}$$

$$=\frac{3}{16}$$

=R.H.S

Q. 25. Prove that

$$\cos 10^0 \cos 30^0 \cos 50^0 \cos 70^0 = \frac{3}{16}$$

Answer : L.H.S

$$=\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ$$

$$=\frac{1}{2}(2\cos 70^\circ \cos 10^\circ)\cos 50^\circ \frac{\sqrt{3}}{2}$$

$$=\frac{\sqrt{3}}{4}\{\cos(70^\circ + 10^\circ) + \cos(70^\circ - 10^\circ)\}\cos 50^\circ$$

$$=\frac{\sqrt{3}}{4}\{\cos 80^\circ \cos 50^\circ + \cos 60^\circ \cos 50^\circ\}$$

$$=\frac{\sqrt{3}}{4}\{cos80^\circ \cos 50^\circ + \frac{1}{2}\cos 50^\circ\}\}$$

$$=\frac{\sqrt{3}}{8}\{2\cos 80^\circ \cos 50^\circ + \cos 50^\circ\}$$

$$=\frac{\sqrt{3}}{8}\{(\cos(80^\circ + 50^\circ) - \cos(80^\circ - 50^\circ) + \cos 50^\circ\}$$

$$=\frac{\sqrt{3}}{8}\{\cos 130^\circ - \cos 30^\circ + \cos 50^\circ\}$$

$$=\frac{\sqrt{3}}{8}\{\cos 130^\circ - \cos 50^\circ + cos30^\circ\}$$

$$=\frac{\sqrt{3}}{8}\{\cos(180^\circ - 50^\circ) - \cos(50^\circ) + \frac{\sqrt{3}}{2}\}$$

$$=\frac{\sqrt{3}}{8}\{\cos 50^\circ - \cos 50^\circ + \frac{\sqrt{3}}{2}\}$$

$$=\frac{3}{16}$$

Q. 26. If $\cos x + \cos y = \frac{1}{3}$ and $\sin x + \sin y = \frac{1}{4}$, prove that $\tan\left(\frac{x+y}{2}\right) = \frac{3}{4}$

Answer :

$\cos x + \cos y = \frac{1}{3}$ --------- i

$\sin x + \sin y = \frac{1}{4}$ ----------ii

dividing ii by I we get,

$$\Rightarrow \frac{\sin x + \sin y}{\cos x + \cos y} = \frac{\frac{1}{4}}{\frac{1}{3}}$$

$$\Rightarrow \frac{\sin x + \sin y}{\cos x + \cos y} = \frac{3}{4}$$

$$\Rightarrow \frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}} = \frac{3}{4}$$

$$\Rightarrow \tan\left(\frac{x+y}{2}\right) = \frac{3}{4}$$

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 27. A. Prove that

$$2\cos 45^0 \cos 15^0 = \frac{\sqrt{3}+1}{2}$$

Answer : L.H.S

$$= 2\cos 45° \cos 15°$$

$$= 2\cos 45° \cos(45° - 30°)$$

$$= 2\frac{1}{\sqrt{2}} (\cos 45° \cos 30° + \sin 45° \sin 30°)$$

$$= \sqrt{2}(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2})$$

$$= \sqrt{2}(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}})$$

$$= \sqrt{2}(\frac{\sqrt{3+1}}{2\sqrt{2}})$$

$$= \frac{\sqrt{3}+1}{\sqrt{2}}$$

Q. 27. B. Prove that

$$2\sin 75^0 \sin 15^0 = \frac{1}{2}$$

Answer : L.H.S

$= 2\sin 75° \sin 15°$

$=2\sin(45° + 30°) \sin(45° - 30°)$

$=\cos(45° - 30° - 45° - 30°) - \cos(45° + 30° + 45° - 30°)$

$=\cos(-60°) - \cos 90°$

$=\cos 60° - 0$

$= \frac{1}{2}$

Q. 27. C. Prove that

$\cos 15^0 - \sin 15 = \frac{1}{\sqrt{2}}$

Answer : L.H.S

$\Rightarrow \cos 15° - \sin 15°$

$\Rightarrow \cos(45° - 30°) - \sin(45° - 30°)$

$\Rightarrow (\cos 45° \cos 30° + \sin 45° \sin 30°) - (\sin 45° \cos 30° - \cos 45° \sin 30°)$

