$$\int \operatorname{cosec}x.\operatorname{cosec}^2 x dx = \operatorname{cosec}x \int .\operatorname{cosec}^2 x dx - \int \left(\frac{d \operatorname{cosec}x}{dx} . \int .\operatorname{cosec}^2 x dx \right) dx$$

$$= \operatorname{cosec}x(-\cot x) - \int (-\operatorname{cosec}x.\cot x)(-\cot x) dx$$

$$= -\operatorname{cosec}x.\cot x - \int \operatorname{cosec}x.\cot^2 x dx$$

We know that $\mathrm{Cot}^2 x = \mathrm{Cosec}^2 x - 1$

$$-\operatorname{cosec}x.\cot x - \int \operatorname{cosec}x(\operatorname{cosec}^2 x - 1) dx$$

$$= -\operatorname{cosec}x.\cot x - \int \operatorname{cosec}^3 x dx + \int \operatorname{cosec}x dx$$

We can write $\int \operatorname{cosec}^3 x dx = I$

$$\Rightarrow \int \operatorname{cosec}^3 x dx - \operatorname{cosec}x.\cot x - \int \operatorname{cosec}^3 x dx + \int \operatorname{cosec}x dx$$

$$\Rightarrow 2\int \operatorname{cosec}^3 x dx = -\operatorname{cosec}x.\cot x + \int \operatorname{cosec}x dx$$

$$\Rightarrow 2\int \operatorname{cosec}^3 x dx = -\operatorname{cosec}x.\cot x + \ln|\sec x + \tan x| + c_1$$

$$\Rightarrow \int \operatorname{cosec}^3 x dx = \frac{-\operatorname{cosec}x.\cot x + \ln|\sec x + \tan x|}{2} + c$$

26. Question

Evaluate the following integrals:

$$\int x \sin^3 x \cos x \, dx$$

Answer

We can write it as $\int x \sin^2 x \sin x \cos x dx$

We also know that 2sinx.cosx = sin2x

$$\int x \sin^2 x \sin x \cos x dx = \frac{1}{2}\int x \sin^2 x \sin 2x dx$$

We also know that $\sin^2 x = \frac{1-\cos 2x}{2}$

$$\frac{1}{2}\int x \sin^2 x \sin 2x dx = \frac{1}{2}\int x.\left(\frac{1-\cos 2x}{2}\right)\sin 2x dx$$

$$= \frac{1}{2}\left[\left(\int \frac{x \sin 2x}{2} dx - \int \frac{x \cos 2x \sin 2x}{2} dx\right)\right]$$

Here Sin4x = 2sin2x.cos2x

$$= \frac{1}{2}\left[\left(\int \frac{x \sin 2x}{2} dx - \frac{1}{4}\int x \sin 4x dx\right)\right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and Sin2x and sin4x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$=\frac{1}{2}\left[\left(\frac{1}{2}\left\{x\int \sin 2xdx - \int\left(\frac{dx}{dx}.\int \sin 2xdx\right)dx\right\}\right)-\left(\frac{1}{4}\left\{x\int \sin 4x - \int\left(\frac{dx}{dx}.\int \sin 4xdx\right)dx\right\}\right)\right]$$

$$=\frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2}+\int\frac{\cos 2x}{2}dx\right\}\right)-\left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4}+\int\frac{\cos 4x}{4}dx\right\}\right)\right]$$

$$=\frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2}+\frac{\sin 2x}{4}\right\}\right)-\left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4}+\frac{\sin 4x}{16}\right\}\right)\right]+c$$

$$=\frac{-x\cos 2x}{8}+\frac{\sin 2x}{16}+\frac{x\cos 4x}{32}-\frac{\sin 4x}{128}+c$$

27. Question

Evaluate the following integrals:

$$\int \sin x \log(\cos x)dx$$

Answer

Let cosx = t

- sinxdx = dt

Now the integral we have is

$$\int \sin x \log(\cos x)dx = -\int \log tdt$$

$$= -\int 1.\log tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$-\int 1.\log tdt = \log t\int 1dt - \int\left(\frac{d\log t}{dt}.\int 1.dt\right)dt$$

$$= -\log t.t + \int\frac{1}{t}.tdt$$

$$= -t\log t + t + c$$

Replacing t with cosx

$$t(-\log t + 1) + c$$

$$= \cos x(1-\log(\cos x)) + c$$

28. Question

Evaluate the following integrals:

$$\int \frac{\log(\log x)}{x} dx$$

Answer

Let logx = t

1/x dx = dt

$$\int \frac{\log(\log x)}{x} dx = \int \log t dt = \int 1.\log t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int 1.\log tdt = \log t\int 1dt - \int\left(\frac{d\log t}{dt}.\int 1.dt\right)dt$$

$$= t.\log t - \int \frac{1}{t} tdt$$

$$= t\log t - t + c$$

Now replacing t with logx

$$\log x.\log(\log x) - \log x + c$$

$$= \log x(\log(\log x) - 1) + c$$

29. Question

Evaluate the following integrals:

$$\int \log(2 + x^2) dx$$

Answer

$$= \int 1.\log(2 + x^2) dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log(2 + x^2)$ is the first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int 1.\log\left(2+x^2\right)dx = \log\left(2+x^2\right)\int 1dx - \int\left(\frac{d\log\left(2+x^2\right)}{dx}.\int 1dx\right)dx$$

$$= \log\left(2+x^2\right).x - \int \frac{1.2x}{2+x^2}.xdx$$

$$= x\log\left(2+x^2\right) - \int \frac{2x^2}{2+x^2}dx$$

$$= x\log\left(2+x^2\right) - 2\int \frac{x^2+2-2}{2+x^2}dx$$

$$= x\log\left(2+x^2\right) - 2\left[\left(\int 1dx\right) - \int \frac{2}{2+x^2}dx\right]$$

$$= x\log\left(2+x^2\right) - 2\left[x - \left(2\int \frac{1}{2+x}\right)dx\right]$$

$$= x\log\left(2+x^2\right) - 2\left[x - 2\left(\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}\right)\right] + c$$

$$= x\log\left(2+x^2\right) - 2x + 2\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}} + c$$

