$2y + x = 39$
$\Rightarrow x = 39 - 2y$
Putting the value of x in (i), we get:
$(39 - 2y)y = 180$
$\Rightarrow 39 - 2y^2 = 180$
$\Rightarrow 39y - 2y^2 - 180 = 0$
$\Rightarrow 2y^2 - 39y + 180 = 0$
$\Rightarrow 2y^2 - (24 + 15)y + 180 = 0$
$\Rightarrow 2y^2 - 24y - 15y + 180 = 0$
$\Rightarrow 2y(y - 12) - 15(y - 12) = 0$
$\Rightarrow (y - 12)(2y - 15) = 0$
$\Rightarrow y = 12 \ or \ y = \frac{15}{2} = 7.5$
If $y = 12, x = 39 - 24 = 15$
If $y = 7.5, x = 39 - 15 = 24$
Thus, the
length and breadth of the garden are (15 m and 12 m) or (24 m and 7.5 m), respectively.

66. The area of a right triangle is 600 cm^2 . If the base of the triangle exceeds the altitude by 10 cm, find the dimensions of the triangle.
Sol:
Let the altitude of the triangle be x cm
Therefore, the base of the triangle will be $(x + 10)cm$
Area of triangle $= \frac{1}{2}x(x + 10) = 600$
$\Rightarrow (x + 10) = 1200$
$\Rightarrow x^2 + 10x - 1200 = 0$
$\Rightarrow x^2 + (40 - 30)x - 1200 = 0$
$\Rightarrow x^2 + 40x - 30x - 1200 = 0$
$\Rightarrow x(x + 40) - 30(x + 40) = 0$
$\Rightarrow (x + 40)(x - 30) = 0$
$\Rightarrow x = -40 \ or \ x = 30$
$\Rightarrow x = 30$ [$\because$ Altitude cannot be negative]
Thus, the altitude and base of the triangle are 30 cm and (30 + 10 = 40) cm, respectively.

$(Hypotenuse)^2 = (Altitude)^2 + (Base)^2$

$\Rightarrow (Hypotenuse)^2 = (30)^2 (40)^2$

$\Rightarrow (Hypotenuse)^2 = 900 + 1600 = 2500$

$\Rightarrow (Hypotenuse)^2 = (50)^2$

$\Rightarrow (Hypotenuse) = 50$

Thus, the dimensions of the triangle are:
Hypotenuse = 50 cm
Altitude = 30 cm
Base = 40 cm

67. The area of right-angled triangle is 96 sq meters. If the base is three time the altitude, find the base.

Sol:

Let the altitude of the triangle be x m.

Therefore, the base will be $3x$ m.

Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Altitude

$\therefore \frac{1}{2} \times 3x \times x = 96 (\because Area = 96\, sq\, m)$

$\Rightarrow \frac{x^2}{2} = 32$

$\Rightarrow x^2 = 64$

$\Rightarrow x = \pm 8$

The value of x cannot be negative

Therefore, the altitude and base of the triangle are 8 m and $(3 \times 8 = 24 m)$, respectively.

68. The area of right -angled triangle is 165 sq meters. Determine its base and altitude if the latter exceeds the former by 7 meters.

Sol:

Let the base be x m.

Therefore, the altitude will be $(x+7)m$.

Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Altitude

$\therefore \frac{1}{2} \times x \times (x+7) = 165$

$\Rightarrow x^2 + 7x = 330$

$\Rightarrow x^2 + 7x - 330 = 0$

$\Rightarrow x^2+(22-15)x-330=0$

$\Rightarrow x^2+22x-15x-330=0$

$\Rightarrow x(x+22)-15(x+22)=0$

$\Rightarrow (x+22)(x-15)=0$

$\Rightarrow x=-22 \ or \ x=15$

The value of x cannot be negative
Therefore, the base is 15 m and the altitude is {(15 + 7) = 22 m}.

69. The hypotenuse of a right=-angled triangle is 20 meters. If the difference between the lengths of the other sides be 4 meters, find the other sides

Sol:

Let one side of the right-angled triangle be x m and the other side be $(x+4)m$.

On applying Pythagoras theorem, we have:

$20^2=(x+4)^2+x^2$

$\Rightarrow 400=x^2+8x+16+x^2$

$\Rightarrow 2x^2+8x-384=0$

$\Rightarrow x^2+4x-192=0$

$\Rightarrow x^2+(16-12)x-192=0$

$\Rightarrow x^2+16x-12x-192=0$

$\Rightarrow x^2(x+16)-12(x+16)=0$

$\Rightarrow (x+16)(x-12)=0$

$\Rightarrow x=-16 \ or \ x=12$

The value of x cannot be negative.
Therefore, the base is 12 m and the other side is {(12 + 4) = 16 m).

70. The length of the hypotenuse of a right-angled triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.

Sol:

Let the base and altitude of the right-angled triangle be x and y cm, respectively
Therefore, the hypotenuse will be $(x + 2)$ cm.

$\therefore (x+2)^2=y^2+x^2 \qquad ....(i)$

Again, the hypotenuse exceeds twice the length of the altitude by 1 cm.