$\Rightarrow (\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}) - (\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2})$

$\Rightarrow \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$$\Rightarrow \frac{2}{2\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}$$

Exercise 15D

Q. 1. A. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$, find the values of

sin 2x

Answer : Given: $\sin x = \frac{\sqrt{5}}{3}$

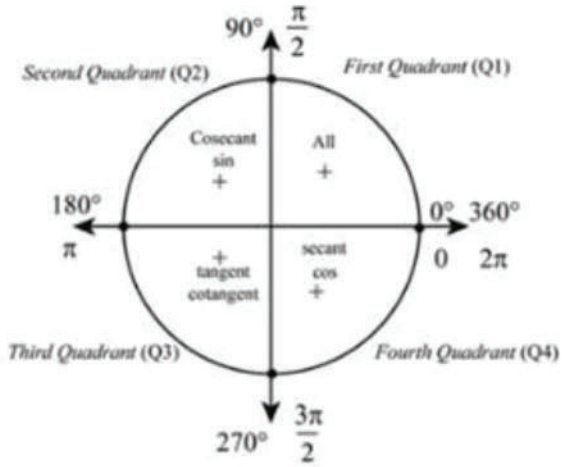
To find: sin2x

We know that,

sin2x = 2 sinx cosx …(i)

Here, we don't have the value of cos x. So, firstly we have to find the value of cosx

We know that,



$\sin^2 x + \cos^2 x = 1$

Putting the values, we get

$$\left(\frac{\sqrt{5}}{3}\right)^2 + \cos^2 x = 1$$

$$\Rightarrow \frac{5}{9} + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \frac{5}{9}$$

$$\Rightarrow \cos^2 x = \frac{9-5}{9}$$

$$\Rightarrow \cos^2 x = \frac{4}{9}$$

$$\Rightarrow \cos x = \sqrt{\frac{4}{9}}$$

$$\Rightarrow \cos x = \pm\frac{2}{3}$$

It is given that $0 < x < \frac{\pi}{2}$

$$\Rightarrow \cos x = \frac{2}{3}$$

Putting the value of sinx and cosx in eq. (i), we get

sin2x = 2sinx cosx

$$\sin 2x = 2 \times \frac{\sqrt{5}}{3} \times \frac{2}{3}$$

$$\therefore \sin 2x = \frac{4\sqrt{5}}{9}$$

Q. 1. B. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$**, find the values of**

cos 2x

Answer :

Given: $\sin x = \frac{\sqrt{5}}{3}$

To find: cos2x

We know that,

$\cos 2x = 1 - 2\sin^2 x$

Putting the value, we get

$$\cos 2x = 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2$$

$$\cos 2x = 1 - 2 \times \frac{5}{9}$$

$$\cos 2x = 1 - \frac{10}{9}$$

$$\cos 2x = \frac{9-10}{9}$$

$$\therefore \cos 2x = -\frac{1}{9}$$

Q. 1. C. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$, **find the values of**

tan 2x

Answer : To find: tan2x

From part (i) and (ii), we have

$$\sin 2x = \frac{4\sqrt{5}}{9}$$

And $\cos 2x = -\frac{1}{9}$

We know that,

$$\tan x = \frac{\sin x}{\cos x}$$

Replacing x by 2x, we get

$$\tan 2x = \frac{\sin 2x}{\cos 2x}$$

Putting the values of sin 2x and cos 2x, we get

$$\tan 2x = \frac{\frac{4\sqrt{5}}{9}}{\frac{-1}{9}}$$

$$\tan 2x = \frac{4\sqrt{5}}{9} \times (-9)$$

∴ tan 2x = -4√5

Q. 2. A. If $\cos x = \frac{-3}{5} \text{ and } \pi < x < \frac{3\pi}{2}$ **, find the values of**

sin 2x

Answer :

