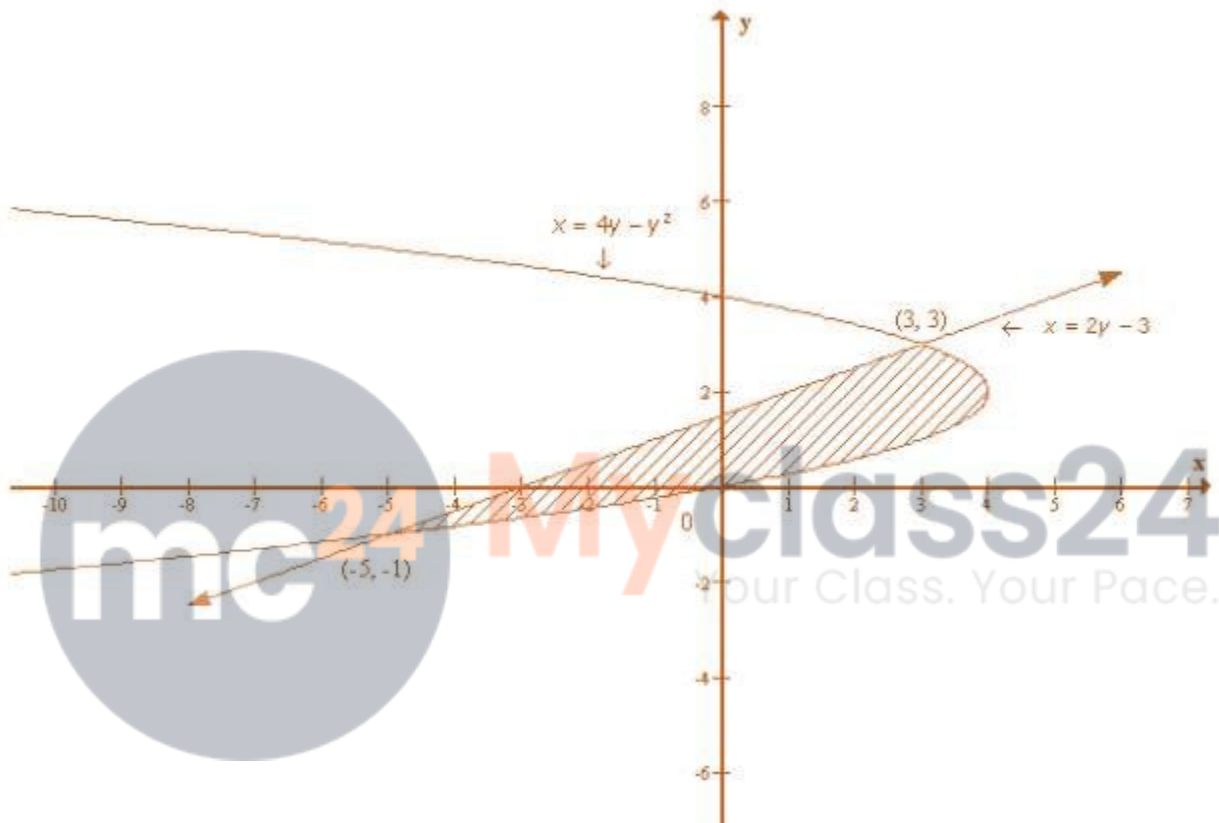


EXERCISE 21.4

Question. 1

Solution:

From the question it is given that, parabola $x = 4y - y^2$ and the line $x = 2y - 3$,
As shown in the figure,