30. Question

Evaluate the following integrals:

$$\int \frac{x}{(1+\sin x)}dx$$

Answer

$$\int \frac{x}{1+\sin x}dx = \int \frac{x(1-\sin x)}{(1+\sin x).(1-\sin x)}dx$$

We can write it as $= \int \frac{x(1-\sin x)}{1-\sin^2 x}dx$

$$= \int \frac{x(1-\sin x)}{\cos^2 x}dx$$

$$= \int x\sec^2 xdx - \int x\tan x\sec xdx$$

Using by part and ILATE

Taking x as first function and $\sec^2 x$ and secxtanx as the second function, we have

$$\int x\sec^2 xdx - \int x\sec x\tan xdx = \left(x\int \sec^2 xdx - \int\left(\frac{dx}{dx}.\int \sec^2 xdx\right)dx\right)$$

$$-\left(x\int \sec x\tan xdx - \int\left(\frac{dx}{dx}.\int \sec x\tan xdx\right)dx\right)$$

$$= \left(x\tan x - \int 1.\tan xdx\right) - \left(x.\sec x - \int 1.\sec xdx\right)$$

$$= x\tan x - \ln|\sec x| - x\sec x + \ln|\sec x + \tan x| + c$$

$$= x(\tan x - \sec x) + \ln\left|\frac{\sec x + \tan x}{\sec x}\right| + c$$

$$= x(\tan x - \sec x) + \ln|1+\sin x| + c$$

31. Question

Evaluate the following integrals:

$$\int\left\{\frac{1}{\log x}-\frac{1}{(\log\ x)^2}\right\}dx$$

Answer

Let us assume logx = t

$X = e^t$

$dx = e^t dt$

Now we have

$$\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx=\int\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt$$

Considering f(x) = 1/t ; f′(x) = - 1/t^2

$$\frac{d}{dt}\left(\frac{1}{t}\right)=-\frac{1}{t^2}$$

By the integral property of $\int\{f(x)+f'(x)\}e^x dx=e^x.f(x)+c$

So the solution of the integral is

$$\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx=e^t\times\frac{1}{t}+c$$

Substituting the value of t as logx

$$=e^{\log x}\times\frac{1}{\log x}+c$$

$$=\frac{x}{\log x}+c$$

32. Question

Evaluate the following integrals:

$$\int e^{-x}\cos\ 2x\ \cos\ 4x\ dx$$

Answer

$$\cos A.\cos B=\frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$

We know that $\Rightarrow \cos 4x.\cos 2x=\frac{1}{2}\left[\cos(4x+2x)+\cos(4x-2x)\right]$

$$=\frac{1}{2}[\cos 6x+\cos 2x]$$

Putting in the original equation

$$\int e^{-x}\cos 2x.\cos 4x dx = \int e^{-x}\left(\frac{1}{2}[\cos 6x + \cos 2x]\right)$$

$$= \frac{1}{2}\left[\left(\int e^{-x}\cos 6x dx\right) + \left(\int e^{-x}\cos 2x dx\right)\right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cos6x and cos2x is first function and e^{-x} as the second function.

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

Solving both parts individually

$$I = \int e^{-x}\cos 6x dx = \cos 6x\int e^{-x}dx - \int\left(\frac{d\cos 6x}{dx}.\int e^{-x}dx\right)dx$$

$$I = \cos 6x.\left(-e^{-x}\right) - \int(-6\sin 6x).\left(-e^{-x}\right)dt$$

$$I = -\cos 6x.e^{-x} - 6\int \sin 6x.e^{-x}dx$$

$$I = -e^{-x}\cos 6x - 6\left[\sin 6x\int e^{-x}dx - \int\left(\frac{d\sin 6x}{dx}.\int e^{-x}dx\right)dx\right]$$

$$I = -e^{-x}\cos 6x - 6\left[\sin 6x\left(-e^{-x}\right) - \int(6\cos 6x).\left(-e^{-x}\right)dt\right]$$

$$I = -e^{-x}\cos 6x - 6\left[-e^{-x}\sin 6x + 6\int e^{-x}\cos 6x dx\right]$$

$$I = -e^{-x}\cos 6x - 6\left[-e^{-x}\sin 6x + 6I\right]$$

$$I = -e^{-x}\cos 6x + 6e^{-x}\sin 6x - 36I$$

$$37I = e^{-x}(6\sin 6x - \cos 6x)$$

$$I = \frac{e^{-x}(6\sin 6x - \cos 6x)}{37}$$

Solving the second part,

$$I=\int e^{-x}\cos 2x dx=\cos 2x\int e^{-x}dx-\int\left(\frac{d\cos 2x}{dx}.\int e^{-x}dx\right)dx$$

$$J=\cos 2x.\left(-e^{-x}\right)-\int\left(-2\sin 2x\right).\left(-e^{-x}\right)dt$$

$$J=-\cos 2x.e^{-x}-2\int\sin 2x.e^{-x}dx$$

$$J=-e^{-x}\cos 2x-2\left[\sin 2x\int e^{-x}dx-\int\left(\frac{d\sin 2x}{dx}.\int e^{-x}dx\right)dx\right]$$

$$J=-e^{-x}\cos 2x-2\left[\sin 2x\left(-e^{-x}\right)-\int\left(2\cos 2x\right).\left(-e^{-x}\right)dt\right]$$

$$J=-e^{-x}\cos 2x-2\left[-e^{-x}\sin 2x+2\int e^{-x}\cos 2xdx\right]$$

$$J=-e^{-x}\cos 2x-2\left[-e^{-x}\sin 2x+2J\right]$$

$$J=-e^{-x}\cos 2x+2e^{-x}\sin 2x-4J$$

$$5J=e^{-x}\left(2\sin 2x-\cos 2x\right)$$

$$J=\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{5}$$

Putting in the obtained equation

$$=\frac{1}{2}\left[\frac{e^{-x}\left(6\sin 6x-\cos 6x\right)}{37}+\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{5}\right]+c$$

$$=\frac{e^{-x}\left(6\sin 6x-\cos 6x\right)}{74}+\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{10}+c$$

$$=e^{-x}\left(\frac{\left(6\sin 6x-\cos 6x\right)}{74}+\frac{\left(2\sin 2x-\cos 2x\right)}{10}\right)+c$$