$\therefore h=(2y+1)$

$\Rightarrow x+2=2y+1$

$\Rightarrow x=2y-1$

Putting the value of x in (i), we get:

$(2y-1+2)^2 = y^2+(2y-1)^2$

$\Rightarrow (2y+1)^2 = y^2+4y^2-4y+1$

$\Rightarrow 4y^2+4y+1 = 5y^2-4y+1$

$\Rightarrow -y^2+8y=0$

$\Rightarrow y^2-8y=0$

$\Rightarrow y(y-8)=0$

$\Rightarrow y=8\ cm$

$\therefore x=16-1=15\ cm$

$\therefore h=16+1=17\ cm$

Thus, the base, altitude and hypotenuse of the triangle are 15 cm, 8 cm and 17 cm, respectively.

71. The hypotenuse of a right-angled triangle is 1 meter less than twice the shortest side. If the third side 1 meter more than the shortest side, find the side, find the sides of the triangle.

Sol:

Let the shortest side be x m.

Therefore, according to the question:

Hypotenuse $=(2x-1)m$

Third side $=(x+1)m$

On applying Pythagoras theorem, we get:

$(2x-1)^2=(x+1)^2+x^2$

$\Rightarrow 4x^2-4x+1=x^2+2x+1+x^2$

$\Rightarrow 2x^2-6x=0$

$\Rightarrow 2x(x-3)=0$

$\Rightarrow x=0\ or\ x=3$

The length of the side cannot be 0; therefore, the shortest side is 3 m.

Therefore,

Hypotenuse $=(2\times 3-1)=5m$

Third side $=(3+1)=4m$

Exercise - 10F

1. Which of the following is a quadratic equation?

(a) $x^3 - 3\sqrt{x} + 2 = 0$ (b) $x + \frac{1}{x} = x^2$

(c) $x^2 + \frac{1}{x^2} = 5$ (d) $2x^2 - 5x = (x-1)^2$

Answer: (d) $2x^2 - 5x = (x-1)^2$

Sol:

A quadratic equation is the equation with degree 2.

$\because 2x^2 - 5x = (x-1)^2$

$\Rightarrow 2x^2 - 5x = x^2 - 2x + 1$

$\Rightarrow 2x^2 - 5x - x^2 + 2x - 1 = 0$

$\Rightarrow x^2 - 3x - 1 = 0,$ which is a quadratic equation

2. Which of the following is a quadratic equation?

(a) $(x^2+1) = (2-x)^2 + 3$ (b) $x^3 - x^2 = (x-1)^3$

(c) $2x^2 + 3 = (5+x)(2x-3)$ (d) None of these

Answer: (b) $x^3 - x^2 = (x-1)^3$

Sol:

$\because x^3 - x^2 = (x-1)^3$

$\Rightarrow x^3 - x^2 = x^3 - 3x^2 + 3x - 1$

$\Rightarrow 2x^2 - 3x + 1 = 0,$ which is a quadratic equation

3. Which of the following is not a quadratic equation?

(a) $3x - x^2 = x^2 + 5$ (b) $(x+2)^2 = 2(x^2 - 5)$

(c) $(\sqrt{2}x + 3)^2 = 2x^2 + 6$ (d) $(x-1)^2 = 3x^2 + x - 2$

Answer: (c) $(\sqrt{2}x + 3)^2 = 2x^2 + 6$

Sol:

$\because (\sqrt{2}x + 3)^2 = 2x^2 + 6$

$\Rightarrow 2x^2 + 9 + 6\sqrt{2}x = 2x^2 + 6$

$\Rightarrow 6\sqrt{2}x + 3 = 0,$ which is not a quadratic equation

4. If $x = 3$ is a solution of the equation $3x^2 + (k-1)x + 9 = 0$, then $k = ?$

(a) 11 (b) –11 (c) 13 (d) –13

Answer: (b) –11

Sol:

It is given that $x = 3$ is a solution of $3x^2 + (k-1)x + 9 = 0$; therefore, we have:

$3(3)^2 + (k-1)\times 3 + 9 = 0$

$\Rightarrow 27 + 3(k-1) + 9 = 0$

$\Rightarrow 3(k-1) = -36$

$\Rightarrow (k-1) = -12$

$\Rightarrow k = -11$

5. If one root of the equation $2x^2 + ax + 6 = 0$ is 2 then a=?

(a) 7 (b) -7 (c) $\frac{7}{2}$ (d) $\frac{-7}{2}$

Answer: (b) -7

Sol:

It is given that one root of the equation $2x^2 + ax + 6 = 0$ is 2.

$\therefore 2\times 2^2 + a\times 2 + 6 = 0$

$\Rightarrow 2a + 14 = 0$

$\Rightarrow a = -7$

6. The sum of the roots of the equation $x^2 - 6x + 2 = 0$ is

(a) 2 (b) -2 (c) 6 (d) -6

Answer: (b) -2

Sol:

Sum of the roots of the equation $x^2 - 6x + 2 = 0$ is

$\alpha + \beta = \frac{-b}{a} = \frac{-(-6)}{1} = 6,$ where α and β are the roots of the equation.

7. If the product of the roots of the equation $x^2 - 3x + k = 10$ is -2 then the value of k is

(a) -2 (b) -8 (c) 8 (d) 12

Answer: (c) 8

Sol:

It is given that the product of the roots of the equation $x^2 - 3x + k = 10$ is -2.