$$x_1 = 4y - y^2$$

$$x_2 = 2y - 3$$

So,

$$2y - 3 = 4y - y^2$$

$$y^2 + 2y - 4y - 3 = 0$$

$$y^2 - 2y - 3 = 0$$

$$y^2 - 3y + y - 3 = 0$$

$$y(y - 3) + 1(y - 3) = 0$$

$$(y - 3)(y + 1) = 0$$

$$y = -1, 3$$

Now, we have to find the area of the bounded region,

$$A = \int_{-1}^3 |x_1 - x_2| dy$$

Now substitute the values of x_1 and x_2 ,

$$= \int_{-1}^3 [(4y - y^2) - (2y - 3)] dy$$

Then,

$$= \int_{-1}^3 (4y - y^2 - 2y + 3) dy$$

Like terms = $4y - 2y = 2y$

$$= \int_{-1}^3 (-y^2 + 2y + 3) dy$$

On integrating we get,

$$= \left[-\frac{y^3}{3} + \frac{2y^2}{2} + 3y \right]_{-1}^3$$

Applying limits, we get,

$$\begin{aligned} &= [- (3^3/3) + 2(3^2)/2 + 3(3)] - [- ((-1)^3/3) + 2(-1^2)/2 + 3(-1)] \\ &= [- 3^2 + 3^2 + 9] - [(1/3) + 1 - 3] \\ &= [9] - [(1/3) - 1 + 3] \\ &= 9 - (1/3) + 2 \\ &= 11 - (1/3) \\ &= (33 - 1)/3 \\ &= 32/3 \text{ square units} \end{aligned}$$

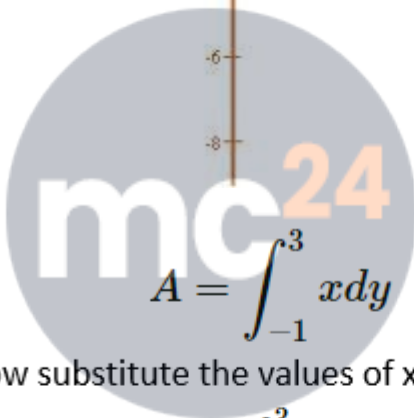
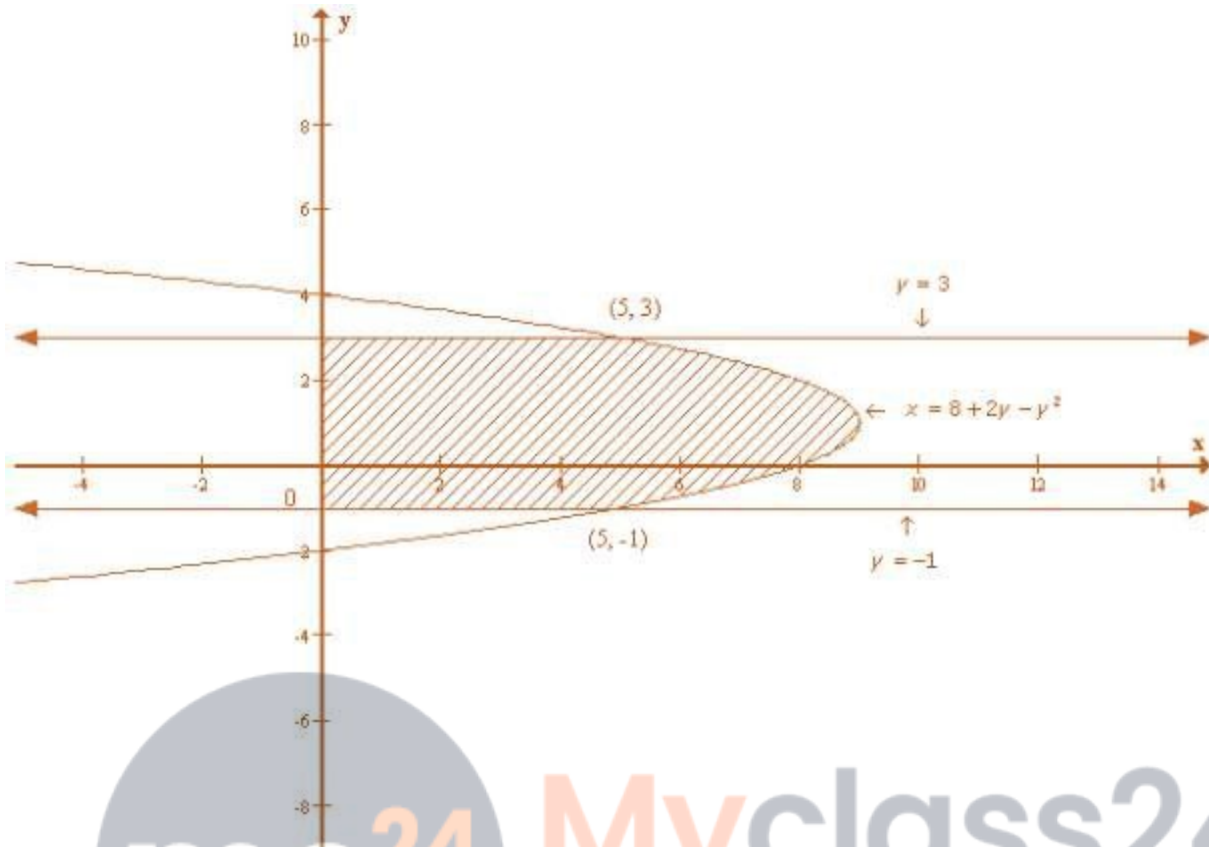
Therefore, the required area is $32/3$ square units.