33. Question

Evaluate the following integrals:

$$\int e^{\sqrt{x}}dx$$

Answer

Let √x = t

$$\frac{1}{2\sqrt{x}}dx=dt$$

$$dx=2\sqrt{x}dt$$

$$\Rightarrow dx=2tdt$$

Replacing in the original equation , we get

$$\int e^{\sqrt{x}}dx=\int e^{t}.2tdt$$

$$=2\int te^{t}dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$2\int te^t dt = 2\left[t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt\right]$$

$$= 2\left[te^t - \int 1.e^t dt\right]$$

$$= 2\left[te^t - e^t\right] + c$$

$$= 2e^t(t-1) + c$$

Replacing t with √x

= $2e^{\sqrt{x}}(\sqrt{x} - 1) + c$

34. Question

Evaluate the following integrals:

$$\int e^{\sin x}\sin 2x\ dx$$

Answer

We can write Sin2x = 2sinx.cosx

$$\int e^{\sin x}\sin 2xdx = 2\int e^{\sin x}.\sin x\cos xdx$$

Let Sinx = t

Cosxdx = dt

$$2\int e^{\sin x}\sin x\cos xdx = 2\int e^t.t.dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$2\int e^t.tdt = 2\left[t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt\right]$$

$$= 2\left[t.e^t - \int 1.e^t dt\right]$$

$$= 2\left[t.e^t - e^t\right] + c$$

$$= 2e^t(t-1) + c$$

Replacing t with sin x

= $2e^{\sin x}(\sin x - 1) + c$

35. Question

Evaluate the following integrals:

$$\int\frac{x\ \sin^{-1}x}{\sqrt{1-x^2}}dx$$

Answer

Let $\sin^{-1}x = t$

$X = \sin t$

$$\frac{1}{\sqrt{1-x^2}}dx = dt$$

Putting this in the original equation, we get

$$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = \int t.\sin t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and sin t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int t.\sin tdt = t\int \sin tdt - \int\left(\frac{dt}{dt}.\int \sin tdt\right)dt$$

$$= t(-\cos t) - \int 1.(-\cos t)dt$$

$$= -t\cos t + \sin t + c$$

We can write $\cos t = \sqrt{1 - \sin^2 t}$

$= - t(\sqrt{1 - \sin^2 t}) + \sin t + c$

Now replacing $\sin^{-1}x = t$

$= - \sin^{-1}x(\sqrt{1 - x^2}) + x + c$

36. Question

Evaluate the following integrals:

$$\int \frac{x^2\tan^{-1}x}{(1+x^2)}dx$$

Answer

Let $\tan^{-1} x = t$ and $x = \tan t$

Differentiating both sides, we get

$$\frac{1}{1+x^2}dx = dt$$

Now we have

$$\int \frac{x^2\tan^{-1}x}{(1+x^2)}dx = \int \tan^2 t.tdt$$

$$\int t.\tan^2 tdt = \int t(\sec^2 t - 1)dt$$

$$= \int t\sec^2 tdt - \int tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and $\sec^2 t$ as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int t\sec^2 tdt - \int tdt = t\int \sec^2 tdt - \int\left(\frac{dt}{dt}.\int \sec^2 tdt\right)dt - \frac{t^2}{2}$$

$$= t.\tan t - \int \tan tdt - \frac{t^2}{2}$$

$$= t.\tan t - \ln|\sec t| - \frac{t^2}{2} + c$$

We know that $\sec t = \sqrt{\tan^2 t + 1}$

$$= \tan^{-1} x.x - \ln|\sqrt{\tan^2 t + 1}| - \frac{\tan^2 x}{2} + c$$

$$= x\tan^{-1} x - \ln|\sqrt{x^2 + 1}| - \frac{\tan^2 x}{2} + c$$

37. Question

Evaluate the following integrals:

$$\int \frac{\log(x+2)}{(x+2)^2}dx$$

Answer

We can write it as $\int \log(x+2).\frac{1}{(x+2)^2}dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log(x + 2)$ is first function and $(x + 2)^{-2}$ as second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \log(x+2).\frac{1}{(x+2)^2}dx = \log(x+2)$$

$$\int \frac{1}{(x+2)^2}dx - \int\left(\frac{d\log(x+2)}{dx}.\int\frac{1}{(x+2)^2}dx\right)dx$$

$$= \log(x+2).\frac{-1}{(x+2)} - \int\frac{1}{x+2}.\frac{-1}{(x+2)}dx$$

$$= -\log(x+2)\frac{1}{(x+2)} + \int\frac{1}{(x+2)^2}dx$$

$$= -\log(x+2)\frac{1}{(x+2)} - \frac{1}{(x+2)} + c$$

38. Question

Evaluate the following integrals:

$$\int x \sin^{-1} x\, dx$$

Answer

Let x = sin t ; t = $\sin^{-1}x$

dx = cos t dt

$$\Rightarrow \int x\sin^{-1}x dx = \int \sin t.\sin^{-1}(\sin t)\cos t dt$$
$$= \int \sin t.t.\cos t dt$$

