Chapter 10. Isosceles Triangles

Exercise 10(A)

Solution 1:

In ΔABC,

$\angle BAC + \angle ACB + \angle ABC = 180^0$

$48^0 + \angle ACB + \angle ABC = 180^0$

But $\angle ACB = \angle ABC$ [AB = AC]

$2\angle ABC = 180^0 - 48^0$

$2\angle ABC = 132^0$

$\angle ABC = 66^0 = \angle ACB$ ......(i)

$\angle ACB = 66^0$

$\angle ACD + \angle DCB = 66^0$

$18^0 + \angle DCB = 66^0$

$\angle DCB = 48^0$ ....(ii)

Now, In ΔDCB,

$\angle DBC = 66^0$ [From (i), Since $\angle ABC = \angle DBC$]

$\angle DCB = 48^0$ [From (ii)]

$\angle BDC = 180^0 - 48^0 - 66^0$

$\angle BDC = 66^0$

Since $\angle BDC = \angle DBC$

Therefore, BC = CD

Equal angles have equal sides opposite to them.

Solution 2:

Given: $\angle ACE = 130^0$; AD = BD = CD

Proof:

(i)

$\angle ACD + \angle ACE = 180^\circ$ [DCE is a st. line]
$\Rightarrow \angle ACD = 180^\circ - 130^\circ$
$\Rightarrow \angle ACD = 50^\circ$
Now, CD = AD
$\Rightarrow \angle ACD = \angle DAC = 50^\circ$.....(i)
[Since angles opposite to equal sides are equal]

In ΔADC,
$\angle ACD = \angle DAC = 50^\circ$
$\angle ACD + \angle DAC + \angle ADC = 180^\circ$
$50^\circ + 50^\circ + \angle ADC = 180^\circ$
$\angle ADC = 180^\circ - 100^\circ$
$\angle ADC = 80^\circ$

(ii)

$\angle ADC = \angle ABD + \angle DAB$ [Exterior angle is equal to sum of opp. interior angles]
But AD = BD
$\therefore \angle DAB = \angle ABD$
$\Rightarrow 80^\circ = \angle ABD + \angle ABD$
$\Rightarrow 2\angle BD = 80^\circ$
$\Rightarrow \angle ABD = 40^\circ = \angle DAB$.........(ii)

(iii)

$\angle BAC = \angle DAB + \angle DAC$
substituting the values from (i) and (ii)
$\angle BAC = 40^\circ + 50^\circ$
$\Rightarrow \angle BAC = 90^\circ$

Solution 3:

$\angle FAB = 128°$ [Given]

$\angle BAC + \angle FAB = 180°$ [FAC is a st. line]

$\Rightarrow \angle BAC = 180° - 128°$

$\Rightarrow \angle BAC = 52°$

In ΔABC,

$\angle A = 52°$

$\angle B = \angle C$ [Given AB = AC and angles opposite to equal sides are equal]

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle B + \angle B = 180°$

$\Rightarrow 52° + 2\angle B = 180°$

$\Rightarrow 2\angle B = 128°$

$\Rightarrow \angle B = 64° = \angle C$............(i)

$\angle B = \angle ADE$ [Given DE || BC]

(i)

Now,

$\angle ADE + \angle CDE + \angle B = 180°$ [ADB is a st. line]

$\Rightarrow 64° + \angle CDE + 64° = 180°$

$\Rightarrow \angle CDE = 180° - 128°$

$\Rightarrow \angle CDE = 52°$

(ii)

Given DE||BC and DC is the transversal.

$\Rightarrow \angle CDE = \angle DCB = 52°$.......(ii)

Also, $\angle ECB = 64°$.........[From (i)]

But,

$\angle ECB = \angle DCE + \angle DCB$

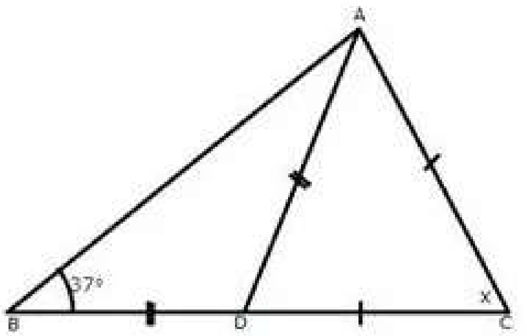
$\Rightarrow 64° = \angle DCE + 52°$

$\Rightarrow \angle DCE = 64° - 52°$

$\Rightarrow \angle DCE = 12°$

Solution 4:

(i) Let the triangle be ABC and the altitude be AD.