The equation can be rewritten as:

$x^2 - 3x + (k-10) = 0$

Product of the roots of a quadratic equation $= \frac{c}{a}$

$\Rightarrow \frac{c}{a} = -2$

$\Rightarrow \frac{(k-10)}{1} = -2$

$\Rightarrow k = 8$

8. The ratio of the sum and product of the roots of the equation $7x^2 - 12x + 18 = 0$ is

(a) $7:12$ (b) $7:18$ (c) $3:2$ (d) $2:3$

Answer : (d) $2:3$

Sol:

Given:

$7x^2 - 12x + 18 = 0$

$\therefore \alpha + \beta = \frac{12}{7}$ *and* $\beta = \frac{18}{7}$, where α and β are the roots of the equation

$\therefore$ Ratio of the sum and product of the roots $= \frac{12}{7} : \frac{18}{7}$

$= 12:18$

$= 2:3$

9. If one root of the equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{3}$ then the other root is

(a) $\frac{-1}{3}$ (b) $\frac{1}{3}$ (c) -3 (d)3

Answer: (d)3

Sol:

Given:

$3x^2 - 10x + 3 = 0$

One root of the equation is $\frac{1}{3}$.

Let the other root be α.

Product of the roots $= \frac{c}{a}$

$\Rightarrow \frac{1}{3} \times \alpha = \frac{3}{3}$

$\Rightarrow \alpha = 3$

10. If one root of $5x^2 + 13x + k = 0$ be the reciprocal of the other root then the value of k is

(a) 0 (b) 1 (c)2(d)5

Answer: (d)5

Sol:

Let the roots of the equation $\frac{-2}{3}$ be α and $\frac{1}{\alpha}$.

$\therefore$ Product of the roots $=\frac{c}{a}$

$\Rightarrow \alpha \times \frac{1}{\alpha}=\frac{k}{5}$

$\Rightarrow a=\frac{k}{5}$

$\Rightarrow k=5$

11. If the sum of the roots of the equation $kx^2+2x+3k=0$ is equal to their product then the value of k

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

Answer: (d) $\frac{-2}{3}$

Sol:

Given:

$kx^2+2x+3k=0$

Sum of the roots = Product of the roots

$\Rightarrow \frac{-2}{k}=\frac{3k}{k}$

$\Rightarrow 3k=-2$

$\Rightarrow k=\frac{-2}{3}$

12. The roots of a quadratic equation are 5 and -2. Then, the equation is

(a) $x^2-3x+10=0$(b) $x^2-3x-10=0$(c) $x^2+3x-10=0$(d) $x^2+3x+10=0$

Answer: (b) $x^2-3x-10=0$

Sol:

It is given that the roots of the quadratic equation are 5 and -2.

Then, the equation is:

$x^2-(5-2)x+5\times(-2)=0$

$\Rightarrow x^2-3x-10=0$

13. If the sum of the roots of a quadratic equation is 6 and their product is 6, the equation is

(a) $x^2-6x+6=0$ (b) $x^2+6x+6=0$ (c) $x^2-6x-6=0$ (d) $x^2+6x+6=0$

Answer: (a) $x^2-6x+6=0$

Sol:

Given:

Sum of roots = 6

Product of roots = 6

Thus, the equation is:

$x^2-6x+6=0$

14. If α *and* β are the roots of the equation $3x^2+8x+2=0$ then $\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=?$

(a) $\frac{-3}{8}$ (b) $\frac{2}{3}$ (c) -4 (d) 4

Answer: (c) -4

Sol:

It is given that α and β are the roots of the equation $3x^2+8x+2=0$

$\therefore \alpha+\beta=-\frac{8}{3}$ *and* $\alpha\beta=\frac{2}{3}$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-\frac{8}{3}}{\frac{2}{3}}=-4$

15. The roots of the equation $ax^2+bx+c=0$ will be reciprocal each other if

(a) a = b (b) b = c (c) c = a (d) none of these

Answer: (c) c = a

Sol:

Let the roots of the equation $(ax^2+bx+c=0)$ be α and $\frac{1}{\alpha}$.

$\therefore$ Product of the roots $=\alpha\times\frac{1}{\alpha}=1$

$\Rightarrow \frac{c}{a}=1$

$\Rightarrow c=a$

16. If the roots of the equation $ax^2+bx+c=0$ are equal then c=?

(a) $\frac{-b}{2a}$ (b) $\frac{b}{2a}$ (c) $\frac{-b^2}{4a}$ (d) $\frac{b^2}{4a}$

Answer: (d) $\frac{b^2}{4a}$

Sol:

It is given that the roots of the equation $\left(ax^2+bx+c=0\right)$ are equal.

$\therefore \left(b^2-4ac\right)=0$

$\Rightarrow b^2=4ac$

$\Rightarrow c=\frac{b^2}{4a}$

17. If the equation $9x^2+6kx+4=0$ has equal roots then k=?

(a) 0 or 0 (b) -2 or 0 (c) 2 or -2 (d) 0 only

Answer: (c) 2 or -2

Sol:

It is given that the roots of the equation $\left(9x^2+6kx+4=0\right)$ are equal.

$\therefore \left(b^2-4ac\right)=0$

$\Rightarrow (6k)^2-4\times 9\times 4=0$

$\Rightarrow 36k^2=144$

$\Rightarrow k^2=4$

$\Rightarrow k=\pm 2$

18. If the equation $x^2+2(k+2)x+9k=0$ has equal roots then k = ?

(a) 1 or 4 (b)-1 or 4 (c) 1 or -4 (d) -1 or –4

Answer: (a) 1 or 4

Sol:

It is given that the roots of the equation $\left(x^2+2(k+2)x+9k=0\right)$ are equal.

