

EXERCISE 6(B)

Question 1.

Find the cardinal number of the following sets:

(i) $A_1 = \{-2, -1, 1, 3, 5\}$

Solution:

$$A_1 = \{-2, -1, 1, 3, 5\}$$

Cardinal number of a set is the number of elements in a set.

The number of elements in the set A_1 is 5.

$$\therefore \text{Cardinal Number of a set } A_1 = 5$$

(ii) $A_2 = \{x : x \in \mathbb{N} \text{ and } 3 \leq x < 7\}$

Solution:

$$A_2 = \{x : x \in \mathbb{N} \text{ and } 3 \leq x < 7\} = \{3, 4, 5, 6\}$$

The number of elements in the set A_2 is 4.

$$\therefore \text{Cardinal number of set } A_2 = 4$$

(iii) $A_3 = \{P : P \in \mathbb{W} \text{ and } 2P - 3 < 8\}$

Solution:

$$\text{Given } 2P - 3 < 8$$

Adding 3 to both sides, we get

$$\rightarrow 2P - 3 + 3 < 8 + 3 \rightarrow 2P < 11$$

Dividing both sides by 2, we get

$$P < 11/2 \rightarrow P < 5.5 \rightarrow P < 5.5 \therefore A_3 = \{0, 1, 2, 3, 4, 5\}$$

The number of elements in the set A_3 is 6.

$$\therefore \text{Cardinal number of set } A_3 = 6$$

(iv) $A_4 = \{b : b \in \mathbb{Z} \text{ and } -7 < 3b - 1 \leq 2\}$

Solution:

$$\text{Given } -7 < 3b - 1$$

Let us take $-7 < 3b - 1$ first,

Adding 1 to both sides, we get

$$\rightarrow -7 + 1 < 3b - 1 + 1 \rightarrow -6 < 3b$$

Dividing both sides by 3, we get

$$\rightarrow -6/3 < b \rightarrow -2 < b$$

Now we take, $3b - 1 \leq 2$

Adding 1 to both sides, we get

$$\rightarrow 3b - 1 + 1 \leq 2 + 1 \rightarrow 3b \leq 3$$

Dividing both sides by 3, we get

$$\rightarrow b \leq 3/3 \rightarrow b \leq 1$$

Hence, we get $-2 < b \leq 1$

$$\therefore \text{Given set } A_4 = \{-1, 0, 1\}$$

The number of elements in the set A_4 is 3.

$$\therefore \text{Cardinal Number of a set } A_4 = 3$$

Question 2:

If $P = \{P : P \text{ is a letter in the word "PERMANENT"}\}$. Find $n(P)$.

Solution:

Given $P = \{P : P \text{ is a letter in the word "PERMANENT"}\}$

i.e. $P = \{p, e, r, m, a, n, t\}$

The number of elements in the set P is 7

Therefore, $n(p) = 7$

Question 3.

State, which of the following sets are finite and which are infinite:

(i) $A = \{x : x \in \mathbb{Z} \text{ and } x < 10\}$

Solution:

$A = \{x : x \in \mathbb{Z} \text{ and } x < 10\} = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$= \{9, 8, 7, 6, 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, \dots\}$

We know that, a set which is not finite is called an infinite set.

$\therefore A = \{x : x \in \mathbb{Z} \text{ and } x < 10\}$ is an infinite set.

(ii) $B = \{x : x \in \mathbb{W} \text{ and } 5x - 3 \leq 20\}$

Solution:

Given $5x - 3 \leq 20$

Adding 3 to both sides, we get

$\rightarrow 5x - 3 + 3 \leq 20 + 3 \rightarrow 5x \leq 23$

Dividing both sides by 5, we get

$\rightarrow x \leq 23/5 \rightarrow x \leq 4.6 \therefore B = \{0, 1, 2, 3, 4\}$

We know that a set with finite number of elements is called a finite set.

$\therefore B = \{x : x \in \mathbb{W} \text{ and } 5x - 3 \leq 20\}$ is a finite set.