Given: $\cos x = \frac{-3}{5}$

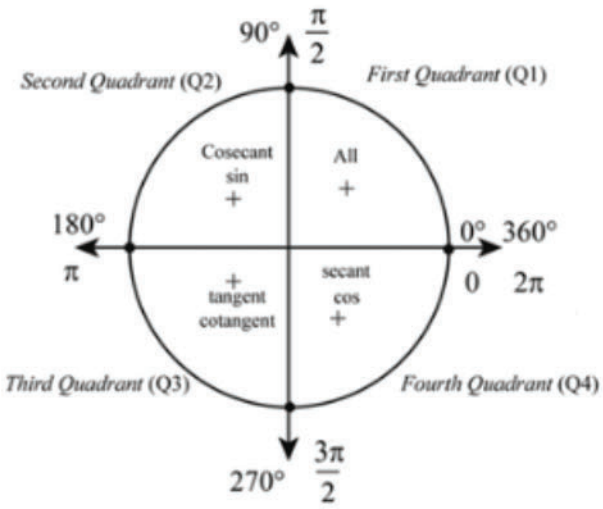
To find: sin2x

We know that,

sin2x = 2 sinx cosx …(i)

Here, we don't have the value of sin x. So, firstly we have to find the value of sinx

We know that,



$\cos^2 x + \sin^2 x = 1$

Putting the values, we get

$$\left(-\frac{3}{5}\right)^2 + \sin^2 x = 1$$

$$\Rightarrow \frac{9}{25} + \sin^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \frac{9}{25}$$

$$\Rightarrow \sin^2 x = \frac{25-9}{25}$$

$$\Rightarrow \sin^2 x = \frac{16}{25}$$

$$\Rightarrow \sin x = \sqrt{\frac{16}{25}}$$

$$\Rightarrow \sin x = \pm\frac{4}{5}$$

It is given that $\pi < x < \frac{3\pi}{2}$

$$\Rightarrow \sin x = -\frac{4}{5}$$

Putting the value of sinx and cosx in eq. (i), we get

sin2x = 2sinx cosx

$$\sin 2x = 2 \times \left(-\frac{4}{5}\right) \times \left(-\frac{3}{5}\right)$$

$$\therefore \sin 2x = \frac{24}{25}$$

Q. 2. B. If $\cos x = \frac{-3}{5} \text{ and } \pi < x < \frac{3\pi}{2}$ **, find the values of**

cos 2x

Answer :

$$\text{Given: } \cos x = \frac{-3}{5}$$

To find: cos2x

We know that,

cos 2x = $2\cos^2 x - 1$

Putting the value, we get

$$\cos 2x = 2\left(-\frac{3}{5}\right)^2 - 1$$

$$\cos 2x = 2 \times \frac{9}{25} - 1$$

$$\cos 2x = \frac{18}{25} - 1$$

$$\cos 2x = \frac{18-25}{25}$$

$$\therefore \cos 2x = -\frac{7}{25}$$

Q. 2. C. If $\cos x = \frac{-3}{5} \text{ and } \pi < x < \frac{3\pi}{2}$ **, find the values of**

tan 2x

Answer : To find: tan2x

From part (i) and (ii), we have

$$\sin 2x = \frac{24}{25}$$

$$\text{and } \cos 2x = -\frac{7}{25}$$

We know that,

$$\tan x = \frac{\sin x}{\cos x}$$

Replacing x by 2x, we get

$$\tan 2x = \frac{\sin 2x}{\cos 2x}$$

Putting the values of sin 2x and cos 2x, we get

$$\tan 2x = \frac{\frac{24}{25}}{\frac{-7}{25}}$$

$$\tan 2x = \frac{24}{25} \times (-\frac{25}{7})$$

$$\therefore \tan 2x = -\frac{24}{7}$$

Q. 3. A. If $\tan x = \frac{-5}{12} \text{ and } \frac{\pi}{2} < x < \pi$ **, find the values of**

sin 2x

Answer :