$\therefore \left(b^2-4ac\right)=0$

$\Rightarrow \left\{2(k+2)\right\}^2-4\times 1\times 9k=0$

$\Rightarrow 4\left(k^2+4k+4\right)-36k=0$

$\Rightarrow 4k^2+16k+16-36k=0$

$\Rightarrow 4k^2-20+16=0$

$\Rightarrow k^2 - 5k + 4 = 0$

$\Rightarrow k^2 - 4k - k + 4 = 0$

$\Rightarrow k(k-4) - (k-4) = 0$

$\Rightarrow (k-4)(k-1) = 0$

$\Rightarrow k = 4 \text{ or } k = 1$

19. If the equation $4x^2 - 3kx + 1 = 0$ has equal roots then value of k=?

(a) $\pm\frac{2}{3}$ (b) $\pm\frac{1}{3}$ (c) $\pm\frac{3}{4}$ (d) $\pm\frac{4}{3}$

Answer: (d) $\pm\frac{4}{3}$

Sol:

It is given that the roots of the equation $(4x^2 - 3kx + 1 = 0)$ are equal.

$\therefore (b^2 - 4ac) = 0$

$\Rightarrow (3k)^2 - 4 \times 4 \times 1 = 0$

$\Rightarrow 9k^2 = 16$

$\Rightarrow k^2 = \frac{16}{9}$

$\Rightarrow k = \pm\frac{4}{3}$

20. The roots of $ax^2 + bx + c = 0, a \neq 0$ are real and unequal, if $(b^2 - 4ac)$ is

(a) > 0 (b) = 0 (c) < 0 (d) none of these

Answer: (a) > 0

Sol:

The roots of the equation are real and unequal when $(b^2 - 4ac) > 0$.

21. In the equation $ax^2 + bx + c = 0$, it is given that $D = (b^2 - 4ac) > 0$. Then, the roots of the equation are

(a) real and equal (b) real and unequal (c) imaginary (d) none of these

Answer: (b) real and unequal

Sol:

We know that when discriminant, $D > 0$, the roots of the given quadratic equation are real and unequal.

22. The roots of the equation $2x^2 - 6x + 7 = 0$ are
(a) real, unequal and rational (b) real, unequal and irrational (c) real and equal (d) imaginary
Answer: (d) imaginary
Sol:
$\because D = (b^2 - 4ac)$
$= (-6)^2 - 4 \times 2 \times 7$
$= 36 - 56$
$= -20 < 0$
Thus, the roots of the equation are imaginary

23. The roots of the equation $2x^2 - 6x + 3 = 0$ are
(a) real, unequal and rational (b) real, unequal and irrational (c) real and equal (d) imaginary
Answer: (b) real, unequal and irrational
Sol:
$\because D = (b^2 - 4ac)$
$= (-6)^2 - 4 \times 2 \times 3$
$= 36 - 24$
$= 12$
12 is greater than
0 and it is not a perfect square; therefore, the roots of the equation are real, unequal and irrational.

24. If the roots of $5x^2 - k + 1 = 0$ are real and distinct then
(a) $-2\sqrt{5} < k < 2\sqrt{5}$ (b) $k > 2\sqrt{5}$ only (c) $k < -2\sqrt{5}$ (d) either $k > 2\sqrt{5}$ or $k < -2\sqrt{5}$
Answer: (d) either $k > 2\sqrt{5}$ *or* $k < -2\sqrt{5}$
Sol:
It is given that the roots of the equation $(5x^2 - k + 1 = 0)$ are real and distinct.
$\therefore (b^2 - 4ac) > 0$
$\Rightarrow (-k)^2 - 4 \times 5 \times 1 > 0$
$\Rightarrow k^2 - 20 > 0$
$\Rightarrow k^2 > 20$
$\Rightarrow k > \sqrt{20}$ *or* $k < -\sqrt{20}$
$\Rightarrow k > 2\sqrt{5}$ *or* $k < -2\sqrt{5}$

25. If the equation $x^2+5kx+16=0$ has no real roots then

(a) $k>\frac{8}{5}$ (b) $k<\frac{-8}{5}$ (c) $\frac{-8}{5}<k<\frac{8}{5}$ (d) None of these

Answer: (c) $\frac{-8}{5}<k<\frac{8}{5}$

Sol:

It is given that the equation $\left(x^2+5kx+16=0\right)$ has no real roots.

$\therefore\left(b^2-4ac\right)<0$

$\Rightarrow(5k)^2-4\times1\times16<0$

$\Rightarrow 25k^2-64<0$

$\Rightarrow k^2<\frac{64}{25}$

$\Rightarrow\frac{-8}{5}<k<\frac{8}{5}$

26. If the equation $x^2-kx+1=0$ has no real roots then

(a) $k<-2$ (b) k > 2 (c) -2 < k < 2 (d) None of these

Answer: c) -2 < k < 2

Sol:

It is given that the equation $x^2-kx+1=0$ has no real roots.

