Exercise 7(B)

1. Solve for x:

(i) $2^{2x+1} = 8$

(ii) $2^{5x-1} = 4 \times 2^{3x+1}$

(iii) $3^{4x+1} = (27)^{x+1}$

(iv) $(49)^{x+4} = 7^2 \times (343)^{x+1}$

Solution:

(i) We have, $2^{2x+1} = 8$

$\Rightarrow 2^{2x+1} = 2^3$

Now, if the bases are equal, then the powers must be equal

On comparing the exponents, we get

$2x + 1 = 3$

$2x = 3 – 1$

$2x = 2$

$x = 2/2$

$x = 1$

(ii) We have, $2^{5x-1} = 4 \times 2^{3x+1}$

$\Rightarrow 2^{5x-1} = 2^2 \times 2^{3x+1}$

$2^{5x-1} = 2^{(3x+1)+2}$

$2^{5x-1} = 2^{3x+3}$

Now, if the bases are equal, then the powers must be equal

On comparing the exponents, we get

$5x – 1 = 3x + 3$

$5x – 3x = 3 + 1$

$2x = 4$

$x = 4/2$

$x = 2$

(iii) We have, $3^{4x+1} = (27)^{x+1}$

$3^{4x+1} = (3^3)^{x+1}$

$3^{4x+1} = (3)^{3x+3}$

Now, if the bases are equal, then the powers must be equal

On comparing the exponents, we get

$4x + 1 = 3x + 3$

$4x – 3x = 3 – 1$

$x = 2$

(iv) We have, $(49)^{x+4} = 7^2 \times (343)^{x+1}$

$(7 \times 7)^{x+4} = 7^2 \times (7 \times 7 \times 7)^{x+1}$

$(7^2)^{x+4} = 7^2 \times (7^3)^{x+1}$

$(7)^{2x+8} = (7)^{3x+3+2}$

$(7)^{2x+8} = (7)^{3x+5}$

Now, if the bases are equal, then the powers must be equal

On comparing the exponents, we get

$2x + 8 = 3x + 5$
$3x - 2x = 8 - 5$
$x = 3$

2. Find x, if:
(i) $4^{2x} = 1/32$
(ii) $\sqrt{(2^{x+3})} = 16$
(iii) $[\sqrt{(3/5)}]^{x+1} = 125/27$
(iv) $[\sqrt[3]{(2/3)}]^{x-1} = 27/8$
Solution:

(i) We have, $4^{2x} = 1/32$
$(2 \times 2)^{2x} = 1/(2 \times 2 \times 2 \times 2 \times 2)$
$(2^2)^{2x} = 1/2^5$
$2^{4x} = 2^{-5}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$4x = -5$
$x = -5/4$

(ii) We have, $\sqrt{(2^{x+3})} = 16$
$(2^{x+3})^{1/2} = (2 \times 2 \times 2 \times 2)$
$2^{(x+3)/2} = 2^4$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$(x + 3)/2 = 4$
$x + 3 = 8$
$x = 8 - 3$
$x = 5$

(iii) We have, $[\sqrt{(3/5)}]^{x+1} = 125/27$
$[(3/5)^{1/2}]^{x+1} = (5 \times 5 \times 5)/(3 \times 3 \times 3)$
$(3/5)^{(x+1)/2} = 5^3/3^3$
$(3/5)^{(x+1)/2} = (5/3)^3$
$(3/5)^{(x+1)/2} = (3/5)^{-3}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$(x + 1)/2 = -3$
$x + 1 = -6$
$x = -6 - 1$
$x = -7$

(iv) We have, $[\sqrt[3]{(2/3)}]^{x-1} = 27/8$
$[(2/3)^{1/3}]^{x-1} = (3 \times 3 \times 3)/(2 \times 2 \times 2)$
$(2/3)^{(x-1)/3} = (3/2)^3$
$(2/3)^{(x-1)/3} = (2/3)^{-3}$

Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$(x - 1)/3 = -3$
$x – 1 = -9$
$x = -9 + 1$
$x = -8$