Given: $\tan x = -\frac{5}{12}$

To find: sin 2x

We know that,

$$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$$

Putting the values, we get

$$\sin 2x = \frac{2\times\left(-\frac{5}{12}\right)}{1+\left(-\frac{5}{12}\right)^2}$$

$$\sin 2x = \frac{\frac{-5}{6}}{1+\frac{25}{144}}$$

$$\sin 2x = \frac{-5}{6\left(\frac{144+25}{144}\right)}$$

$$\sin 2x = \frac{-5\times144}{6\times169}$$

$$\sin 2x = \frac{-5\times24}{169}$$

$$\sin 2x = -\frac{120}{169}$$

Q. 3. B. If $\tan x = \frac{-5}{12}$ and $\frac{\pi}{2} < x < \pi$, find the values of

cos 2x

Answer :

Given: $\tan x = -\frac{5}{12}$

To find: cos 2x

We know that,

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

Putting the values, we get

$$\cos 2x = \frac{1-\left(-\frac{5}{12}\right)^2}{1+\left(-\frac{5}{12}\right)^2}$$

$$\cos 2x = \frac{1-\frac{25}{144}}{1+\frac{25}{144}}$$

$$\cos 2x = \frac{\frac{144-25}{144}}{\left(\frac{144+25}{144}\right)}$$

$$\cos 2x = \frac{\frac{119}{144}}{\frac{169}{144}}$$

$$\cos 2x = \frac{119}{169}$$

Q. 3. C. If $\tan x = \frac{-5}{12}$ and $\frac{\pi}{2} < x < \pi$, find the values of

tan 2x

Answer :

Given: $\tan x = -\frac{5}{12}$

To find: tan 2x

We know that,

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Putting the values, we get

$$\tan 2x = \frac{2\times\left(-\frac{5}{12}\right)}{1-\left(-\frac{5}{12}\right)^2}$$

$$\tan 2x = \frac{-\frac{5}{6}}{1-\frac{25}{144}}$$

$$\tan 2x = \frac{-5}{6\left(\frac{144-25}{144}\right)}$$

$$\tan 2x = \frac{-5\times144}{6\times119}$$

$$\tan 2x = \frac{-5\times24}{119}$$

$$\tan 2x = -\frac{120}{119}$$

Q. 4. A. If Sin X = $\frac{1}{6}$, find the value of sin 3x.

Answer : Sin X = $\frac{1}{6}$

Given: Sin X = $\frac{1}{6}$

To find: sin 3x

We know that,

sin 3x = 3 sinx – $\sin^3 x$

Putting the values, we get

$$\sin 3x = 3 \times \left(\frac{1}{6}\right) - \left(\frac{1}{6}\right)^3$$

$$\sin 3x = \frac{1}{6}\left[3 - \left(\frac{1}{6}\right)^2\right]$$

$$\sin 3x = \frac{1}{6}\left[3 - \frac{1}{36}\right]$$

$$\sin 3x = \frac{1}{6}\left[\frac{108-1}{36}\right]$$

$$\sin 3x = \frac{107}{216}$$

Q. 4. B. If Cos X = $\frac{-1}{2}$, find the value of cos 3x.

Answer : Given: Cos X = $\frac{-1}{2}$

To find: cos 3x

We know that,

cos 3x = $4\cos^3 x - 3\cos x$

Putting the values, we get

$$\cos 3x = 4 \times \left(-\frac{1}{2}\right)^3 - 3 \times \left(-\frac{1}{2}\right)$$

$$\cos 3x = 4 \times \left(-\frac{1}{8}\right) + \frac{3}{2}$$

$$\cos 3x = \left[-\frac{1}{2} + \frac{3}{2}\right]$$

$$\cos 3x = \left[\frac{-1+3}{2}\right]$$

$$\cos 3x = \frac{2}{2}$$

cos 3x = 1

Q. 5. Prove that

$$\frac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x$$

Answer :

To Prove: $\frac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x$

Taking LHS,

$= \frac{\cos 2x}{\cos x - \sin x}$

$= \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$ [$\because \cos 2x = \cos^2 x - \sin^2 x$]

Using, $(a^2 - b^2) = (a - b)(a + b)$

$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x - \sin x)}$

= cos x + sin x

= RHS

∴ LHS = RHS

Q. 6. Prove that

$$\frac{\sin 2x}{1 + \cos 2x} = \tan x$$

Answer : To Prove: $\frac{\sin 2x}{1 + \cos 2x} = \tan x$

Taking LHS,

$= \frac{\sin 2x}{1 + \cos 2x}$

$= \frac{2 \sin x \cos x}{1 + \cos 2x}$ [$\because$ sin 2x = 2 sinx cosx]