$\therefore\left(b^2-4ac\right)<0$

$\Rightarrow(-k)^2-4\times1\times1<0$

$\Rightarrow k^2<4$

$\Rightarrow -2<k<2$

27. For what value of k, the equation $kx^2-6x-2=0$ has real roots?

(a) $k\le\frac{-9}{2}$ (b) $k\ge\frac{-9}{2}$ (c) $k\le-2$ (d)None of these

Answer: (b) $k\ge\frac{-9}{2}$

Sol:

It is given that the roots of the equation $\left(kx^2-6x-2=0\right)$ are real.

$\therefore D\ge0$

$\Rightarrow\left(b^2-4ac\right)\ge0$

$\Rightarrow(-6)^2-4\times k\times(-2)\ge0$

$\Rightarrow 36+8k \geq 0$

$\Rightarrow k \geq \frac{-36}{8}$

$\Rightarrow k \geq \frac{-9}{2}$

28. The sum of a number and its reciprocal is $2\frac{1}{20}$. The number is

(a) $\frac{5}{4}$ or $\frac{4}{5}$ (b) $\frac{4}{3}$ or $\frac{3}{4}$ (c) $\frac{5}{6}$ or $\frac{6}{5}$ (d) $\frac{1}{6}$ or 6

Answer: (a) $\frac{5}{4}$ or $\frac{4}{5}$

Sol:

Let the required number be x.

According to the question:

$x+\frac{1}{x}=\frac{41}{20}$

$\Rightarrow \frac{x^2+1}{x}=\frac{41}{20}$

$\Rightarrow 20x^2-41x+20=0$

$\Rightarrow 20x^2-25x-16x+20=0$

$\Rightarrow 5x(4x-5)-4(4x-5)=0$

$\Rightarrow (4x-5)(5x-4)=0$

$\Rightarrow x=\frac{5}{4}$ or $x=\frac{4}{5}$

29. The perimeter of a rectangle is 82m and its area is $400m^2$. The breadth of the rectangle is

(a) 25 m (b) 20 m (c) 16 m (d) 9m

Answer: (c) 16 m

Sol:

Let the length and breadth of the rectangle be l and b.

Perimeter of the rectangle $=82m$

$\Rightarrow 2\times(l+b)=82$

$\Rightarrow l+b=41$

$\Rightarrow l=(41-b)$ ....(i)

Area of the rectangle $=400\ m^2$

$\Rightarrow l\times b=400m^2$

$\Rightarrow (41-b)b = 400$ (using (i))

$\Rightarrow 41b - b^2 = 400$

$\Rightarrow b^2 - 41b + 400 = 0$

$\Rightarrow b^2 - 25b - 16b + 400 = 0$

$\Rightarrow b(b-25) - 16(b-25) = 0$

$\Rightarrow (b-25)(b-16) = 0$

$\Rightarrow b = 25$ or $b = 16$

If $b = 25$, we have:

$l = 41 - 25 = 16$

Since, l cannot be less than b,

$\therefore b = 16m$

30. The length of a rectangular field exceeds its breadth by 8 m and the area of the field is $240m^2$. The breadth of the field is

(a) 20 m (b) 30 m (c) 12 m (d) 16 m

Sol:

Let the breadth of the rectangular field be x m.

$\therefore$ Length of the rectangular field $= (x+8)m$

Area of the rectangular field $= 240\,m^2$ (Given)

$\therefore (x+8) \times x = 240$ (Area=Length $\times$ Breadth)

$\Rightarrow x^2 + 8x - 240 = 0$

$\Rightarrow x^2 + 20x - 12x - 240 = 0$

$\Rightarrow x(x+20) - 12(x+20) = 0$

$\Rightarrow (x+20)(x-12) = 0$

$\Rightarrow x + 20 = 0$ or $x - 12 = 0$

$\Rightarrow x = -20$ or $x = 12$

$\therefore x = 12$ (Breadth cannot be negative)

Thus, the breadth of the field is 12 m

Hence, the correct answer is option C.

31. The roots of the quadratic equation $2x^2 - x - 6 = 0.$ are

(a) $-2, \frac{3}{2}$ (b) $2, \frac{-3}{2}$ (c) $-2, \frac{-3}{2}$ (d) $2, \frac{3}{2}$

Answer: (b) $2, \frac{-3}{2}$

Sol:

The given quadratic equation is $2x^2 - x - 6 = 0$.

$2x^2 - x - 6 = 0$

$\Rightarrow 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x-2) + 3(x-2) = 0$

$\Rightarrow (x-2)(2x+3) = 0$

$\Rightarrow x - 2 = 0 \ or \ 2x + 3 = 0$

$\Rightarrow x = 2 \ or \ x = \frac{-3}{2}$

Thus, the roots of the given equation are 2 and $\frac{-3}{2}$

Hence, the correct answer is option B.

32. The sum of two natural numbers is 8 and their product is 15., Find the numbers.

Sol:

Let the required natural numbers be x and $(8-x)$.

It is given that the product of the two numbers is 15.

$\therefore x(8-x) = 15$

$\Rightarrow 8x - x^2 = 15$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 5x - 3x + 15 = 0$

$\Rightarrow x(x-5) - 3(x-5) = 0$

$\Rightarrow (x-5)(x-3) = 0$

$\Rightarrow x - 5 = 0 \ or \ x - 3 = 0$

$\Rightarrow x = 5 \ or \ x = 3$

Hence, the required numbers are 3 and 5.