3. Solve:
(i) $4^{x-2} - 2^{x+1} = 0$
(ii) $3^{x^2} : 3^x = 9 : 1$
Solution:

(i) We have,
$4^{x-2} - 2^{x+1} = 0$
$(2^2)^{x-2} – 2^{x+1} = 0$
$2^{2x-4} – 2^{x+1} = 0$
$2^{2x-4} = 2^{x+1}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$2x – 4 = x + 1$
$2x – x = 4 + 1$
$x = 5$

(ii) We have,
$3^{x^2} : 3^x = 9 : 1$
$\frac{3^{x^2}}{3^x} = \frac{9}{1}$
$\Rightarrow 3^{x^2} = 9 \times 3^x$
$\Rightarrow 3^{x^2} = 3^2 \times 3^x$
$\Rightarrow 3^{x^2} = 3^{x+2}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$x^2 = x+ 2$
$x^2 – x – 2 = 0$
On factorization, we get
$x^2 – 2x + x – 2 = 0$
$x(x – 2 + 1(x – 2) = 0$
$(x + 1)(x - 2) = 0$
So, either $(x + 1) = 0$ or $(x - 2) = 0$
Thus, $x = -1$ or 2

4. Solve:
(i) $8 \times 2^{2x} + 4 \times 2^{x+1} = 1 + 2^x$

(ii) $2^{2x} + 2^{x+2} - 4 \times 2^3 = 0$
(iii) $(\sqrt{3})^{x-3} = (\sqrt[4]{3})^{x+1}$
Solution:

(i) We have, $8 \times 2^{2x} + 4 \times 2^{x+1} = 1 + 2^x$
$8 \times (2^x)^2 + 4 \times (2^x) \times 2^1 = 1 + 2^x$
Let us substitute $2^x = t$
Then,
$8 \times t^2 + 4 \times t \times 2 = 1 + t$
$8t^2 + 8t = 1 + t$
$8t^2 + 8t - t - 1 = 0$
$8t^2 + 7t - 1 = 0$
$8t^2 + 8t - t - 1 = 0$
$8t(t + 1) - 1(t + 1) = 0$
$(8t - 1)(t + 1) = 0$
So, either $8t - 1 = 0$ or $t + 1 = 0$
Thus, $t = 1/8$ or -1
Now, we have
$2^x = t$
So,
$2^x = 1/8$ or $2^x = -1$
The equation, $2^x = -1$ is not possible
Hence, for $2^x = 1/8$
$2^x = 1/(2 \times 2 \times 2)$
$2^x = 1/2^3$
$2^x = 2^{-3}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$x = -3$

(ii) We have,
$2^{2x} + 2^{x+2} - 4 \times 2^3 = 0$
$2^{2x} + 2^{x+2} - 2^2 \times 2^3 = 0$
$(2^x)^2 + 2^x.2^2 - 2^{3+2} = 0$
$(2^x)^2 + 2^x.2^2 - 2^5 = 0$
Now, let's assume $2^x = t$
So, the above equation becomes
$(t)^2 + t.2^2 - 2^5 = 0$
$t^2 + 4t - 32 = 0$
On factorization, we have
$t^2 + 8t - 4t - 32 = 0$
$t(t + 8) - 4(t + 8) = 0$
$(t - 4)(t + 8) = 0$
So, either $(t - 4) = 0$ or $(t + 8) = 0$
Thus, $t = 4$ or -8
Now, we have $t = 2^x$

So,
$2^x = 4$ or $2^x = -8$
The equation, $2^x = -8$ is not possible
Hence, for
$2^x = 4$
$2^x = 2^2$
On comparing the exponents, we get
$x = 2$

(iii) $(\sqrt{3})^{x-3} = (\sqrt[4]{3})^{x+1}$
$(3^{1/2})^{x-3} = (3^{1/4})^{x+1}$
$3^{(x-3)/2} = 3^{(x+1)/4}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$(x - 3)/2 = (x + 1)/4$
$2(x - 3) = (x + 1)$
$2x – 6 = x + 1$
$2x – x = 6 + 1$
$x = 7$