Question. 2

Solution:

From the question it is given that, parabola $x = 8 + 2y - y^2$ and the line $y = -1, y = 3$

As shown in the figure,



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$$A = \int_{-1}^3 x dy$$

Now substitute the values of x_1 and x_2 ,

$$= \int_{-1}^3 (8 + 2y - y^2) dy$$

On integrating we get,

$$= \left[8y + y^2 - \frac{y^3}{3} \right]_{-1}^3$$

Applying limits, we get,

$$\begin{aligned} &= [8(3) + (3^2) - (3)^3/3] - [8(-1) + (-1)^2 - (-1)^3/3] \\ &= [24 + 9 - 9] - [-8 + 1 + (1/3)] \\ &= [24] - [-7 + 1/3] \\ &= 24 + 7 - (1/3) \\ &= 31 - (1/3) \\ &= (93 - 1)/3 \end{aligned}$$

$$= 92/3 \text{ square units}$$

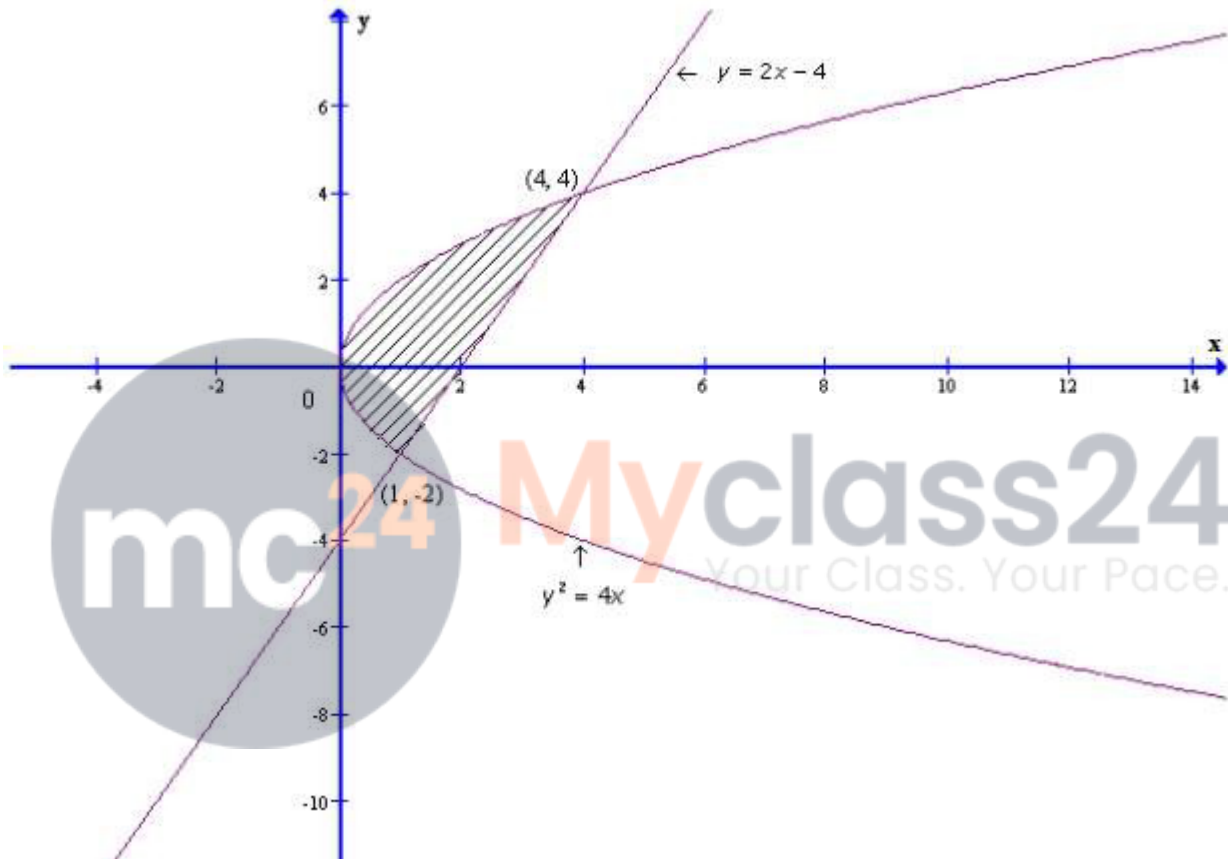
Therefore, the required area is $92/3$ square units.