$= \frac{2 \sin x \cos x}{2 \cos^2 x}$ [$\because$ 1 + cos 2x = 2 $\cos^2$x]

$= \frac{\sin x}{\cos x}$

$= \tan x \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 7. Prove that

$$\frac{\sin 2x}{1-\cos 2x} = \cot x$$

Answer :

To Prove: $\frac{\sin 2x}{1-\cos 2x} = \tan x$

Taking LHS,

$= \frac{\sin 2x}{1-\cos 2x}$

$= \frac{2\sin x \cos x}{1-\cos 2x}$ [$\because$ sin 2x = 2 sinx cosx]

$= \frac{2\sin x \cos x}{2\sin^2 x}$ [$\because$ 1 – cos 2x = 2 $\sin^2$x]

$= \frac{\cos x}{\sin x}$

$= \cot x \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 8. Prove that

$$\frac{\tan 2x}{1+\sec 2x} = \tan x$$

Answer :

To Prove: $\frac{\tan 2x}{1+\sec 2x} = \tan x$

Taking LHS,

$$= \frac{\frac{\sin 2x}{\cos 2x}}{1+\frac{1}{\cos 2x}} \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \ \& \sec\theta = \frac{1}{\cos\theta}\right]$$

$$= \frac{\sin 2x}{\cos 2x\left(\frac{\cos 2x+1}{\cos 2x}\right)}$$

$$= \frac{\sin 2x}{1+\cos 2x}$$

$$= \frac{2\sin x\cos x}{1+\cos 2x} [\because \sin 2x = 2 \sin x \cos x]$$

$$= \frac{2\sin x\cos x}{2\cos^2 x} [\because 1 + \cos 2x = 2\cos^2 x]$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

= RHS

∴ LHS = RHS

Hence Proved

Q. 9. Prove that

sin 2x(tan x + cot x) = 2

Answer : To Prove: sin 2x(tan x + cot x) = 2

Taking LHS,

sin 2x(tan x + cot x)

We know that,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \ \& \ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \sin 2x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \sin 2x\left(\frac{\sin x(\sin x)+\cos x(\cos x)}{\cos x \sin x}\right)$$

$$= \sin 2x\left(\frac{\sin^2 x+\cos^2 x}{\cos x \sin x}\right)$$

We know that,

sin 2x = 2 sinx cosx

$$= 2\sin x\cos x\left(\frac{\sin^2 x+\cos^2 x}{\cos x \sin x}\right)$$

= 2(sin^{2}x + cos^{2}x)

= 2 × 1 [∵ cos^2 θ + sin^2 θ = 1]

= 2

= RHS

∴ LHS = RHS

Hence Proved

Q. 10. Prove that

cosec 2x + cot 2x = cot x

Answer : To Prove: cosec 2x + cot 2x = cot x

Taking LHS,

= cosec 2x + cot 2x …(i)

We know that,

$$\text{cosec}\,x = \frac{1}{\sin x} \;\&\; \cot x = \frac{\cos x}{\sin x}$$

Replacing x by 2x, we get

$$\text{cosec}\,2x = \frac{1}{\sin 2x} \;\&\; \cot 2x = \frac{\cos 2x}{\sin 2x}$$

So, eq. (i) becomes

$$= \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$$

$$= \frac{1+\cos 2x}{\sin 2x}$$

$$= \frac{2\cos^2 x}{\sin 2x}$$ [∵ 1 + cos 2x = 2 cos^{2}x]

$$= \frac{2\cos^2 x}{2\sin x \cos x}$$ [∵ sin 2x = 2 sinx cosx]

$$= \frac{\cos x}{\sin x}$$

$$= \cot x \left[\because \cot x = \frac{\cos x}{\sin x}\right]$$

= RHS

Hence Proved

Q. 11. Prove that

cos 2x + 2sin^{2}x = 1

Answer :

To Prove: cos 2x + 2sin^{2}x = 1

Taking LHS,

$= \cos 2x + 2\sin^2 x$

$= (2\cos^2 x - 1) + 2\sin^2 x$ [$\because 1 + \cos 2x = 2\cos^2 x$]