We know that sin 2t = 2 sint×cost

We have $\int t\cos t\sin t dt = \frac{1}{2}\int t\sin 2t dt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and sin 2t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int t\sin 2t dt = \frac{1}{2}\left(t\int \sin 2t dt - \int\left[\frac{dt}{dt}.\int \sin 2t dt\right]dt\right)$$

$$= \frac{1}{2}\left(t.\frac{-\cos 2t}{2} + \int\frac{\cos 2t}{2}dt\right)$$

$$= \frac{1}{2}\left(\frac{-t\cos 2t}{2} + \frac{\sin 2t}{4}\right) + c$$

$$= \frac{-t\cos 2t}{4} + \frac{\sin 2t}{8} + c$$

We know that $\cos 2t = 1 - 2\sin^2 t$, sin2t = 2sint×cost and $\cos t = \sqrt{1 - \sin^2 t}$

Replacing in above equation

$$= \frac{-t\left(1-2\sin^2 t\right)}{4} + \frac{2\sin t \times \cos t}{8} + c$$

$$= \frac{-t\left(1-2\sin^2 t\right)}{4} + \frac{\sqrt{1-\sin^2 t}}{4}.\sin t + c$$

$$= \frac{-\sin^{-1} x\left(1-2x^2\right)}{4} + \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \frac{1}{2}x^2 \sin^{-1} x - \frac{\sin^{-1} x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c$$

$$= \frac{1}{2}x^2 \sin^{-1} x - \frac{\sin^{-1} x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c$$

39. Question

Evaluate the following integrals:

$$\int x \cos^{-1} x\,dx$$

Answer

Let x = cos t ; t = $\cos^{-1}x$

dx = - sin t dt

$$\int x\cos^{-1} x dx = -\int \cos t.\cos^{-1}(\cos t)\sin t dt$$

$$= -\int \cos t.t.\sin t.dt$$

We know that sin 2t = 2 sint×cost

We have $-\int t\cos t\sin t dt = \frac{-1}{2}\int t\sin 2t dt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking first function to the one which comes first in the list.

Here t is first function and sin 2t as second function.

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\frac{-1}{2}\int t\sin 2t dt = \frac{-1}{2}\left(t\int \sin 2t dt - \int\left[\frac{dt}{dt}.\int \sin 2t dt\right]dt\right)$$

$$= \frac{-1}{2}\left(t.\frac{-\cos 2t}{2} + \int \frac{\cos 2t}{2}dt\right)$$

$$= \frac{-1}{2}\left(\frac{-t\cos 2t}{2} + \frac{\sin 2t}{4}\right) + c$$

$$= \frac{t\cos 2t}{4} - \frac{\sin 2t}{8} + c$$

We know that cos2t = $2\cos^2 t$ - 1 and sin2t = 2sint×cost and sint = $\sqrt{1 - \cos^2 t}$

Replacing in above equation

$$= \frac{t(2\cos^2 t - 1)}{4} - \frac{2\sin t \times \cos t}{8} + c$$

$$= \frac{t(2\cos^2 t - 1)}{4} - \frac{\sqrt{1-\cos^2 t}}{4}.\cos t + c$$

$$= \frac{\cos^{-1} x(2x^2 - 1)}{4} - \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \frac{1}{2}x^2\cos^{-1}x - \frac{\cos^{-1}x}{4} - \frac{1}{4}x\sqrt{1-x^2} + c$$

$$= \frac{1}{2}x^2\cos^{-1}x + \frac{\sin^{-1}x}{4} - \frac{1}{4}x\sqrt{1-x^2} + c$$

40. Question

Evaluate the following integrals:

$$\int \cot^{-1} x \, dx$$

Answer

We can write it as $\int \cot^{-1} x.1dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cot $^{-1}$x is first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \cot^{-1} x.1dx = \cot^{-1} x\int 1dx - \int\left(\frac{d\cot^{-1}x}{dx}.\int 1dx\right)dx$$

$$= \cot^{-1}x.x - \int\frac{-1}{1+x^2}.x.dx$$

$$= x\cot^{-1}x + \int\frac{x}{1+x^2}dx$$

Let 1 + x^2 = t

2xdx = dt

Xdx = dt/2

$$\Rightarrow \int \cot^{-1} xdx = x\cot^{-1}x + \int\frac{dt}{2t}$$

$$= x\cot^{-1}x + \frac{\log t}{2} + c$$

Now replacing t with 1 + x^2

= xcot $^{-1}$x + log(1 + x^2)/2 + c

41. Question

Evaluate the following integrals:

$\int x \cot^{-1} x \, dx$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x$,

$\therefore \int x \cot^{-1} x \, dx$

$= \cot^{-1} x \int x dx - \int \left\{\frac{d}{dx}(\cot^{-1} x) \int x dx\right\} dx$

$= \frac{x^2 \cot^{-1} x}{2} - \int \frac{1}{(1+x^2)} \times \frac{x^2}{2} dx$

$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{(1+x^2)} dx$

$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} \int \frac{1+x^2-x^2}{(1+x^2)} dx$

$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} \int 1 - \frac{1}{(1+x^2)} dx$

$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2}[x - \tan^{-1} x] + c$, where c is the integrating constant

42. Question

Evaluate the following integrals:

$\int x^2 \cot^{-1} x \, dx$

[CBSE 2006C]

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x^2$,

$\therefore \int x^2 \cot^{-1} x \, dx$

$= \cot^{-1} x \int x^2 dx - \int \left\{\frac{d}{dx}(\cot^{-1} x) \int x^2 dx\right\} dx$