(iii) $P = \{y : y = 3x - 2, x \in \mathbb{N} \text{ \& } x > 5\}$

Solution:

Given $y = 3x - 2$

$x = 6, 7, 8, 9, \dots$ [$\because x > 5$]

When $x = 6, y = 3 \times 6 - 2 = 18 - 2 = 16$

When $x = 7, y = 3 \times 7 - 2 = 21 - 2 = 19$

When $x = 8, y = 3 \times 8 - 2 = 24 - 2 = 22$

When $x = 9, y = 3 \times 9 - 2 = 27 - 2 = 25 \therefore P = \{16, 19, 22, 25, \dots\}$

We know that, a set which is not finite is called an infinite set.

$\therefore P = \{y : y = 3x - 2, x \in \mathbb{N} \text{ \& } x > 5\}$ is an infinite set.

(iv) $M = \{r : r = 3/n ; n \in \mathbb{W} \text{ and } 6 < n \leq 15\}$

Solution:

Given $r = 3/n$

$n = 7, 8, 9, 10, 11, 12, 13, 14, 15$ [$\because 6 < n \leq 15$]

When $n = 7, r = 3/7$

When $n = 8, r = 3/8$

When $n = 9, r = 3/9$

When $n = 10, r = 3/10$

When $n = 11$, $r = 3/11$

When $n = 12$, $r = 3/12$

When $n = 13$, $r = 3/13$

When $n = 14$, $r = 3/14$

When $n = 15$, $r = 3/15 \therefore M = \{3/7, 3/8, 3/9, 3/10, 3/11, 3/12, 3/13, 3/14, 3/15\}$

We know that a set with finite number of elements is called a finite set.

$\therefore M = \{r : r = 3/n ; n \in W \text{ and } 6 < n \leq 15\}$ is a finite set.

Question 4.

Find, which of the following sets singleton sets are:

(i) The set of points of intersection of two non-parallel straight lines in the same plane

A set, which has only one element in it, is called a SINGLETON or unit set.

Solution:

The set of points of intersection of two non-parallel straight lines in the same plane.

Therefore, the set has only one element in it.

Then, the given set is singleton set.

(ii) $A = \{x : 7x - 3 = 11\}$

Solution:

Given $7x - 3 = 11$

$\rightarrow 7x = 11 + 3 \rightarrow 7x = 14 \rightarrow x = 14/7 = 2 \therefore A = \{2\}$

The set A has only one element in it.

Hence the given set A is a singleton set.

(iii) $B = \{y : 2y + 1 < 3 \text{ and } y \in W\}$

Solution:

Given $2y + 1 < 3$

Subtracting 1 from both sides, we get

$\rightarrow 2y + 1 - 1 < 3 - 1 \rightarrow 2y < 2$

(Dividing both sides by 2)

$\rightarrow y < 2/2 \rightarrow y < 1 \therefore B = \{0\}$

The set B has only one element in it.

Hence the given set B is a singleton set.

Question 5.

Find, which of the following sets are empty:

(i) The set of points of intersection of two parallel lines.

Solution:

“The set of points of intersection of two parallel lines” is an empty set \because two parallel lines do not intersect anywhere

(ii) $A = \{x : x \in N \text{ and } 5 < x < 6\}$

Solution:

$A = \{x : x \in N \text{ and } 5 < x \leq 6\}$

As, $5 < x \leq 6 \therefore x = 6 \therefore A = \{6\}$

The set A has one element in it.

Hence the given set A is not an empty set.

(iii) $B = \{x : x^2 + 4 = 0, x \in \mathbb{N}\}$

Solution:

Given $x^2 + 4 = 0 \rightarrow x^2 = -4 \rightarrow x = \sqrt{-4}$ which is not a natural number.

But $x \in \mathbb{N} \therefore B = \{\}$

\therefore Given set B is an empty set.

(iv) $C = \{\text{Even numbers between 6 \& 10}\}$

Solution:

Given $C = \{\text{Even numbers between 6 and 10}\}$

$\therefore C = \{8\}$

The set C has one element in it.

Hence, the given set C is not an empty set.

(v) $D = \{\text{prime numbers between 7 \& 11}\}$

Solution:

$D = \{\text{Prime numbers between 7 and 11}\}$

There is no prime number between 7 and 11.