33. Show the x= -3 is a solution of $x^2 + 6x + 9 = 0$

Sol:

The given equation is $x^2 + 6x + 9 = 0$

Putting $x = -3$ in the given equation, we get

$LHS = (-3)^2 + 6 \times (-3) + 9 = 9 - 18 + 9 = 0 = RHS$

$\therefore x = -3$ is a solution of the given equation.

34. Show that x= -2 is a solution of $3x^2+13x+14=0$.

Sol:

The given equation is $3x^2+13x+14=0$.

Putting $x=-2$ in the given equation, we get

$LHS-3\times(-2)^2+13\times(-2)+14=12-26+14=0=RHS$

$\therefore x=-2$ is a solution of the given equation.

35. If $x=\frac{-1}{2}$ is a solution of the quadratic equation $3x^2+2kx-3=0$. Find the value of k.

Sol:

It is given that $x=\frac{-1}{2}$ is a solution of the quadratic equation $3x^2+2kx-3=0$.

$\therefore 3\times\left(\frac{-1}{2}\right)^2+2k\times\left(\frac{-1}{2}\right)-3=0$

$\Rightarrow \frac{3}{4}-k-3=0$

$\Rightarrow k=\frac{3-12}{4}=-\frac{9}{4}$

Hence, the value of k is $-\frac{9}{4}$.

36. Find the roots of the quadratic equation $2x^2-x-6=0$.

Sol:

The given quadratic equation is $2x^2-x-6=0$.

$2x^2-x-6=0$

$\Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow 2x(x-2)+3(x-2)=0$

$\Rightarrow (x-2)(2x+3)=0$

$\Rightarrow x-2=0 \text{ or } 2x+3=0$

$\Rightarrow x=2 \text{ or } x=-\frac{3}{2}$

Hence, the roots of the given equation are 2 and $-\frac{3}{2}$.

37. Find the solution of the quadratic equation $3\sqrt{3}x^2+10x+\sqrt{3}=0$.

Sol:

The given quadratic equation is $3\sqrt{3}x^2+10x+\sqrt{3}=0$

$3\sqrt{3}x^2+10x+\sqrt{3}=0$

$\Rightarrow 3\sqrt{3}x^2+9x+x+\sqrt{3}=0$

$\Rightarrow 3\sqrt{3}x\left(x+\sqrt{3}\right)+1\left(x+\sqrt{3}\right)=0$

$\Rightarrow \left(x+\sqrt{3}\right)\left(3\sqrt{3}x+1\right)=0$

$\Rightarrow x+\sqrt{3}=0 \ or \ 3\sqrt{3}x+1=0$

$\Rightarrow x=-\sqrt{3} \ or \ x=-\frac{1}{3\sqrt{3}}=-\frac{\sqrt{3}}{9}$

Hence, $-\sqrt{3}$ and $-\frac{\sqrt{3}}{9}$ are the solutions of the given equation.

38. If the roots of the quadratic equation $2x^2+8x+k=0$ are equal then find the value of k.

Sol:

It is given that the roots of the quadratic equation $2x^2+8x+k=0$ are equal.

$\therefore D=0$

$\Rightarrow 8^2-4\times 2\times k=0$

$\Rightarrow 64-8k=0$

$\Rightarrow k=8$

Hence, the value of k is 8.

39. If the quadratic equation $px^2-2\sqrt{5}px+15=0$ has two equal roots then find the value of p.

Sol:

It is given that the quadratic equation $px^2-2\sqrt{5}px+15=0$ has two equal roots.

$\therefore D=0$

$\Rightarrow \left(-2\sqrt{5}p\right)^2-4\times p\times 15=0$

$\Rightarrow 20p^2-60p=0$

$\Rightarrow 20p\left(p-3\right)=0$

$\Rightarrow p=0 \ or \ p-3=0$

$\Rightarrow p=0 \ or \ p=3$

For $p=0$, we get $15=0$, which is not true.

$\therefore p\neq 0$

Hence, the value of p is 3.

40. If 1 is a root of the equation $ay^2 + ay + 3 = 0.$ and $y^2 + y + b = 0.$ then find the value of ab.

Sol:

It is given that $y = 1$ is a root of the equation $ay^2 + ay + 3 = 0.$

$\therefore a \times (1)^2 + a \times 1 + 3 = 0$

$\Rightarrow a + a + 3 = 0$

$\Rightarrow 2a + 3 = 0$

$\Rightarrow a = -\frac{3}{2}$

Also, $y = 1$ is a root of the equation $y^2 + y + b = 0.$

$\therefore (1)^2 + 1 + b = 0$

$\Rightarrow 1 + 1 + b = 0$

$\Rightarrow b + 2 = 0$

$\Rightarrow b = -2$

$\therefore ab = \left(-\frac{3}{2}\right) \times (-2) = 3$

Hence, the value of ab is 3.

41. If one zero of the polynomial $x^2 - 4x + 1$ is $\left(2 + \sqrt{3}\right)$, write the other zero.

Sol:

Let the other zero of the given polynomial be α.

Now,

Sum of the zeroes of the given polynomial $= \frac{-(-4)}{1} = 4$

$\therefore \alpha + \left(2 + \sqrt{3}\right) = 4$

$\Rightarrow \alpha = 4 - 2 - \sqrt{3} = 2 - \sqrt{3}$

Hence, the other zero of the given polynomial is $\left(2 - \sqrt{3}\right)$.

