NCERT Solutions for Class-XII Maths

Chapter-1
Exercise- 3.1

1. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

(i) The order of the matrix
(ii) The number of elements,
(iii) Write the elements a_{13}, a_{21}, a_{33}, a_{24}, a_{23}

1. (i) In the given matrix, the number of rows is 3 and the number of columns is 4.
Therefore, the order of the matrix is 3 × 4.
(ii) Since the order of the matrix is 3 × 4, there are 3 × 4 = 12 elements in it.
(iii) $a_{13} = 19$, $a_{21} = 35$, $a_{33} = -5$, $a_{24} = 12$, $a_{23} = \frac{5}{2}$

2. If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

2. It is known that if a matrix is of the order m × n, then it has mn elements.
Therefore, to find all the possible orders of a matrix having 24 elements, we had to find all the ordered pairs of natural numbers whose product is 24.
The ordered pairs are: (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), and (6, 4).
Therefore, the possible orders of a matrix having 24 elements are;
1 × 24, 24 × 1, 2 × 12, 12 × 2, 3 × 8, 8 × 3, 4 × 6, 6 × 4
(1, 13) and (13, 1) are the ordered pairs of natural numbers whose product is 13.
Therefore, the possible orders of a matrix having 13 elements are 1 × 13 and 13 × 1.

3. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

3. We know that if a matrix is of the order m × n, it has mn elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18. The ordered pairs are: (1, 18), (18, 1), (2, 9), (9, 2), (3, 6,), and (6, 3)
Hence, the possible orders of a matrix having 18 elements are:
1 × 18, 18 × 1, 2 × 9, 9 × 2, 3 × 6, and 6 × 3
(1, 5) and (5, 1) are the ordered pairs of natural numbers whose product is 5.
Hence, the possible orders of a matrix having 5 elements are 1 × 5 and 5 × 1.

4. Construct a 2 × 2 matrix, A = [a_{ij}], whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$ **(ii)** $a_{ij} = \frac{i}{j}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

4. (i) In general, a 2 × 2 matrix is given by A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$a_{ij} = \frac{(i+j)^2}{2}, i, j = 1, 2$

Therefore,

$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2$

$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$

$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$

$a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$

Therefore, the required matrix is A = $\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

(ii) In general, a 2 × 2 matrix is given by A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$a_{ij} = \frac{i}{j}, i, j = 1, 2$

Therefore,

$a_{11} = \frac{1}{1} = 1$

$a_{12} = \frac{1}{2}$

$a_{21} = \frac{2}{1} = 2$

$a_{22} = \frac{2}{2} = 1$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$.

(iii In general, a 2 × 2 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$a_{ij} = \frac{(i+2j)^2}{2}, i, j = 1, 2$

Therefore,

$a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2}$

$a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2}$

$a_{21} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$

$a_{22} = \frac{(2+4)^2}{2} = \frac{36}{2} = 18$

Therefore, the required matrix is $A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

5. Construct a 3 × 4 matrix, whose elements are given by

(i) $a_{ij} = \frac{1}{2}|-3i + j|$ (ii) $a_{ij} = 2i - j$

5. (i) $a_{ij} = \frac{1}{2}|-3i + j|$

In general a 3 × 4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

$a_{11} = \frac{1}{2}|-3 \times 1 + 1| = 1$

$a_{12} = \frac{1}{2}|-3 \times 1 + 2| = \frac{1}{2}$

$a_{13} = \frac{1}{2}|-3 \times 1 + 3| = 0$

$a_{14} = \frac{1}{2}|-3 \times 1 + 4| = \frac{1}{2}$

$$a_{21} = \frac{1}{2}|-3\times2+1| = \frac{5}{2}$$
$$a_{22} = \frac{1}{2}|-3\times2+2| = 2$$
$$a_{23} = \frac{1}{2}|-3\times2+3| = \frac{3}{2}$$
$$a_{24} = \frac{1}{2}|-3\times2+4| = 1$$

$$a_{31} = \frac{1}{2}|-3\times3+1| = 4$$
$$a_{32} = \frac{1}{2}|-3\times3+2| = \frac{7}{2}$$
$$a_{33} = \frac{1}{2}|-3\times3+3| = 3$$
$$a_{34} = \frac{1}{2}|-3\times3+4| = \frac{5}{2}$$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{1} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