$\therefore D = \{\}$

The set D has no element in it.

Hence, the given set D is an empty set or null set.

Question 6.

(i) Are the sets $A = \{4, 5, 6\}$ and $B = \{x : x^2 - 5x - 6 = 0\}$ disjoint?

(ii) Are the sets $A = \{b, c, d, e\}$ and $B = \{x : x \text{ is a letter in the word 'MASTER'}\}$ joint?

Note:

(i) Two sets are said to be joint sets, if they have atleast one element in common.

(ii) Two sets are said to be disjoint, if they have no element in common.

Solution:

(i) $A = \{4, 5, 6\}$

$B = \{x : x^2 - 5x - 6 = 0\}$ $x^2 - 5x - 6 = 0 \rightarrow x^2 - 6x + x - 6 = 0$

$\rightarrow x(x - 6) + 1(x - 6) = 0 \rightarrow (x - 6)(x + 1) = 0$ $x - 6 = 0 \rightarrow x = 6$ $x + 1 = 0 \rightarrow x = -1$

$\therefore B = \{6, -1\}$

\therefore Set A and B have the element 6 in common. So these sets are joint sets.

Hence set A and set B are not disjoint.

(ii) $A = \{b, c, d, e\}$

$B = \{x : x \text{ is a letter in the word "MASTER"}\}$

$\therefore B = \{m, a, s, t, e, r\}$

The element 'e' is common in both the sets A and B.

Hence set A and set B are joint sets.

Question 7.

State, whether the following pairs of sets are equivalent or not:

(i) $A = \{x : x \in \mathbb{N} \text{ and } 11 \geq 2x - 1\}$ and $B = \{y : y \in \mathbb{W} \text{ and } 3 \leq y \leq 9\}$

(ii) Set of integers and set of natural numbers.

(iii) Set of whole numbers and set of multiples of 3.

(iv) $P = \{5, 6, 7, 8\}$ and $M = \{x : x \in W \text{ and } x < 4\}$

Note: Two sets are said to be equivalent, if they contain the same number of elements.

Solution:

(i) $A = \{x : x \in N \text{ and } 11 \geq 2x - 1\}$ $11 \geq 2x - 1$

Adding 1 on both sides, we get

$\rightarrow 11 + 1 \geq 2x - 1 + 1 \rightarrow 12 \geq 2x$

Dividing both sides by 2, we get

$\rightarrow 12/2 \geq x \rightarrow 12/2 \geq x \rightarrow 6 \geq x \therefore A = \{1, 2, 3, 4, 5, 6\}$

\therefore Cardinal number of set A, $n(A) = 6$

$B = \{y : y \in W \text{ and } 3 \leq y \leq 9\} \therefore 3 \leq y \leq 9$

$B = \{3, 4, 5, 6, 7, 8, 9\}$

Cardinal number of set B, $n(B) = 7$

Set A and set B are not equivalent.

(ii) We know that, Set of integers has infinite number of elements.

Set of natural numbers has infinite number of elements.

Set of integers and set of natural numbers are equivalent \therefore both sets have infinite number of elements.

(iii) Set of whole numbers has infinite number of elements.

Set of multiples of 3, has infinite number of elements.

Set of whole numbers and set of multiples of 3 are equivalent \therefore both sets have infinite number of elements.

(iv) $P = \{5, 6, 7, 8\}$

Cardinal number of set P, $n(P) = 4$

$M = \{x : x \in W \text{ and } x \leq 4\}$

$M = \{0, 1, 2, 3, 4\}$

Cardinal number of set M, $n(M) = 5$

These sets are not equivalent, \therefore both sets do not have same number of elements.

Question 8.

State, whether the following pairs of sets are equal or not:

(i) $A = \{2, 4, 6, 8\}$ and

$B = \{2n : n \in N \text{ and } n < 5\}$

(ii) $M = \{x : x \in W \text{ and } x + 3 < 8\}$ $N = \{y : y = 2n - 1, n \in N \text{ and } n < 5\}$

(iii) $E = \{x : x^2 + 8x - 9 = 0\}$

$F = \{1, -9\}$

(iv) $A = \{x : x \in N, x < 3\}$ $B = \{y : y^2 - 3y + 2 = 0\}$

Note: Two sets are equal, if both the sets have same (identical) elements.