42. If one root of the quadratic equation $3x^2 - 10x + k = 0.$ is reciprocal of the other , find the value of k.

Sol:

Let α and β be the roots of the equation $3x^2 - 10x + k = 0.$

$\therefore \alpha = \frac{1}{\beta}$ (Given)

$\Rightarrow \alpha\beta = 1$

$\Rightarrow \frac{k}{3} = 1$ (Product of the roots $= \frac{c}{a}$)

$\Rightarrow k = 3$

Hence, the value of k is 3.

43. If the roots of the quadratic equation $px(x-2)+=0$ are equal, find the value of p.

Sol:

It is given that the roots of the quadratic equation $px^2 - 2px + 6 = 0$ are equal.

$\therefore D = 0$

$\Rightarrow (-2p)^2 - 4 \times p \times 6 = 0$

$\Rightarrow 4p^2 - 24p = 0$

$\Rightarrow 4p(p-6) = 0$

$\Rightarrow p = 0 \text{ or } p - 6 = 0$

$\Rightarrow p = 0 \text{ or } p = 6$

For $p = 0$, we get $6 = 0$, which is not true.

$\therefore p \neq 0$

Hence, the value of p is 6.

44. Find the value of k so that the quadratic equation $x^2 - 4kx + k = 0$ has equal roots.

Sol:

It is given that the quadratic equation $x^2 - 4kx + k = 0$ has equal roots.

$\therefore D = 0$

$\Rightarrow (-4k)^2 - 4 \times 1 \times k = 0$

$\Rightarrow 16k^2 - 4k = 0$

$\Rightarrow 4k(4k-1) = 0$

$\Rightarrow k = 0 \text{ or } 4k - 1 = 0$

$\Rightarrow k = 0 \text{ or } k = \frac{1}{4}$

Hence, 0 and $\frac{1}{4}$ are the required values of k.

45. Find the value of k for which the quadratic equation $9x^2-3kx+k=0$ has equal roots.

Sol:

It is given that the quadratic equation $9x^2-3kx+k=0$ has equal roots.

$\therefore D=0$

$\Rightarrow (-3k)^2-4\times 9\times k=0$

$\Rightarrow 9k^2-36k=0$

$\Rightarrow 9k(k-4)=0$

$\Rightarrow k=0 \text{ or } k-4=0$

$\Rightarrow k=0 \text{ or } k=4$

Hence, 0 and 4 are the required values of k.

46. Solve $x^2-(\sqrt{3}+1)x+\sqrt{3}=0$

Sol:

$x^2-(\sqrt{3}+1)x+\sqrt{3}=0$

$\Rightarrow x^2-\sqrt{3}x-x+\sqrt{3}=0$

$\Rightarrow x(x-\sqrt{3})-1(x-\sqrt{3})=0$

$\Rightarrow (x-\sqrt{3})(x-1)=0$

$\Rightarrow x-\sqrt{3}=0 \text{ or } x-1=0$

$\Rightarrow x=\sqrt{3} \text{ or } x=1$

Hence, 1 and $\sqrt{3}$ are the roots of the given equation.

47. Solve $2x^2+ax-a^2=0$

Sol:

$2x^2+ax-a^2=0$

$\Rightarrow 2x^2+2ax-ax-a^2=0$

$\Rightarrow 2x(x+a)-a(x+a)=0$

$\Rightarrow (x+a)(2x-a)=0$

$\Rightarrow x+a=0 \text{ or } 2x-a=0$

$\Rightarrow x=-a \text{ or } x=\frac{a}{2}$

Hence, $-a$ and $\frac{a}{2}$ are the roots of the given equation.

48. Solve $3x^2+5\sqrt{5}x-10=0$

Sol:

$3x^2+5\sqrt{5}x-10=0$

$\Rightarrow 3x^2+6\sqrt{5}x-\sqrt{5}x-10=0$

$\Rightarrow 3x\left(x+2\sqrt{5}\right)-\sqrt{5}\left(x+2\sqrt{5}\right)=0$

$\Rightarrow \left(x+2\sqrt{5}\right)\left(3x-\sqrt{5}\right)=0$

$\Rightarrow x+2\sqrt{5}=0 \text{ or } 3x-\sqrt{5}=0$

$\Rightarrow x=-2\sqrt{5} \text{ or } x=\dfrac{\sqrt{5}}{3}$

Hence, $-2\sqrt{5}$ and $\dfrac{\sqrt{5}}{3}$ are the roots of the given equation.

49. Solve $\sqrt{3}x^2+10x-8\sqrt{3}=0$

Sol:

$\sqrt{3}x^2+10x-8\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2+12x-2x-8\sqrt{3}=0$

$\Rightarrow \sqrt{3}x\left(x+4\sqrt{3}\right)-2\left(x+4\sqrt{3}\right)=0$

$\Rightarrow \left(x+4\sqrt{3}\right)\left(\sqrt{3}x-2\right)=0$

$\Rightarrow x+4\sqrt{3}=0 \text{ or } \sqrt{3}x-2=0$

$\Rightarrow x=-4\sqrt{3} \text{ or } x=\dfrac{2}{\sqrt{3}}=\dfrac{2\sqrt{3}}{3}$

Hence, $-4\sqrt{3}$ and $\dfrac{2\sqrt{3}}{3}$ are the roots of the given equation.