(ii) $a_{ij} = 2i - j$

In general a 3 × 4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{223} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

$$a_{11} = 2\times1-1 = 1$$
$$a_{12} = 2\times1-2 = 0$$
$$a_{13} = 2\times1-3 = -1$$
$$a_{14} = 2\times1-4 = -2$$

$$a_{21} = 2\times2-1 = 3$$
$$a_{22} = 2\times2-2 = 3$$
$$a_{23} = 2\times2-3 = 1$$
$$a_{24} = 2\times2-4 = 0$$

$a_{31} = 2 \times 3 - 1 = 5$

$a_{32} = 2 \times 3 - 2 = 4$

$a_{33} = 2 \times 3 - 3 = 3$

$a_{34} = 2 \times 3 - 4 = 2$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

6. Find the value of x, y, and z from the following equation:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

6. (i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

Since, the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding element, we have:
x =1, y =4 and z = 5

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Since, the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding element, we have:
x + y = 6, xy = 8, 5 + z = 5
Now, 5 + z = 5
=> z = 0
We know that $(x - y)^2 = (x + y)^2 - 4xy$
=> $(x - y)^2 = 36 - 32 = 4$
=> x – y = ±2
Now, when x – y = 2, x + y = 6 , we get, x = 4 and y = 2
And, when x – y = -2, x + y = 6 , we get, x = 2 and y = 4
Therefore, x = 4, y = 2 and z = 0 or x = 2, y = 4 and z = 0

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Since, the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding element, we have:

x + y + z = 9

x + z = 5

y + z = 7

From above equations we get,

y +5 = 9

⇨ y = 4

Then, putting the value of y we get

4 + z = 7

⇨ z = 3

Therefore, x + z = 5

⇨ x = 2

Therefore, x =2, y = 4 and z = 3.

7. Find the value of a, b, c, and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

7. $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

As the two matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$a - b = -1$ …… (1)

$2a - b = 0$ …… (2)

$2a + c = 5$ ……. (3)

$3c + d = 13$ ….. (4)

From (2), we have:

$b = 2a$

Then, from (1), we have:

$a - 2a = -1 \Rightarrow a = 1 \Rightarrow b = 2$

Now, from (3), we have:

$2 \times 1 + c = 5 \Rightarrow c = 3$

From (4) we have:

$3 \times 3 + d = 13$

$\Rightarrow 9 + d = 13 \Rightarrow d = 4$

$\therefore a = 1, b = 2, c = 3$ and $d = 4$.

8. $A = \left[a_{ij}\right]_{m \times n}$ is a square matrix, if

(A) $m < n$ (B) $m > n$

(C) $m = n$ (D) None of these

8. The correct option is (C).

Explanation: We know that if a given matrix is said to be square matrix if the number of rows is equal to the number of columns.

Therefore, $A = [a_{ij}]_{m \times n}$ is a square matrix, if m = n.

9. Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x = \frac{-1}{3}, y = 7$ (B) Not possible to find

(C) $y = 7, x = \frac{-2}{3}$ (D) $x = \frac{-1}{3}, y = \frac{-2}{3}$

9. The correct answer is B.

It is given that $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating the corresponding elements, we get:

$3x + 7 = 0 \Rightarrow x = \frac{-7}{3}$ and $5 = y - 2 \Rightarrow y = 7$

$y + 1 = 8 \Rightarrow y = 7$ and $2 - 3x = 4 \Rightarrow x = \frac{-2}{3}$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x, which is not possible.

Hence, it is not possible to find the values of x and y for which the given matrices are equal.

10. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:

(A) 27 (B) 18

(C) 81 (D) 512

10. The correct option is (D).

Explanation: The given matrix of the order 3 × 3 has 9 elements with each entry 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Hence, the required number of possible matrices is $2^9 = 512$.