

EXERCISE 22.2

Question. 1

Solution:

From the question it is given that,

$$y^2 = (x - c)^3 \quad \dots \text{ [equation (i)]}$$

Now, differentiate the equation (i) with respect x,

$$2y \, dy/dx = 3(x - c)^2$$

By cross multiplication we get,

$$(x - c)^2 = (2y/3) (dy/dx)$$

Transferring square to Right Hand Side (RHS),

$$(x - c) = \{(2y/3) (dy/dx)\}^{1/2}$$

Then, substitute the value of (x - c) in equation (i),

$$y^2 = [\{(2y/3) (dy/dx)\}^{1/2}]^3$$

$$y^2 = \{(2y/3) (dy/dx)\}^{3/2}$$

Squaring on both the side we get,

$$(y^2)^2 = [\{(2y/3) (dy/dx)\}^{3/2}]^2$$

$$y^4 = [\{(2y/3) (dy/dx)\}^3]$$

$$y^4 = (2y/3)^3 (dy/dx)^3$$

$$y^4 = (8y^3/27) (dy/dx)^3$$

By cross multiplication we get,

$$27y^4/y^3 = 8 (dy/dx)^3$$

$$27y = 8 (dy/dx)^3$$

Question. 2

Solution:

From the question it is given that,

$$y = e^{mx} \quad \dots \text{ [equation (i)]}$$

Now, differentiate the equation (i) with respect x,

$$dy/dx = me^{mx}$$

We know that, from equation (i) $y = e^{mx}$

So, applying log on both side we get,

$$\log y = mx$$

$$m = \log y/x$$

Now, substitute the value of m and e^{mx} is equation (i) we get,

$$dy/dx = (\log y/x)y$$

By cross multiplication we get,

$$x(dy/dx) = y \log y$$

Question. 3(i)

Solution:

From the question it is given that,

$$y^2 = 4ax \quad \dots \text{ [equation (i)]}$$

Now, differentiate the equation (i) with respect x,

$$2y (dy/dx) = 4a \quad \dots \text{ [equation (ii)]}$$

From equation (i), $a = y^2/4x$

Then, substitute the value of a in equation (ii),

$$2y (dy/dx) = 4(y^2/4x)$$

$$2y (dy/dx) = y^2/x$$

By cross multiplication we get,

$$2x (dy/dx) = y^2/y$$

$$2x (dy/dx) = y$$

Question. 3(ii)

Solution:

From the question it is given that,

$$y = cx + 2c^2 + c^3 \quad \dots \text{ [equation (i)]}$$

Now, differentiate the equation (i) with respect x,

$$dy/dx = c$$

Then, substitute the value of c in equation (i),

$$y = (dy/dx) x + 2(dy/dx)^2 + (dy/dx)^3$$

Question. 3(iii)

Solution:

From the question it is given that,

$$xy = a^2 \quad \dots \text{ [equation (i)]}$$

Now, differentiate the equation (i) with respect x,

$$x dy/dx + y(1) = 0$$

$$x dy/dx + y = 0$$

Question. 3(iv)

Solution:

From the question it is given that,

$$y = ax^2 + bx + c \quad \dots \text{ [equation (i)]}$$

Now, differentiate the equation (i) with respect x,

$$dy/dx = 2ax + b$$

Then, the above equation is again differentiating with respect to x we get,

$$d^2y/dx^2 = 2a$$

The above equation is again differentiating with respect to x we get

$$d^3y/dx^3 = 0$$

Question. 4

Solution:

From the question it is given that,

$$y = Ae^{2x} + Be^{-2x} \quad \dots \text{ [equation (i)]}$$

Now, differentiate the equation (i) with respect x,

$$dy/dx = 2Ae^{2x} - 2Be^{-2x}$$

Then, the above equation is again differentiating with respect to x we get,

$$d^2y/dx^2 = 4Ae^{2x} + 4Be^{-2x}$$

Taking common terms outside,

$$d^2y/dx^2 = 4 (Ae^{2x} + Be^{-2x})$$

From equation (i) we have $y = Ae^{2x} + Be^{-2x}$

Therefore, $d^2y/dx^2 = 4y$

Question. 5

Solution:

From the question it is given that,

$$x = A \cos nt + B \sin nt \quad \dots \text{ [equation (i)]}$$

Now, differentiate the equation (i) with respect t,

$$dx/dt = -An \sin nt + Bn \cos nt$$

Then, the above equation is again differentiating with respect to t we get,

$$d^2x/dt^2 = -An^2 \cos nt - Bn^2 \sin nt$$

Taking common terms outside,

$$d^2x/dt^2 = -n^2 (A \cos nt + B \sin nt)$$

From equation (i) we have $x = A \cos nt + B \sin nt$

$$d^2x/dt^2 = -n^2x$$

By transposing we get,

$$d^2x/dt^2 + n^2x = 0$$