50. Solve $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

Sol:

$\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2-3\sqrt{2}x+\sqrt{2}x-2\sqrt{3}=0$

$\Rightarrow \sqrt{3}x\left(x-\sqrt{6}\right)+\sqrt{2}\left(x-\sqrt{6}\right)=0$

$\Rightarrow \left(x-\sqrt{6}\right)\left(\sqrt{3}+\sqrt{2}\right)=0$

$\Rightarrow x-\sqrt{6}=0 \text{ or } \sqrt{3}x+\sqrt{2}=0$

$$\Rightarrow x=\sqrt{6} \text{ or } x=-\frac{\sqrt{2}}{\sqrt{3}}=-\frac{\sqrt{6}}{3}$$

Hence, $\sqrt{6}$ and $-\frac{\sqrt{6}}{3}$ are the roots of the given equation.

51. Solve $4\sqrt{3}x^2+5x-2\sqrt{3}=0$

Sol:

$$4\sqrt{3}x^2+5x-2\sqrt{3}=0$$

$$\Rightarrow 4\sqrt{3}x^2+8x-3x-2\sqrt{3}=0$$

$$\Rightarrow 4x\left(\sqrt{3}x+2\right)-\sqrt{3}\left(\sqrt{3}x+2\right)=0$$

$$\Rightarrow \left(\sqrt{3}x+2\right)\left(4x-\sqrt{3}\right)=0$$

$$\Rightarrow \sqrt{3}x+2=0 \text{ or } 4x-\sqrt{3}=0$$

$$\Rightarrow x=-\frac{2}{\sqrt{3}}=-\frac{2\sqrt{3}}{3} \text{ or } x=\frac{\sqrt{3}}{4}$$

Hence, $-\frac{2\sqrt{3}}{3}$ and $\frac{\sqrt{3}}{4}$ are the roots of the given equation.

52. Solve $4x^2+4bx-\left(a^2-b^2\right)=0$

Sol:

$$4x^2+4bx-\left(a^2-b^2\right)=0$$

$$\Rightarrow 4x^2+4bx-(a-b)(a+b)=0$$

$$\Rightarrow 4x^2+2\left[(a+b)-(a-b)\right]x-(a-b)(a+b)=0$$

$$\Rightarrow 4x^2+2(a+b)x-2(a-b)x-(a-b)(a+b)=0$$

$$\Rightarrow 2x\left[2x+(a+b)\right]-(a-b)\left[2x+(a+b)\right]=0$$

$$\Rightarrow \left[2x+(a+b)\right]\left[2x-(a-b)\right]=0$$

$$\Rightarrow 2x+(a+b)=0 \text{ or } 2x-(a-b)=0$$

$$\Rightarrow x=-\frac{a+b}{2} \text{ or } x=\frac{a-b}{2}$$

Hence, $-\frac{a+b}{2}$ and $\frac{a-b}{2}$ are the roots of the given equation.

53. Solve $x^2+5x-(a^2+a-6)=0$

Sol:

$x^2+5x-(a^2+a-6)=0$

$\Rightarrow x^2+5x-(a+3)(a-2)=0$

$\Rightarrow x^2+[(a+3)-(a-2)]x-(a+3)(a-2)=0$

$\Rightarrow x^2+(a+3)x-(a-2)x-(a+3)(a-2)=0$

$\Rightarrow x[x+(a+3)]-(a-2)[x+(a+3)]=0$

$\Rightarrow [x+(a+3)][x-(a-2)]=0$

$\Rightarrow x+(a+3)=0 \ or \ x-(a-2)=0$

$\Rightarrow x=-(a+3) \ or \ x=(a-2)$

Hence, $-(a+3)$ and $(a-2)$ are the roots of the given equation.

54. Solve $x^2+6x-(a^2+2a-8)=0$

Sol:

$x^2+6x-(a^2+2a-8)=0$

$\Rightarrow x^2+6x-(a+4)(a-2)=0$

$\Rightarrow x^2+[(a+4)-(a-2)]x-(a+4)(a-2)=0$

$\Rightarrow x^2+(a+4)x-(a-2)x-(a+4)(a-2)=0$

$\Rightarrow x[x+(a+4)]-(a-2)[x+(a+4)]=0$

$\Rightarrow [x+(a+4)][x-(a-2)]=0$

$\Rightarrow x+(a+4)=0 \ or \ x-(a-2)=0$

$\Rightarrow x=-(a+4) \ or \ x=(a-2)$

Hence, $-(a+4)$ and $(a-2)$ are the roots of the given equation.

55. Solve $x^2-4ax+4a^2-b^2=0$

Sol:

$x^2-4ax+4a^2-b^2=0$

$\Rightarrow x^2-4ax+(2a+b)(2a-b)=0$

$\Rightarrow x^2-[(2a+b)+(2a-b)]x+(2a+b)(2a-b)=0$

$\Rightarrow x^2-(2a+b)x-(2a-b)x+(2a+b)(2a-b)=0$

$$\Rightarrow x\left[x-(2a+b)\right]-(2a-b)\left[x-(2a+b)\right]=0$$

$$\Rightarrow \left[x-(2a+b)\right]\left[x-(2a-b)\right]=0$$

$$\Rightarrow x-(2a+b)=0 \ or \ x-(2a-b)=0$$

$$\Rightarrow x=(2a+b) \ or \ x=(2a-b)$$

Hence, $(2a+b)$ and $(2a-b)$ are the roots of the given equation.