In ΔABD,

$\angle DBA = \angle DAB = 37°$ [Given BD = AD and angles opposite to equal sides are equal]

Now,

$\angle CDA = \angle DBA + \angle DAB$ [Exterior angle is equal to the sum of opp. interior angles]

$\therefore \angle CDA = 37° + 37°$

$\Rightarrow \angle CDA = 74°$

Now in ΔADC,

$\angle CDA = \angle CAD = 74°$ [Given CD = AC and angles opposite to equal sides are equal]

Now,

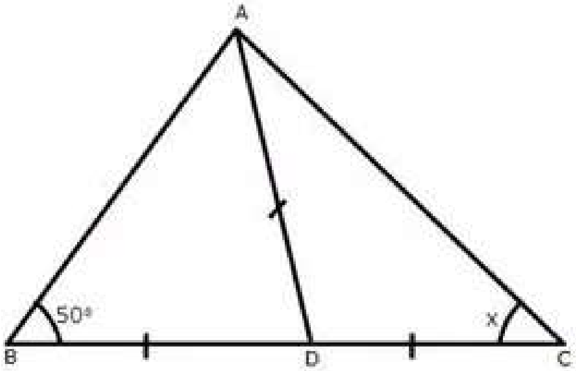
$\angle CAD + \angle CDA + \angle ACD = 180°$

$\Rightarrow 74° + 74° + x = 180°$

$\Rightarrow x = 180° - 148°$

$\Rightarrow x = 32°$

(ii) Let triangle be ABC and altitude be AD.



In ΔABD,

$\angle DBA = \angle DAB = 50°$ [Given BD = AD and angles opposite to equal sides are equal]

Now,

$\angle CDA = \angle DBA + \angle DAB$ [Exterior angle is equal to the sum of opp. interior angles]

$\therefore \angle CDA = 50° + 50°$

$\Rightarrow \angle CDA = 100°$

In ΔADC,

$\angle DAC = \angle DCA = x$ [Given AD = DC and angles opposite to equal sides are equal]

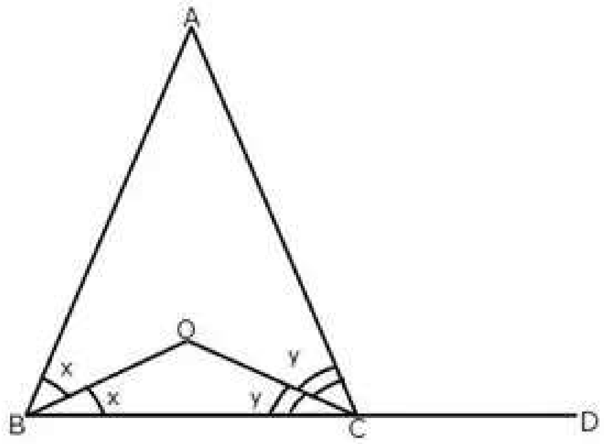
$\therefore \angle DAC + \angle DCA + \angle ADC = 180°$

$\Rightarrow x + x + 100° = 180°$

$\Rightarrow 2x = 80°$

$\Rightarrow x = 40°$

Solution 5:



Let $\angle ABO = \angle OBC = x$ and $\angle ACO = \angle OCB = y$

In ΔABC,

$\angle BAC = 180° - 2x - 2y$.........(i)

Since $\angle B = \angle C$ [AB = AC]

$\frac{1}{2}B = \frac{1}{2}C$

$\Rightarrow x = y$

Now,

$\angle ACD = 2x + \angle BAC$ [Exterior angle is equal to sum of opp. interior angles]

$= 2x + 180° - 2x - 2y$ [From (i)]

$\angle ACD = 180° - 2y$.........(ii)

In ΔOBC,

$\angle BOC = 180° - x - y$

$\Rightarrow \angle BOC = 180° - y - y$ [Already proved]

$\Rightarrow \angle BOC = 180° - 2y$....(iii)

From (i) and (ii)

$\angle BOC = \angle ACD$

Solution 6:

Given: $\angle PLN = 110^\circ$

(i) We know that the sum of the measure of all the angles of a quadrilateral is 360°.