$= 2(\cos^2 x + \sin^2 x) - 1$

$= 2(1) - 1$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

$= 2 - 1$

$= 1$

$= RHS$

$\therefore LHS = RHS$

Hence Proved

Q. 12. Prove that

$(\sin x - \cos x)^2 = 1 - \sin 2x$

Answer : To Prove: $(\sin x - \cos x)^2 = 1 - \sin 2x$

Taking LHS,

$= (\sin x - \cos x)^2$

Using,

$(a - b)^2 = (a^2 + b^2 - 2ab)$

$= \sin^2 x + \cos^2 x - 2\sin x \cos x$

$= (\sin^2 x + \cos^2 x) - 2\sin x \cos x$

$= 1 - 2\sin x \cos x$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

$= 1 - \sin 2x$ [$\because \sin 2x = 2 \sin x \cos x$]

$= RHS$

$\therefore LHS = RHS$

Hence Proved

Q. 13. Prove that

cot x – 2cot 2x = tan x

Answer : To Prove: cot x – 2cot 2x = tan x

Taking LHS,

= cot x – 2cot 2x …(i)

We know that,

$$\cot x = \frac{\cos x}{\sin x}$$

Replacing x by 2x, we get

$$\cot 2x = \frac{\cos 2x}{\sin 2x}$$

So, eq. (i) becomes

$$= \frac{\cos x}{\sin x} - 2\left(\frac{\cos 2x}{\sin 2x}\right)$$

$$= \frac{\cos x}{\sin x} - 2\left(\frac{\cos 2x}{2\sin x \cos x}\right)$$ [∵ sin 2x = 2 sinx cosx]

$$= \frac{\cos x}{\sin x} - \left(\frac{\cos 2x}{\sin x \cos x}\right)$$

$$= \frac{\cos x(\cos x) - \cos 2x}{\sin x \cos x}$$

$$= \frac{\cos^2 x - \cos 2x}{\sin x \cos x}$$

$$= \frac{\cos^2 x - [2\cos^2 x - 1]}{\sin x \cos x}$$ [∵ 1 + cos 2x = 2 $\cos^2$x]

$$= \frac{\cos^2 x - 2\cos^2 x + 1}{\sin x \cos x}$$

$$= \frac{-\cos^2 x + 1}{\sin x \cos x}$$

$= \frac{1-\cos^2 x}{\sin x \cos x}$

$= \frac{\cos^2 x+\sin^2 x-\cos^2 x}{\sin x \cos x}$ [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

$= \frac{\sin^2 x}{\sin x \cos x}$

$= \frac{\sin x}{\cos x}$

$= \tan x \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 14. Prove that

$$(\cos^4 x + \sin^4 x) = \frac{1}{2}(2 - \sin^2 2x)$$

Answer :

To Prove: $\cos^4 x + \sin^4 x = \frac{1}{2}(2 - \sin^2 2x)$

Taking LHS,

= $\cos^4 x + \sin^4 x$

Adding and subtracting $2\sin^2 x \cos^2 x$, we get

= $\cos^4 x + \sin^4 x + 2\sin^2 x \cos^2 x - 2\sin^2 x \cos^2 x$

We know that,

$a^2 + b^2 + 2ab = (a + b)^2$

= $(\cos^2 x + \sin^2 x) - 2\sin^2 x \cos^2 x$

= (1) – $2\sin^2 x \cos^2 x$ [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

= 1 – $2\sin^2 x \cos^2 x$

Multiply and divide by 2, we get

$$= \frac{1}{2}[2 \times (1 - 2\sin^2 x \cos^2 x)]$$

$$= \frac{1}{2}[2 - 4\sin^2 x \cos^2 x]$$

$$= \frac{1}{2}[2 - (2\sin x \cos x)^2]$$

$$= \frac{1}{2}[2 - (\sin 2x)^2]$$ [∵ sin 2x = 2 sinx cosx]

$$= \frac{1}{2}(2 - \sin^2 2x)$$

= RHS

∴ LHS = RHS

Hence Proved

Q. 15. Prove that

$$\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = \frac{1}{2}(2 + \sin 2x)$$