5. Find the values of m and n if:
$4^{2m} = (\sqrt[3]{16})^{-6/n} = (\sqrt{8})^2$
Solution:

We have, $4^{2m} = (\sqrt[3]{16})^{-6/n} = (\sqrt{8})^2$
Now, considering
$4^{2m} = (\sqrt{8})^2$
$(2^2)^{2m} = (8^{1/2})^2$
$2^{4m} = 8^{1/2 \times 2}$
$2^{4m} = 8$
$\Rightarrow 2^{4m} = 2^3$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$4m = 3$
$m = 3/4$

Now, from the given considering
$(\sqrt[3]{16})^{-6/n} = (\sqrt{8})^2$
$(16^{1/3})^{-6/n} = (8^{1/2})^2$
$(16)^{1/3 \times -6/n} = 8^{1/2 \times 2}$
$(16)^{-2/n} = 8$
$(2 \times 2 \times 2 \times 2)^{-2/n} = (2 \times 2 \times 2)$
$(2)^{4 \times -2/n} = 2^3$
$(2)^{-8/n} = 2^3$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get

$-8/n = 3$
$n = -8/3$

Therefore, the value of m and n are ¾ and -8/3

6. Solve x and y if:
$(\sqrt{32})^x \div 2^{y+1} = 1$ and $8^y - 16^{4 - x/2} = 0$
Solution:

Consider the equation, $(\sqrt{32})^x \div 2^{y+1} = 1$
$(\sqrt{(2 \times 2 \times 2 \times 2 \times 2)})^x \div 2^{y+1} = 1$
$(\sqrt{2^5})^x \div 2^{y+1} = 1$
$(2^5)^{1/2 \times x} \div 2^{y+1} = 1$
$2^{5x/2} \div 2^{y+1} = 1$
$(2^{5x/2})/(2^{y+1}) = 1$
$2^{5x/2 - (y+1)} = 2^0$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$5x/2 - (y+1) = 0$
$5x/2 - y - 1 = 0$
$5x - 2y - 2 = 0$ … (i)

Next, let's consider $8^y - 16^{4 - x/2} = 0$
$(2 \times 2 \times 2)^y - (2 \times 2 \times 2 \times 2)^{4 - x/2} = 0$
$(2^3)^y - (2^4)^{4 - x/2} = 0$
$2^{3y} - 2^{16 - 2x} = 0$
$2^{3y} = 2^{16 - 2x}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$3y = 16 - 2x$
$2x + 3y - 16 = 0$ … (ii)
On solving equations (i) and (ii),
By manipulating by (i) × 3 + (ii) × 2, we have
$15x - 6y - 6 = 0$
$4x + 6y - 32 = 0$
– – – – – – – – – – – – – –
$19x - 38 = 0$
$x = 38/19$
$x = 2$
Now, substituting the value of x in (i)
$5(2) - 2y - 2 = 0$
$10 - 2y - 2 = 0$
$8 = 2y$
$y = 8/2$
$y = 4$

Therefore, the values of x and y are 2 and 4 respectively

7. Prove that:
(i) $(x^a/x^b)^{a+b-c}.\ (x^b/x^c)^{b+c-a}.\ (x^c/x^a)^{c+a-b} = 1$
(ii) $x^{a(b-c)}/x^{b(a-c)} \div (x^b/x^a)^c = 1$
Solution:

(i) Taking L.H.S, we have
$(x^a/x^b)^{a+b-c}.\ (x^b/x^c)^{b+c-a}.\ (x^c/x^a)^{c+a-b}$
$= (x^a/x^b)^{a+b-c}.\ (x^b/x^c)^{b+c-a}.\ (x^c/x^a)^{c+a-b}$
$= x^{(a-b)(a+b-c)}.\ x^{(b-c)(b+c-a)}.\ x^{(c-a)(c+a-b)}$
$= x^{a^2+ab-ac-ab-b^2+bc} \times x^{b^2+bc-ab-cb-c^2+ac} \times x^{c^2+ac-bc-ac-a^2+ab}$
$= x^{a^2-ac-b^2+bc+b^2-ab-c^2+ac+c^2-bc-a^2+ab}$
$= x^0$
$= 1$
= R.H.S

