

SECTION B: Long Answer Type Questions

Q.36 A new system of units is proposed in which unit of mass is α kg, unit of length β m and unit of time γ s. How much will 5J measure in this new system?

Answer: Using $n_1 u_1 = n_2 u_2$: $n_2 = n_1 \times (M_1/M_2) \times (L_1/L_2)^2 \times (T_2/T_1)^2$ $n_2 = 5 \times (1/\alpha) \times (1/\beta)^2 \times (\gamma/1)^2$
 $n_2 = 5\gamma^2/(\alpha\beta^2)$

Q.37 The volume of a liquid flowing out per second of a pipe of length l and radius r is written by a student as $V = (\pi pr^4)/(8\eta l)$ where p is the pressure difference between the two ends of the pipe and η is coefficient of viscosity of the liquid having dimensional formula $[ML^{-1}T^{-1}]$. Check whether the equation is dimensionally correct.

Answer: LHS: $[V] = [L^3T^{-1}]$ RHS: $[ML^{-1}T^{-2}][L^4]/([ML^{-1}T^{-1}][L]) = [L^3T^{-1}]$ Since LHS = RHS, the equation is dimensionally correct.

Q.38 A physical quantity X is related to four measurable quantities a , b , c and d as follows $X = a^2 b^3 c^5 / d^2$. The percentage error in the measurement of a , b , c and d are 1%, 2%, 3% and 4%, respectively. What is the percentage error in quantity X ? If the value of X calculated on the basis of the above relation is 2.763, to what value should you round off the result.

Answer: $\Delta X/X \times 100 = |2 \times 1\% + 3 \times 2\% + (5/2) \times 3\% + 2 \times 4\%| = |2 + 6 + 7.5 + 8|\% = 23.5\%$
 Mean absolute error = $\pm 0.235 \approx \pm 0.24$ X should be rounded to 2 significant digits: $X = 2.8$

Q.39 In the expression $P = El^2 m^{-5} G^{-2}$, E , m , l and G denote energy, mass, angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity.

Answer: $[P] = [ML^2T^{-2}] \times [ML^2T^{-1}]^2 \times [M]^{-5} \times [M^{-1}L^3T^{-2}]^{-2} = [ML^2T^{-2}] \times [M^2L^4T^{-2}] \times [M^{-5}] \times [M^2L^{-6}T^4] = [M^{1+2-5+2}L^{2+4-6}T^{-2-2+4}] = [M^0L^0T^0]$ Therefore, P is dimensionless.

Q.40 If velocity of light c , Planck's constant h and gravitational constant G are taken as fundamental quantities, then express mass, length and time in terms of dimensions of these quantities.

Answer: Given: $[h] = [ML^2T^{-1}]$, $[c] = [LT^{-1}]$, $[G] = [M^{-1}L^3T^{-2}]$

For mass: $m \propto c^x h^y G^z$ Solving: $y - z = 1$, $x + 2y + 3z = 0$, $-x - y - 2z = 0$ Result: $m = k_1 \sqrt{ch/G}$

For length: $L \propto c^x h^y G^z$
 Result: $L = k_2 (hG/c^3)^{1/2}$

For time: $T \propto c^x h^y G^z$ Result: $T = k_3 (hG/c^5)^{1/2}$

Q.41 An artificial satellite is revolving around a planet of mass M and radius R , in a circular orbit of radius r . From Kepler's third law about the period of a satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit r . Show using dimensional analysis, that $T = (k/R)\sqrt{r^3/g}$, where k is a dimensionless constant and g is acceleration due to gravity.

Answer: By Kepler's third law: $T^2 \propto r^3 \Rightarrow T \propto r^{3/2}$ Let $T \propto r^{3/2} R^a g^b$ $[T] = [L]^{3/2} \times [L]^a \times [LT^{-2}]^b$ Comparing dimensions: $a + b + 3/2 = 0$ and $-2b = 1$ Solving: $b = -1/2$, $a = -1$ Therefore: $T = (k/R)\sqrt{r^3/g}$

Q.42 [Oleic Acid Experiment Question - Complex multi-part]

Answer: (a) Oleic acid doesn't dissolve in water, hence dissolved in alcohol (b) Lycopodium powder helps visualize the circular area where oleic acid spreads (c) Volume of oleic acid in each mL = $(1/20) \times (1/20) = 1/400$ mL (d) Using burette to measure n drops in 1 mL (e) Volume of oleic acid in one drop = $1/(400n)$ mL

Q.43 (a) How many astronomical units (AU) make 1 parsec?

Answer: 1 parsec = $(3600 \times 180)/\pi$ AU = 206,265 AU $\approx 2 \times 10^5$ AU

(b) Angular size of sun-like star at 2 parsecs with 100× magnification

Answer: At 1 parsec: 1.5×10^{-5} arcmin With 100× magnification: 1.5×10^{-3} arcmin Due to atmospheric fluctuations: still appears ~ 1 arcmin

(c) Mars size through telescope Answer: Mars diameter at closest approach with 100× magnification = 30' This exceeds atmospheric resolution limit, so appears magnified.

Q.44 Einstein's mass-energy relation emerging out of his famous theory of relativity relates mass (m) to energy (E) as $E = mc^2$, where c is speed of light in vacuum.

(a) Show that the energy equivalent of 1u is 931.5 MeV.

Answer: $E = mc^2 = (1.67 \times 10^{-27})(3 \times 10^8)^2 = 1.67 \times 9 \times 10^{-11}$ J $E = (1.67 \times 9 \times 10^{-11})/(1.6 \times 10^{-13})$ MeV = 931.5 MeV

(b) Correct the dimensionally incorrect relation 1u = 931.5 MeV

Answer: $1u \times c^2 = 931.5$ MeV