Answer :

To Prove: $\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = \frac{1}{2}(2 + \sin 2x)$

Taking LHS,

$$= \frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} \ldots (i)$$

We know that,

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So, $\cos^3 x - \sin^3 x = (\cos x - \sin x)(\cos^2 x + \cos x \sin x + \sin^2 x)$

So, eq. (i) becomes

$$= \frac{(\cos x - \sin x)(\cos^2 x + \cos x \sin x + \sin^2 x)}{\cos x - \sin x}$$

$= \cos^2 x + \cos x \sin x + \sin^2 x$

$= (\cos^2 x + \sin^2 x) + \cos x \sin x$

$= (1) + \cos x \sin x$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

$= 1 + \cos x \sin x$

Multiply and Divide by 2, we get

$$= \frac{1}{2}[2 \times (1 + \cos x \sin x)]$$

$$= \frac{1}{2}[2 + 2 \sin x \cos x]$$

$$= \frac{1}{2}[2 + \sin 2x]$$ [$\because \sin 2x = 2 \sin x \cos x$]

= RHS

$\therefore$ LHS = RHS

Hence Proved

Q. 16. Prove that

$$\frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$$

Answer :

To prove: $\frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$

Taking LHS,

$= \frac{1-\cos 2x+\sin x}{\sin 2x+\cos x}$

$= \frac{(1-\cos 2x)+\sin x}{\sin 2x+\cos x}$

We know that,

1 – cos 2x = 2 $\sin^2 x$ & sin 2x = 2 sinx cosx

$= \frac{2\sin^2 x+\sin x}{2\sin x\cos x+\cos x}$

Taking sinx common from the numerator and cosx from the denominator

$= \frac{\sin x(2\sin x+1)}{\cos x(2\sin x+1)}$

$= \frac{\sin x}{\cos x}$

$= \tan x \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 17. Prove that

$$\cos x\cos 2x\cos 4x\cos 8x = \frac{\sin 16x}{16\sin x}$$

Answer :

To Prove: $\cos x\cos 2x\cos 4x\cos 8x = \frac{\sin 16x}{16\sin x}$

Taking LHS,

= cosx cos2x cos4x cos8x

Multiply and divide by 2sinx, we get

$$= \frac{1}{2 \sin x} [2 \sin x \cos x \cos 2x \cos 4x \cos 8x]$$

$$= \frac{1}{2 \sin x} [(2 \sin x \cos x) \cos 2x \cos 4x \cos 8x]$$

$$= \frac{1}{2 \sin x} [\sin 2x \cos 2x \cos 4x \cos 8x]$$ [∵ sin 2x = 2 sinx cosx]

Multiply and divide by 2, we get

$$= \frac{1}{2 \times 2 \sin x} [(2 \sin 2x \cos 2x) \cos 4x \cos 8x]$$

We know that,

sin 2x = 2 sinx cosx

Replacing x by 2x, we get

sin 2(2x) = 2 sin(2x) cos(2x)

or sin 4x = 2 sin 2x cos 2x

$$= \frac{1}{4 \sin x} [\sin 4x \cos 4x \cos 8x]$$

Multiply and divide by 2, we get

$$= \frac{1}{2 \times 4 \sin x} [2 \sin 4x \cos 4x \cos 8x]$$

We know that,

sin 2x = 2 sinx cosx

Replacing x by 4x, we get

sin 2(4x) = 2 sin(4x) cos(4x)

or sin 8x = 2 sin 4x cos 4x

$$= \frac{1}{8 \sin x} [\sin 8x \cos 8x]$$

Multiply and divide by 2, we get

$= \frac{1}{2 \times 8 \sin x} [2 \sin 8x \cos 8x]$

We know that,

sin 2x = 2 sinx cosx

Replacing x by 8x, we get

sin 2(8x) = 2 sin(8x) cos(8x)

or sin 16x = 2 sin 8x cos 8x

$= \frac{1}{16 \sin x} [\sin 16x]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 18. A. Prove that

$$2 \sin 22\frac{1^0}{2} \cos 22\frac{1^0}{2} = \frac{1}{\sqrt{2}}$$

Answer :