(ii) Taking L.H.S, we have
$x^{a(b-c)}/x^{b(a-c)} \div (x^b/x^a)^c$
$= x^{a(b-c) - b(a-c)} \div x^{c(b-a)}$
$= x^{a(b-c) - b(a-c)}/x^{c(b-a)}$
$= x^{a(b-c) - b(a-c) - c(b-a)}$
$= x^{ab-ac-ba+bc-cb+ac}$
$= x^0$
$= 1$
= R.H.S

8. If $a^x = b$, $b^y = c$ and $c^z = a$, prove that: $xyz = 1$.
Solution:

We have, $a^x = b$, $b^y = c$ and $c^z = a$
Now, considering
$a^x = b$
On raising to the power yz on both sides, we get
$(a^x)^{yz} = (b)^{yz}$
$(a)^{xyz} = (b^y)^z$
$(a)^{xyz} = (c)^z$ [As, $b^y = c$]
$a^{xyz} = a$
$a^{xyz} = a^1$ [As, $c^z = a$]
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
Hence, $xyz = 1$

9. If $a^x = b^y = c^z$ and $b^2 = ac$, prove that: $y = 2az/(x + z)$.
Solution:

Let's assume $a^x = b^y = c^z = k$
So,
$a = k^{1/x}$; $b = k^{1/y}$ and $c = k^{1/z}$
Now,
It's given that $b^2 = ac$
$\Rightarrow (k^{1/y})^2 = (k^{1/x}) \times (k^{1/z})$
$(k^{2/y}) = k^{1/x + 1/z}$
$k^{2/y} = k^{(z+x)/xz}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
2/y = (z + x)/xz
2xz = y(z + x)
Hence,
y = 2xz/(x + z)

10. If $5^{-p} = 4^{-q} = 20^r$, show that: 1/p + 1/q + 1/r = 0.
Solution:

Let's assume $5^{-p} = 4^{-q} = 20^r = k$
Then, as
$5^{-p} = k \Rightarrow 5 = k^{-1/p}$
$4^{-q} = k \Rightarrow 4 = k^{-1/q}$
$20^r = k \Rightarrow 20 = k^{1/r}$
Now, we know
5 x 4 = 20
$(k^{-1/p}) \times (k^{-1/q}) = k^{1/r}$
$k^{-1/p - 1/q} = k^{1/r}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
-1/p – 1/q = 1/r
Hence,
1/p + 1/q + 1/r = 0

11. If m ≠ n and $(m + n)^{-1} (m^{-1} + n^{-1}) = m^x n^y$, show that: x + y + 2 = 0
Solution:

Given equation,
$(m + n)^{-1} (m^{-1} + n^{-1}) = m^x n^y$
$1/(m + n) \times (1/m + 1/n) = m^x n^y$
$1/(m + n) \times (m + n)/mn = m^x n^y$
$1/mn = m^x n^y$
$m^{-1} n^{-1} = m^x n^y$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
x = -1 and y = -1

Substituting the values of x and y in the equation $x + y + 2 = 0$, we have
$(-1) + (-1) + 2 = 0$
$0 = 0$
L.H.S = R.H.S
Therefore, $x + y + 2 = 0$

12. If $5^{x+1} = 25^{x-2}$, find the value of $3^{x-3} \times 2^{3-x}$
Solution:

We have, $5^{x+1} = 25^{x-2}$
$5^{x+1} = (5^2)^{x-2}$
$5^{x+1} = 5^{2x-4}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$x + 1 = 2x - 4$
$2x - x = 4 + 1$
$x = 5$
Hence, the value of
$3^{x-3} \times 2^{3-x} = 3^{5-3} \times 2^{3-5}$
$= 3^2 \times 2^{-2}$
$= 9 \times 1/4$
$= 9/4$