$= \frac{x^3 \cot^{-1} x}{3} - \int \frac{1}{(1+x^2)} \times \frac{x^3}{3} dx$

$= \frac{x^3 \cot^{-1} x}{3} - \frac{1}{3} \int \frac{x^3}{(1+x^2)} dx$

Taking $(1+x^2)=a$,

$2xdx=da$ i.e. $xdx=da/2$

Again, x^2=a-1

$$\therefore \frac{1}{3}\int \frac{x^2 \times xdx}{(1+x^2)}$$

$$=\frac{1}{3}\int \frac{(a-1)da}{2a}$$

$$=\frac{1}{6}\int \left(1-\frac{1}{a}\right)da$$

$$=\frac{1}{6}(a-\ln a)$$

Replacing the value of a, we get,

$$\therefore \frac{1}{6}(a-\ln a)$$

$$=\frac{1}{6}[(1+x^2)-\ln|x^2+1|+c_1$$

$$=\frac{x^2}{6}-\frac{\ln|x^2+1|}{6}+\left(c_1+\frac{1}{6}\right)$$

$$=\frac{x^2}{6}-\frac{\ln|x^2+1|}{6}+c$$

The total integration yields as

$=\frac{x^3 \cot^{-1}x}{3}+\frac{x^2}{6}-\frac{\ln|x^2+1|}{6}+c$, where c is the integrating constant

43. Question

Evaluate the following integrals:

$$\int \sin^{-1}\sqrt{x}\ dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \sin^{-1}\sqrt{x}dx$$

$$=\sin^{-1}\sqrt{x}\int dx-\int\left\{\frac{d}{dx}(\sin^{-1}\sqrt{x})\int dx\right\}dx$$

$$=x\sin^{-1}\sqrt{x}-\int \frac{1}{2\sqrt{x}\sqrt{1-x}}\times xdx$$

$$=x\sin^{-1}\sqrt{x}-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}\,da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= \mathbf{x\sin^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c}$, where c is the integrating constant

44. Question

Evaluate the following integrals:

$$\int \cos^{-1}\sqrt{x}\,dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \cos^{-1}\sqrt{x}\,dx$$

$$= \cos^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right)\int dx\right\}dx$$

$$= x\cos^{-1}\sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}} \times x\,dx$$

$$= x\cos^{-1}\sqrt{x} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}\,da$$

$= -\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$

Replacing the value of a, we get,

$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$

$= -\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c$

The total integration yields as

$= \mathbf{x\cos^{-1}\sqrt{x}-\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c}$, where c is the integrating constant

45. Question

Evaluate the following integrals:

$\int \cos^{-1}\left(4x^3-3x\right)dx$

Answer

Formula to be used – We know , cos3x = $4\cos^3x$-3cosx

$\therefore \int \cos^{-1}(4x^3-3x)\,dx$

Assuming x = cosa, $4\cos^3a$-3cosa=cos3a

And, dx = -sinada

Hence, a=$\cos^{-1}x$

Again, sina=$\sqrt{(1-x^2)}$

$\therefore \int \cos^{-1}(4x^3-3x)\,dx$

$= \int \cos^{-1}(\cos 3a)\{-\sin a da\}$

$= -3\int a\sin a da$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = a and $f_2(x)$ = sina,

$\therefore -3\int a\sin a da$

$= -3\left[a\int \sin a da - \int\left\{\frac{d}{dx}a\int \sin a da\right\}da\right]$

$= 3a\cos a - \int \cos a da$

$= 3a\cos a - \sin a + c$

Replacing the value of a we get,

$\therefore 3a\cos a - \sin a + c$

$= \mathbf{3x\cos^{-1}x - \sqrt{1-x^2} + c}$, where c is the integrating constant

46. Question

Evaluate the following integrals:

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and $f_2(x) = 1$,

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$$

$$= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\int dx - \int\left[\frac{d}{dx}\left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}\int dx\right]dx$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \int\left[\frac{\frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}}\right]dx$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \int\frac{-4x^2dx}{(1+x^2)^2\times\frac{1}{1+x^2}\times 2x}$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) - \int\frac{2xdx}{1+x^2}$$

Now,

$$\int\frac{2xdx}{1+x^2}$$

$$= \int\frac{d(1+x^2)}{1+x^2}$$

$$= \ln(1+x^2) + c$$

Again, we know,

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

$$\Rightarrow 2x = \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$$

Replacing x by tanx, it is obtained that,

$$2\tan x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

So, the final integral yielded is

$\mathbf{2xtanx - ln(1 + x^2) + c}$, where c is the integrating constant

47. Question

Evaluate the following integrals:

$$\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

Answer

Formula to be used – We know, $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

Assuming x = tana,

$$\frac{2tana}{1-tan^2a} = tan2a$$

And, dx = sec^2ada

Hence, a=$tan^{-1}x$

Now, sec^2a-tan^2a=1 , so,seca=$\sqrt{(1+x^2)}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

$$= \int \tan^{-1}(tan2a)\{sec^2ada\}$$

$$= 2\int asec^2ada$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = a and $f_2(x)$ = sec^2a,

$$\therefore 2\int asec^2ada$$

$$= 2\left[a\int sec^2ada - \int\left\{\frac{d}{dx}a\int sec^2ada\right\}da\right]$$

$$= 2atana - \int tanada$$

$$= 2atana - \ln|seca| + c$$

Replacing the value of a we get,

$$\therefore 2atana - \ln|seca| + c$$

$= \mathbf{2x\tan^{-1}x - \ln\sqrt{1+x^2} + c}$, where c is the integrating constant

48. Question

Evaluate the following integrals:

$\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$

Answer

Formula to be used – We know, $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

$\therefore \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$

Assuming x = tana,

$\frac{3\tan a - \tan^3 a}{1-3\tan^2 a} = \tan 3a$

And, dx = $\sec^2 a da$

Hence, a=$\tan^{-1}x$

Now, $\sec^2 a - \tan^2 a = 1$, so, seca=$\sqrt{(1+x^2)}$

$\therefore \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$

$= \int \tan^{-1}(\tan 3a)\{\sec^2 a da\}$

$= 3\int a\sec^2 a da$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = a and $f_2(x) = \sec^2 a$,