Solution:

(i) $A = \{2, 4, 6, 8\}$

$B = \{2n : n \in N \text{ and } n < 5\}$

$n = 1, 2, 3, 4$ [$\therefore n < 5$]

When $n = 1$, $2n = 2 \times 1 = 2$

When $n = 2$, $2n = 2 \times 2 = 4$

When $n = 3$, $2n = 2 \times 3 = 6$

When $n = 4$, $2n = 2 \times 4 = 8 \therefore B = \{2, 4, 6, 8\}$

Now we see that elements of sets A and B are same (identical)

∴ Sets A and B are equal.

(ii) $M = \{x : x \in W \text{ and } x + 3 < 8\}$

$$x + 3 < 8$$

Subtracting 3 from both sides, we get

$$\rightarrow x < 8 - 3 \rightarrow x < 5 \therefore M = \{0, 1, 2, 3, 4\} \quad N = \{y : y = 2n - 1, n \in N \text{ and } n < 5\}$$

$$y = 2n - 1$$

$$n = 1, 2, 3, 4 \quad [\because n < 5]$$

$$\text{When } n = 1, y = 2 \times 1 - 1 \rightarrow y = 2 - 1 = 1$$

$$\text{When } n = 2, y = 2 \times 2 - 1 \rightarrow y = 4 - 1 = 3$$

$$\text{When } n = 3, y = 2 \times 3 - 1 \rightarrow y = 6 - 1 = 5$$

$$\text{When } n = 4, y = 2 \times 4 - 1 \rightarrow y = 8 - 1 = 7 \therefore N = \{1, 3, 5, 7\}$$

The elements of sets M and N are not same (identical).

∴ Sets M and N are not equal.

(iii) $E = \{x : x^2 + 8x - 9 = 0\}$

$$x^2 + 8x - 9 = 0$$

$$\rightarrow x^2 + 9x - x - 9 = 0 \rightarrow x(x + 9) - 1(x + 9) = 0 \rightarrow (x - 1)(x + 9) = 0$$

$$\therefore \text{Either } x + 9 = 0 \text{ or } x - 1 = 0 \quad x + 9 = 0 \rightarrow x = -9 \quad x - 1 = 0 \rightarrow x = 1 \therefore E = \{-9, 1\}$$

$$F = \{1, -9\}$$

The elements of sets E and F are same (identical)

∴ Sets E and F are equal.

(iv) $A = \{x : x \in N, x < 3\}$

$$A = \{1, 2\}$$

$$B = \{y : y^2 - 3y + 2 = 0\} \quad y^2 - 3y + 2 = 0 \rightarrow y^2 - 2y - y + 2 = 0 \rightarrow y(y - 2) - 1(y - 2) = 0$$

$$\rightarrow (y - 2)(y - 1) = 0 \therefore \text{Either } y - 2 = 0 \text{ or } y - 1 = 0 \quad y - 2 = 0 \rightarrow y = 2 \quad y - 1 = 0 \rightarrow y = 1$$

$$\therefore B = \{1, 2\}$$

The elements of sets A and B are same (identical).

∴ Sets A and B are equal.

Question 9.

State whether each of the following sets is a finite set or an infinite set:

(i) The set of multiples of 8.

(ii) The set of integers less than 10.

(iii) The set of whole numbers less than 12.

(iv) $\{x : x = 3n - 2, n \in W, n \leq 8\}$

(v) $\{x : x = 3n - 2, n \in Z, n \leq 8\}$

(vi) $\{x : x = (n - 2)/(n + 1), n \in w\}$

Solution:

(i) The set of multiples of 8

$$= \{8, 16, 24, 32, \dots\}$$

It is an infinite set.

(ii) The set of integers less than 10

$$= \{9, 8, 7, 6, 5, 4, 3, 2, 1, -1, -2, \dots\}$$

It is an infinite set.