Question. 3

Solution:

From the question it is given that, parabola $y^2 = 4x$ and the line $y = 2x - 4$,

As shown in the figure,



So,

Now, we have to find the points of intersection,

$$2x - 4 = \sqrt{4x}$$

Squaring on both side,

$$(2x - 4)^2 = (\sqrt{4x})^2$$

$$4x^2 + 16 - 16x = 4x$$

$$4x^2 + 16 - 16x - 4x = 0$$

$$4x^2 + 16 - 20x = 0$$

Dividing both side by 4 we get,

$$x^2 - 5x + 4 = 0$$

$$x^2 - 4x - x + 4 = 0$$

$$x(x - 4) - 1(x - 4) = 0$$

$$(x - 4)(x - 1) = 0$$

$$x = 4, 1$$

$$A = \int_{-2}^4 (x_1 - x_2) dy.$$

Given, $y = 2x - 4$

$$x_1 = (y + 4)/2$$

$$y^2 = 4x$$

$$x_2 = y^2/4$$

Now substitute the values of x_1 and x_2 ,

$$= \int_{-2}^4 \left[\left(\frac{y + 4}{2} \right) - \left(\frac{y^2}{4} \right) \right] dy$$

On integrating we get,

$$= \left[\frac{y^2}{4} + 2y - \frac{y^3}{12} \right]_{-2}^4$$

Applying limits, we get,

$$= \left[\frac{4^2}{4} + 2(4) - \frac{4^3}{12} \right] - \left[\frac{(-2)^2}{4} + 2(-2) - \frac{(-2)^3}{12} \right]$$

$$= [4 + 8 - (64/12)] - [1 - 4 + (8/12)]$$

$$= [12 - (16/3)] - [-3 + (2/3)]$$

$$= 12 - (16/3) + 3 - (2/3)$$

$$= 15 - 18/3$$

$$= 15 - 6$$

$$= 9 \text{ square units}$$

Therefore, the required area is 9 square units.