$\therefore 3\int a\sec^2 a da$

$= 3\left[a\int \sec^2 a da - \int\left\{\frac{d}{dx}a\int \sec^2 a da\right\}da\right]$

$= 3a\tan a - \frac{3}{2}\int \tan a da$

$= 3a\tan a - \frac{3}{2}\ln|\sec a| + c$

Replacing the value of a we get,

$\therefore 3a\tan a - \frac{3}{2}\ln|\sec a| + c$

$= \mathbf{3x\tan^{-1}x - \frac{3}{2}\ln\sqrt{1+x^2} + c}$, where c is the integrating constant

49. Question

Evaluate the following integrals:

$\int \frac{\sin^{-1}x}{x^2}dx$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = 1/x^2$,

$$\therefore \int \frac{\sin^{-1}x}{x^2}dx$$

$$= \sin^{-1}x\int\frac{1}{x^2}dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int\frac{1}{x^2}dx\right\}dx$$

$$= \frac{-\sin^{-1}x}{x} - \int\frac{1}{\sqrt{1-x^2}}\times(-\frac{1}{x})dx$$

$$= \frac{-\sin^{-1}x}{x} + \int\frac{1}{x\sqrt{1-x^2}}dx$$

Taking x= sina, dx = cosada

Hence, coseca=1/x

Now, $\text{cosec}^2a - \cot^2 a = 1$ so $\cot a = \sqrt{(1-x^2)}/x$

$$\therefore \int\frac{1}{x\sqrt{1-x^2}}dx$$

$$= \int\frac{1}{\text{sinacosa}}(\text{cosada})$$

$$= \int \text{cosecada}$$

$$= \ln|\text{coseca} - \text{cota}| + c$$

Replacing the value of a, we get,

$$\therefore \ln|\text{coseca} - \text{cota}| + c$$

$$= \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c$$

The total integration yields as

$= \frac{-\sin^{-1}x}{x} + \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c$, where c is the integrating constant

50. Question

Evaluate the following integrals:

$$\int\frac{\tan x\sec^2 x}{(1-\tan^2 x)}dx$$

Answer

Say, tanx = a

Hence, $\sec^2 x dx = da$

$$\therefore \int\frac{\tan x\sec^2 x}{1-\tan^2 x}dx$$

$= \int \frac{ada}{1-a^2}$

Now, taking $1-a^2 = k$, $-2ada=dk$ i.e. $ada=-dk/2$

$\therefore \int \frac{ada}{1-a^2}$

$= \int \frac{-dk}{2k}$

$= -\frac{1}{2}\ln|k| + c$

Replacing the value of k,

$-\frac{1}{2}\ln|k| + c$

$= -\frac{1}{2}\ln|1-a^2| + c$

Replacing the value of a,

$-\frac{1}{2}\ln|1-a^2| + c$

$= -\frac{1}{2}\mathbf{ln}|\mathbf{1} - \mathbf{tan^2x}| + \mathbf{c}$, where c is the integrating constant

51. Question

Evaluate the following integrals:

$\int e^{3x} \sin 4x \, dx$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin 4x$ and $f_2(x) = e^{3x}$,

$\therefore \int e^{3x}\sin4xdx$

$= \sin4x\int e^{3x}dx - \int\left\{\frac{d}{dx}(\sin4x)\int e^{3x}dx\right\}dx$

$= \frac{e^{3x}\sin4x}{3} - \int 4\cos4x \times \frac{e^{3x}}{3}dx$

$= \frac{e^{3x}\sin4x}{3} - \frac{4}{3}\int e^{3x}\cos4xdx$

$= \frac{e^{3x}\sin4x}{3} - \frac{4}{3}\left[\cos4x\int e^{3x}dx - \int\left\{\frac{d}{dx}(\cos4x)\int e^{3x}dx\right\}dx\right]$

$= \frac{e^{3x}\sin4x}{3} - \frac{4e^{3x}\cos4x}{9} - \frac{4}{3}\int 4\sin4x \times \frac{e^{3x}}{3}dx$

$= \frac{e^{3x}\sin4x}{3} - \frac{4e^{3x}\cos4x}{9} - \frac{16}{9}\int e^{3x}\sin4xdx$

$$\therefore \left(1+\frac{16}{9}\right)\int e^{3x}\sin 4x dx=\frac{e^{3x}\sin 4x}{3}-\frac{4e^{3x}\cos 4x}{9}+c_1$$

$$\Rightarrow \frac{25}{9}\int e^{3x}\sin 4x dx=\frac{3e^{3x}\sin 4x-4e^{3x}\cos 4x}{9}+c_1$$

$\Rightarrow \int e^{3x}\sin 4x dx=\frac{e^{3x}}{25}(\mathbf{3\sin 4x-4\cos 4x})+\mathbf{c}$, where c is the integrating constant

52. Question

Evaluate the following integrals:

$$\int e^{2x}\sin x\, dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = sinx and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\sin x dx$$

$$=\sin x\int e^{2x}dx-\int\left\{\frac{d}{dx}(\sin x)\int e^{2x}dx\right\}dx$$

$$=\frac{e^{2x}\sin x}{2}-\int \cos x\times\frac{e^{2x}}{2}dx$$

$$=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\int e^{2x}\cos x dx$$

$$=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x\int e^{2x}dx-\int\left\{\frac{d}{dx}(\cos x)\int e^{2x}dx\right\}dx\right]$$

$$=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}-\frac{1}{2}\int \sin x\times\frac{e^{2x}}{2}dx$$

$$=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}-\frac{1}{4}\int e^{2x}\sin x dx$$

$$\therefore \left(1+\frac{1}{4}\right)\int e^{2x}\sin x dx=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}+c_1$$

$$\Rightarrow \frac{5}{4}\int e^{2x}\sin x dx=\frac{2e^{2x}\sin x-e^{2x}\cos x}{4}+c_1$$