13. If $4^{x+3} = 112 + 8 \times 4^x$, find the value of $(18x)^{3x}$.
Solution:

We have,
$4^{x+3} = 112 + 8 \times 4^x$
$4^x . 4^3 = 112 + 8 \times 4^x$
Let's assume $4^x = t$
Then,
$t . 4^3 = 112 + 8 \times t$
$64t = 112 + 8t$
$64t - 8t = 112$
$56t = 112$
$t = 112/56$
$t = 2$
But we have taken $4^x = t$
So, $4^x = 2$
$(2^2)^x = 2^1$
$2^{2x} = 2^1$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$2x = 1$
$x = 1/2$

Now, the value of $(18x)^{3x}$ will be
$= (18 \times ½)^{3 \times 1/2}$
$= (9)^{3/2}$
$= (3^2)^{3/2}$
$= 3^3$
$= 27$

14. Solve for x:
(i) $4^{x-1} \times (0.5)^{3 - 2x} = (1/8)^{-x}$
Solution:

We have,
$4^{x-1} \times (0.5)^{3 - 2x} = (1/8)^{-x}$
$(2^2)^{x-1} \times (1/2)^{3 - 2x} = (1/2^3)^{-x}$
$(2)^{2x-1} \times (2)^{-(3 - 2x)} = (2^{-3})^{-x}$
$(2)^{2x-2} \times (2)^{2x-3} = (2)^{3x}$
$2^{(2x-2) + (2x-3)} = (2)^{3x}$
$2^{4x-5} = 2^{3x}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$4x – 5 = 3x$
$4x – 3x = 5$
$x = 5$

(ii) $(a^{3x + 5})^2 \times (a^x)^4 = a^{8x + 12}$
Solution:

We have,
$(a^{3x + 5})^2 \times (a^x)^4 = a^{8x + 12}$
$a^{6x + 10} \times a^{4x} = a^{8x + 12}$
$a^{6x + 10 + 4x} = a^{8x + 12}$
$a^{10x + 10} = a^{8x + 12}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$10x + 10 = 8x + 12$
$10x – 8x = 12 – 10$
$2x = 2$
$x = 1$

(iii) $(81)^{3/4} – (1/32)^{-2/5} + x(1/2)^{-1} \times 2^0 = 27$
Solution:

We have,
$(81)^{3/4} – (1/32)^{-2/5} + x(1/2)^{-1} \times 2^0 = 27$
$(3^4)^{3/4} – (1/2^5)^{-2/5} + x(1/2)^{-1} \times 2^0 = 27$
$(3^4)^{3/4} – (2^{-5})^{-2/5} + x(2^{-1})^{-1} \times 2^0 = 27$

$3^3 – 2^2 + 2x \times 1 = 27$
$27 – 4 + 2x = 27$
$2x + 23 = 27$
$2x = 27 – 23$
$2x = 4$
$x = 4/2$
$x = 2$

(iv) $2^{3x + 3} = 2^{3x + 1} + 48$
Solution:

We have,
$2^{3x + 3} = 2^{3x + 1} + 48$
$2^{3x + 3} - 2^{3x + 1} = 48$
$2^{3x}(2^3 - 2^1) = 48$
$2^{3x}(8 - 2) = 48$
$2^{3x} \times 6 = 48$
$2^{3x} = 48/6$
$2^{3x} = 8$
$2^{3x} = 2^3$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$3x = 3$
$x = 1$

(v) $3(2^x + 1) - 2^{x+2} + 5 = 0$
Solution:

We have,
$3(2^x + 1) - 2^{x+2} + 5 = 0$
$3 \times 2^x + 3 - 2^x.2^2 + 5 = 0$
$2^x(3 - 2^2) + 5 + 3 = 0$
$2^x(3 - 4) + 8 = 0$
$-2^x + 8 = 0$
$2^x = 8$
$2^x = 2^3$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$x = 3$

(vi) $9^{x+2} = 720 + 9^x$
Solution:

We have,
$9^{x+2} = 720 + 9^x$
$9^{x+2} - 9^x = 720$

$9^x (9^2 - 1) = 720$
$9^x (81 - 1) = 720$
$9^x (80) = 720$
$9^x = 9$
$9^x = 9^1$
Therefore, $x = 1$