Question. 4

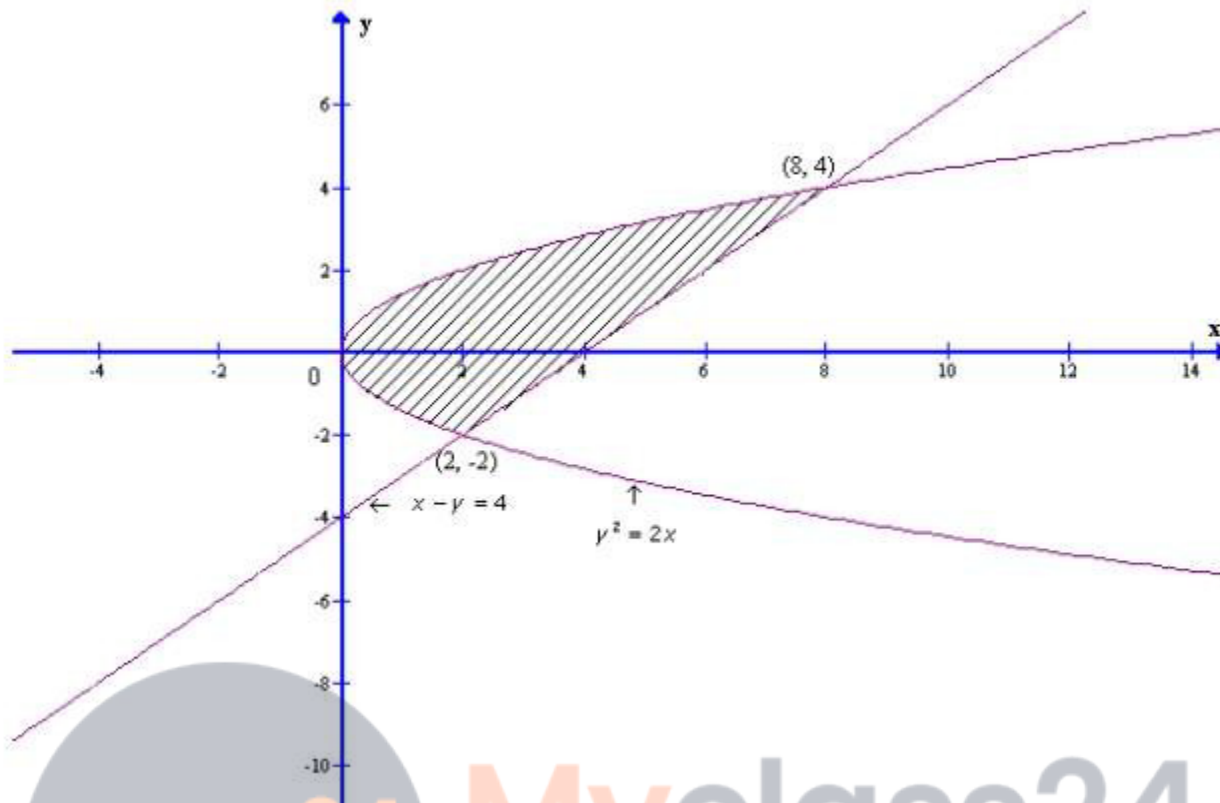
Solution:

From the question it is given that, parabola $y^2 = 2x$ and the line $x - y = 4$,

As shown in the figure,

$$y^2 = 2x \quad \dots \text{ [equation (i)]}$$

$$x = y + 4 \quad \dots \text{ [equation (ii)]}$$



Now, we have to find the points of intersection,

So,
 $y^2 = 2(y + 4)$

$$y^2 = 2y + 8$$

Transposing we get,

$$y^2 - 2y - 8 = 0$$

$$y^2 - 4y + 2y - 8 = 0$$

$$y(y - 4) + 2(y - 4) = 0$$

$$(y - 4)(y + 2) = 0$$

$$y = 4, -2$$

$$A = \int_{-2}^4 x_{\text{line}} dy - \int_{-2}^4 x_{\text{parabola}} dy$$

Now substitute the values,

$$= \int_{-2}^4 (y + 4) dy - \int_{-2}^4 \frac{y^2}{2} dy$$

On integrating we get,

$$= \left| 4y + \frac{y^2}{2} - \frac{1}{6}y^3 \right|_{-2}^4$$

Applying limits, we get,

$$= 4(4 - (-2)) + \frac{1}{2}(4^2 - (-2)^2) - (1/6)(4^3 - (-2)^3)$$

$$= 4(4 + 2) + \frac{1}{2}(16 - 4) - (1/6)(64 + 8)$$

$$= 4(6) + \frac{1}{2}(12) - 1/6(72)$$

$$= 24 + 6 - 12$$

$$= 30 - 12$$

$$= 18 \text{ square units}$$

Therefore, the required area is 18 square units.



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