$\Rightarrow \int e^{2x}\sin x dx=\frac{e^{2x}}{5}(\mathbf{2\sin x-\cos x})+\mathbf{c}$, where c is the integrating constant

53. Question

Evaluate the following integrals:

$$\int e^{2x}\sin x\cos x\, dx$$

Answer

$$\int e^{2x}\sin x\cos x dx$$

$=\frac{1}{2}\int e^{2x}\times 2\sin x\cos x dx$

$=\frac{1}{2}\int e^{2x}\sin 2x dx$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)=\sin 2x$ and $f_2(x)=e^{2x}$,

$\therefore\int e^{2x}\sin 2x dx$

$=\sin 2x\int e^{2x}dx-\int\left\{\frac{d}{dx}(\sin 2x)\int e^{2x}dx\right\}dx$

$=\frac{e^{2x}\sin 2x}{2}-\int 2\cos 2x\times\frac{e^{2x}}{2}dx$

$=\frac{e^{2x}\sin 2x}{2}-\int e^{2x}\cos 2x dx$

$=\frac{e^{2x}\sin 2x}{2}-\left[\cos 2x\int e^{2x}dx-\int\left\{\frac{d}{dx}(\cos 2x)\int e^{2x}dx\right\}dx\right]$

$=\frac{e^{2x}\sin 2x}{2}-\frac{e^{2x}\cos 2x}{2}-\int 2\sin 2x\times\frac{e^{2x}}{2}dx$

$=\frac{e^{2x}\sin 2x}{2}-\frac{e^{2x}\cos 2x}{2}-\int e^{2x}\sin x dx$

$\therefore(1+1)\int e^{2x}\sin 2x dx=\frac{e^{2x}\sin 2x}{2}-\frac{e^{2x}\cos 2x}{2}+c_1$

$\Rightarrow 2\int e^{2x}\sin 2x dx=\frac{e^{2x}\sin 2x-e^{2x}\cos 2x}{2}+c_1$

$\Rightarrow\int e^{2x}\sin 2x dx=\frac{e^{2x}}{4}(\sin 2x-\cos 2x)+c'$

$\therefore\frac{1}{2}\int e^{2x}\sin 2x dx$

$=\frac{1}{2}\times\left[\frac{e^{2x}}{4}(\sin 2x-\cos 2x)+c'\right]$

$=\boldsymbol{\frac{e^{2x}}{8}(\sin 2x-\cos 2x)+c}$, where c is the integrating constant

54. Question

Evaluate the following integrals:

$\int e^{2x}\cos(3x+4)dx$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos(3x+4)$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\cos(3x+4)\,dx$$

$$= \cos(3x+4)\int e^{2x}dx - \int\left\{\frac{d}{dx}\cos(3x+4)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \int 3\sin(3x+4)\times\frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}\int e^{2x}\sin(3x+4)dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}\left[\sin(3x+4)\int e^{2x}dx - \int\left\{\frac{d}{dx}\sin(3x+4)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} - \frac{3}{2}\int 3\cos(3x+4)\times\frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} - \frac{9}{4}\int e^{2x}\cos(3x+4)dx$$

$$\therefore \left(1+\frac{9}{4}\right)\int e^{2x}\cos(3x+4)dx = \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} + c_1$$

$$\Rightarrow \frac{13}{4}\int e^{2x}\cos(3x+4)\,dx = \frac{2e^{2x}\cos(3x+4) + 3e^{2x}\sin(3x+4)}{4} + c_1$$

$\Rightarrow \int e^{2x}\cos(3x+4)\,dx = \frac{e^{2x}}{13}(2\cos(3x+4) + 3\sin(3x+4)) + c$, where c is the integrating constant

55. Question

Evaluate the following integrals:

$$\int e^{-x}\cos x\,dx$$

Answer

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos x$ and $f_2(x) = e^{-x}$,

$$\therefore \int e^{-x}\cos x\,dx$$

$$= \cos x\int e^{-x}dx - \int\left\{\frac{d}{dx}\cos x\int e^{-x}dx\right\}dx$$

$$= -e^{-x}\cos x - \int e^{-x}\sin x\,dx$$

$$= -e^{-x}\cos x - \left[\sin x\int e^{-x}dx - \int\left\{\frac{d}{dx}\sin x\int e^{-x}dx\right\}dx\right]$$

$$= -e^{-x}\cos x - \left[-e^{-x}\sin x + \int e^{-x}\cos x\,dx\right]$$

$= -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x dx$

$\therefore (1+1)\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$

$\Rightarrow 2\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$

$\Rightarrow \int e^{-x}\cos x dx = \frac{e^{-x}}{2}(\mathbf{\sin x - \cos x}) + \mathbf{c}$, where c is the integrating constant

56. Question

Evaluate the following integrals:

$\int e^x(\sin x + \cos x)dx$

Answer

$\int e^x(\sin x + \cos x)dx$

$= \int e^x \sin x dx + \int e^x \cos x dx$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \sin x dx + \int e^x \cos x dx$

$= \sin x \int e^x dx - \int\left[\frac{d}{dx}(\sin x)\int e^x dx\right]dx + \int e^x \cos x dx$

$= e^x \sin x - \int e^x \cos x dx + \int e^x \cos x dx + c$

$= \mathbf{e^x \sin x + c}$, where c is the integrating constant

57. Question

Evaluate the following integrals:

$\int e^x(\cot x - \mathrm{cosec}^2 x)dx$

Answer

$\int e^x(\cot x - \mathrm{cosec}^2 x)dx$

$= \int e^x \cot x dx + \int e^x \mathrm{cosec}^2 x dx$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \cot x dx + \int e^x \operatorname{cosec}^2 x dx$

$= \cot x \int e^x dx - \int \left[\frac{d}{dx}(\cot x) \int e^x dx\right] dx + \int e^x \operatorname{cosec}^2 x dx$