To Prove: $2 \sin 22\frac{1}{2}^\circ \cos 22\frac{1}{2}^\circ = \frac{1}{\sqrt{2}}$

Taking LHS,

$= 2 \sin 22\frac{1}{2}^\circ \cos 22\frac{1}{2}^\circ$ ...(i)

We know that,

2sinx cosx = sin 2x

Here, $x = 22\frac{1}{2} = \frac{45}{2}$

So, eq. (i) become

$= \sin 2\left(\frac{45}{2}\right)$

= sin 45°

$= \frac{1}{\sqrt{2}} \left[\because \sin(45^\circ) = \frac{1}{\sqrt{2}}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 18. B. Prove that

$$2\cos^2 15^0 - 1 = \frac{\sqrt{3}}{2}$$

Answer :

To Prove: $2\cos^2 15° - 1 = \frac{\sqrt{3}}{2}$

Taking LHS,

= 2 $\cos^2$ 15° - 1 …(i)

We know that,

1 + cos 2x = 2 $\cos^2$x

Here, x = 15°

So, eq. (i) become

= [1 + cos 2(15°)] – 1

= 1 + cos 30° - 1

$= \cos 30° \left[\because \cos(30°) = \frac{\sqrt{3}}{2}\right]$

$= \frac{\sqrt{3}}{2}$

= RHS

∴ LHS = RHS

Hence Proved

Q. 18. C. Prove that

$$8\cos^3 20^0 - 6\cos 20^0 = 1$$

Answer : To Prove: 8 $\cos^3$ 20° - 6 cos 20° = 1

Taking LHS,

= 8 $\cos^3$ 20° - 6 cos 20°

Taking 2 common, we get

= 2(4 $\cos^3$ 20° - 3 cos 20°) …(i)

We know that,

cos 3x = 4$\cos^3$x – 3 cosx

Here, x = 20°

So, eq. (i) becomes

= 2[cos 3(20°)]

= 2[cos 60°]

$= 2 \times \frac{1}{2} \left[\because \cos(60°) = \frac{1}{2}\right]$

= 1

= RHS

∴ LHS = RHS

Hence Proved

Q. 18. D. Prove that

$$3\sin 40° - \sin^3 40° = \frac{\sqrt{3}}{2}$$

Answer :

To prove: $3 \sin 40° - \sin^3 40° = \frac{\sqrt{3}}{2}$

Taking LHS,

= 3 sin 40° - $\sin^3$ 40° …(i)

We know that,

sin 3x = 3 sinx – $\sin^3$x

Here, x = 40°

So, eq. (i) becomes

= sin 3(40°)

= sin 120°

= sin (180° - 60°)

= sin 60° [∵sin (180° - θ) = sin θ]

$$= \frac{\sqrt{3}}{2} \left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$$

= RHS

∴ LHS = RHS

Hence Proved

Q. 19. A. Prove that

$$\sin^2 24^0 - \sin^2 6^0 = \frac{(\sqrt{5}-1)}{8}$$

Answer :

To Prove: $\sin^2 24° - \sin^2 6° = \frac{\sqrt{5}-1}{8}$

Taking LHS,

= $\sin^2 24° - \sin^2 6°$

We know that,

$\sin^2 A - \sin^2 B = \sin(A + B) \sin(A - B)$

= $\sin(24° + 6°) \sin(24° - 6°)$

= $\sin 30° \sin 18°$ …(i)

Now, we will find the value of sin 18°

Let x = 18°

so, 5x = 90°

Now, we can write

2x + 3x = 90°

so 2x = 90° - 3x

Now taking sin both the sides, we get

sin2x = sin(90° - 3x)

sin2x = cos3x [as we know, sin(90°- 3x) = Cos3x]

We know that,

sin2x = 2sinxcosx

$Cos3x = 4\cos^3 x - 3\cos x$

$2\sin x\cos x = 4\cos^3 x - 3\cos x$

$\Rightarrow 2\sin x\cos x - 4\cos^3 x + 3\cos x = 0$

$\Rightarrow \cos x (2\sin x - 4\cos^2 x + 3) = 0$