(iii) The set of whole numbers less than 12

$$= \{11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0\}$$

It is a finite set.

$$(iv) \{x : x = 3n - 2, n \in W, n \leq 8\}$$

Substituting the value of $n = (0, 1, 2, 3, 4, 5, 6, 7, 8)$ we get
 $= \{-2, 1, 4, 7, 10, 13, 16, 19, 22\}$

It is a finite set.

$$(v) \{x : x = 3n - 2, n \in Z, n \leq 8\}$$

Substituting the value of $n = (0, 1, 2, 3, 4, 5, 6, 7, 8)$ we get
 $x = \{22, 19, 16, 13, 10, 7, 4, 1, -2, -5, \dots\}$

It is an infinite set.

$$(vi) \{x : x = n - 2n + 1, n \in W\} \{-2, -12, 0, 14, 25, \dots\}$$

It is an infinite set.

Question 10.

Answer, whether the following statements are true or false. Give reasons.

(i) The set of even natural numbers less than 21 and the set of odd natural numbers less than 21 are equivalent sets.

(ii) If $E = \{\text{factors of } 16\}$ and $F = \{\text{factors of } 20\}$, then $E = F$.

(iii) The set $A = \{\text{integers less than } 20\}$ is a finite set.

(iv) If $A = \{x : x \text{ is an even prime number}\}$, then set A is empty.

(v) The set of odd prime numbers is the empty set.

(vi) The set of squares of integers and the set of whole numbers are equal sets.

(vii) In $n(P) = n(M)$, then $P \rightarrow M$

(viii) If set $P = \text{set } M$, then $n(P) = n(M)$

(ix) $n(A) = n(B) \Rightarrow (A = B)$

Solution:

(i) Set of even natural number less than 21 = $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

\therefore Cardinal Number of this set = 10

Set of odd natural numbers less than 21 = $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

\therefore Cardinal number of this set = 10

The cardinal numbers of both these sets = 10

\therefore The given statement is True.

(ii) $E = \{\text{Factors of } 16\}$

$$1 \times 16 = 16, 2 \times 8 = 16, 4 \times 4 = 16$$

$$E = \{1, 2, 4, 8, 16\}$$

$F = \{\text{Factors of } 20\}$

$$1 \times 20 = 20, 2 \times 10 = 20, 4 \times 5 = 20$$

$$F = \{1, 2, 4, 5, 10, 20\}$$

The elements of set E and set F are not same (identical)

Therefore, the given statement is false.

(iii) $A = \{\text{Integers less than } 20\}$

$$A = \{19, 18, 17, 16, \dots, 0, -1, -2, -3, \dots\}$$

The set A has infinite elements, so it is not a finite set.

Therefore, the given statement is false.

(iv) $A = \{x : x \text{ is an even prime number}\} = \{2\}$

The given set A has only one element in it, so it is a singleton set.

Therefore, it is not an empty set.

Hence, the given statement is false.

(v) Set of odd prime numbers = $\{3, 5, 7, 11, 13, 17, 19, 23, \dots\}$

The given set has infinite number of element, so it is an infinite set.

Therefore it is not an empty set.

Hence, the given statement is false.

(vi)

Integer	Square of Integer	Whole No.
0	$(0)^2 = 0$	0
± 1	$(\pm 1)^2 = 1$	1
± 2	$(\pm 2)^2 = 4$	2
± 3	$(\pm 3)^2 = 9$	3
± 4	$(\pm 4)^2 = 16$	4
± 5	$(\pm 5)^2 = 25$	5
.	.	.
.	.	.
.	.	.
.	.	.

Set of squares of integers = $\{0, 1, 4, 9, 16, 25, \dots\}$

Set of whole number = $\{0, 1, 2, 3, 4, 5, 6, 7, \dots\}$

Therefore, these two sets are not equal sets.

Hence, the given statement is false.

(vii) $n(P) = n(M)$ means the number of elements of set P = Number of elements of set M.

\therefore Sets P and M are equivalent.

Hence, the given statement is true.

(viii) Set P = Set M

It means sets P and M are equal. Equal sets are equivalent also.

\therefore Number of elements of set P = Number of elements of set M

Hence, the given statement is true.

(xi) $n(A) = n(B)$

i.e. Number of elements of set A = Number of elements of set B

\therefore Given sets are equivalent but not equal.

Hence, the given statement is false.