$= e^x \cot x - \int e^x \operatorname{cosec}^2 x dx + \int e^x \operatorname{cosec}^2 x dx + c$

$= \mathbf{e^x \cot x + c}$, where c is the integrating constant

58. Question

Evaluate the following integrals:

$\int e^x \sec x(1 + \tan x) dx$

Answer

$\int e^x \sec x(1 + \tan x) dx$

$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \sec x dx + \int e^x \sec x \tan x dx$

$= \sec x \int e^x dx - \int \left[\frac{d}{dx}(\sec x) \int e^x dx\right] dx + \int e^x \sec x \tan x dx$

$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c$

$= \mathbf{e^x \sec x + c}$, where c is the integrating constant

59. Question

Evaluate the following integrals:

$\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2}\right) dx$

Answer

$\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2}\right) dx$

$= \int e^x \tan^{-1} x dx + \int \frac{e^x}{1+x^2} dx$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan^{-1} x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \tan^{-1} x\, dx + \int \frac{e^x}{1+x^2} dx$

$= \tan^{-1} x \int e^x dx - \int \left[\frac{d}{dx}(\tan^{-1} x) \int e^x dx\right] dx + \int \frac{e^x}{1+x^2} dx$

$= e^x \tan^{-1} x - \int \frac{e^x}{1+x^2} dx + \int \frac{e^x}{1+x^2} dx + c$

$= \mathbf{e^x \tan^{-1} x + c}$, where c is the integrating constant

60. Question

Evaluate the following integrals:

$\int e^x (\cot x + \log \sin x) dx$

Answer

$\int e^x(\cot x + \log\sin x)dx$

$= \int e^x \cot x\, dx + \int e^x \log\sin x dx$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logsinx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int e^x \cot x\, dx + \int e^x \log\sin x dx$

$= \int e^x \cot x\, dx + \log\sin x \int e^x dx - \int \left[\frac{d}{dx}(\log\sin x) \int e^x dx\right]$

$= \int e^x \cot x\, dx + e^x \log\sin x - \int e^x \cot x\, dx + c$

$= \mathbf{e^x \log|\sin x| + c}$, where c is the integrating constant

61. Question

Evaluate the following integrals:

$\int e^x (\tan x - \log \cos x) dx$

Answer

$\int e^x(\tan x + \log\cos x)dx$

$= \int e^x \tan x\, dx + \int e^x \log\cos x dx$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logcosx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int e^x \tan x\, dx - \int e^x \log\cos x dx$

$= \int e^x \tan x\, dx - \log\cos x \int e^x dx + \int \left[\frac{d}{dx}(\log\cos x) \int e^x dx\right]$

$= \int e^x \tan x\, dx - e^x \log\cos x - \int e^x \tan x\, dx + c$

$= \mathbf{e^x \log|\sec x| + c}$, where c is the integrating constant

62. Question

Evaluate the following integrals:

$\int e^x \left[\sec x + \log(\sec x + \tan x)\right] dx$

Answer

$\int e^x [\sec x + \log(\sec x + \tan x)] dx$

$= \int e^x \sec x\, dx + \int e^x \log(\sec x + \tan x) dx$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logcosx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int e^x \sec x\, dx + \int e^x \log(\sec x + \tan x) dx$

$= \int e^x \sec x dx + \log(\sec x + \tan x) \int e^x dx - \int \left[\frac{d}{dx}(\log(\sec x + \tan x)) \int e^x dx\right]$

$= \int e^x \sec x\, dx + e^x \log(\sec x + \tan x) - \int \frac{e^x \tan x \times (\sec^2 x + \sec x \tan x)\, dx}{\sec x + \tan x} + c$

$= \int e^x \sec x\, dx + e^x \log(\sec x + \tan x) - \int e^x \sec x\, dx + c$

$= \mathbf{e^x \log|\sec x + \tan x| + c}$, where c is the integrating constant

63. Question

Evaluate the following integrals:

$\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x}\right) dx$

Answer

$\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x}\right) dx$

$= \int e^x (\sec^2 x + \tan x) dx$

$= \int e^x \sec^2 x dx + \int e^x \tan x dx$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY

PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = tanx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \sec^2 x dx + \int e^x \tan x dx$$

$$= \int e^x \sec^2 x dx + \tan x \int e^x dx - \int\left[\frac{d}{dx}(\tan x)\int e^x dx\right]$$

$$= \int e^x \sec^2 x\, dx + e^x \tan x - \int e^x \sec^2 x\, dx + c$$

$= \mathbf{e^x \tan x + c}$, where c is the integrating constant

64. Question

Evaluate the following integrals:

$$\int e^x\left(\frac{\sin x \cos x - 1}{\sin^2 x}\right)dx$$

Answer

$$\int e^x\left(\frac{\sin x \cos x - 1}{\sin^2 x}\right)dx$$

$$= \int e^x(\cot x - \mathrm{cosec}^2 x)dx$$

$$= \int e^x \cot x dx - \int e^x \mathrm{cosec}^2 x dx$$

Tip - If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = cotx and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \cot x dx - \int e^x \mathrm{cosec}^2 x dx$$

$$= \cot x \int e^x dx - \int\left\{\frac{d}{dx}(\cot x)\int e^x dx\right\}dx - \int e^x \mathrm{cosec}^2 x dx$$

$$= e^x \cot x + \int e^x \mathrm{cosec}^2 x dx - \int e^x \mathrm{cosec}^2 x dx + c$$

$= \mathbf{e^x \cot x + c}$, where c is the integrating constant

65. Question

Evaluate the following integrals:

$$\int e^x\left(\frac{\cos x + \sin x}{\cos^2 x}\right)dx$$

Answer

$$\int e^x\left(\frac{\cos x + \sin x}{\cos^2 x}\right)dx$$