In quad. PQNL,

$\angle QPL + \angle PLN + \angle LNQ + \angle NQP = 360^\circ$

$\Rightarrow 90^\circ + 110^\circ + \angle LNQ + 90^\circ = 360^\circ$

$\Rightarrow \angle LNQ = 360^\circ - 290^\circ$

$\Rightarrow \angle LNQ = 70^\circ$

$\Rightarrow \angle LNM = 70^\circ$ ....(i)

In ΔLMN,

LM = LN [Given]

$\therefore \angle LNM = \angle LMN$ [angles opp. to equal sides are equal]

$\Rightarrow \angle LMN = 70^\circ$.........(ii) [From (i)]

(ii)

In ΔLMN,

$\angle LMN + \angle LNM + \angle MLN = 180^\circ$

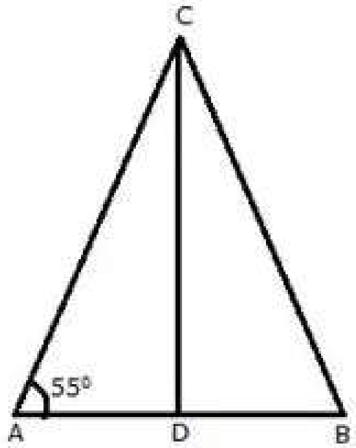
But, $\angle LNM = \angle LMN = 70^\circ$ [From (i) and (ii)]

$\therefore 70^\circ + 70^\circ + \angle MLN = 180^\circ$

$\Rightarrow \angle MLN = 180^\circ - 140^\circ$

$\Rightarrow \angle MLN = 40^\circ$

Solution 7:



In ΔABC,

$AC = BC$ [Given]

$\therefore \angle CAB = \angle CBD$ [angles opp. to equal sides are equal]

$\Rightarrow \angle CBD = 55^\circ$

In ΔABC,

$\angle CBA + \angle CAB + \angle ACB = 180^\circ$

but, $\angle CAB = \angle CBA = 55^\circ$

$\Rightarrow 55^\circ + 55^\circ + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 180^\circ - 110^\circ$

$\Rightarrow \angle ACB = 70^\circ$

Now,

In ΔACD and ΔBCD,

$AC = BC$ [Given]

$CD = CD$ [Common]

$AD = BD$ [Given : CD bisects AB]

$\therefore \Delta ACD \cong \Delta BCD$

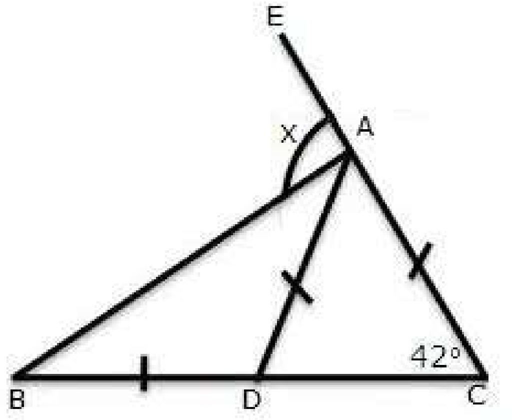
$\Rightarrow \angle DCA = \angle DCB$

$\Rightarrow \angle DCB = \dfrac{\angle ACB}{2} = \dfrac{70^\circ}{2}$

$\Rightarrow \angle DCB = 35^\circ$

Solution 8:

Let us name the figure as following:



In ΔABC,

$AD = AC$ [Given]

$\therefore \angle ADC = \angle ACD$ [angles opp. to equal sides are equal]

$\Rightarrow \angle ADC = 42°$

Now,

$\angle ADC = \angle DAB + \angle DBA$ [Exterior angle is equal to the sum of opp. interior angles]

But,

$\angle DAB = \angle DBA$ [Given : BD = DA]

$\therefore \angle ADC = 2\angle DBA$

$\Rightarrow 2\angle DBA = 42°$

$\Rightarrow \angle DBA = 21°$

For x:

$x = \angle CBA + \angle BCA$ [Exterior angle is equal to the sum of opp. interior angles]

We know that,

$\angle CBA = 21°$

$\angle BCA = 42°$

$\therefore x = 21° + 42°$

$\Rightarrow x = 63°$

Solution 9:

In ΔABD and ΔDBC,

$BD = BD$ [Common]

$\angle BDA = \angle BDC$ [each equal to $90°$]

$\angle ABD = \angle DBC$ [BD bisects $\angle ABC$]

$\therefore \Delta ABD \cong \Delta DBC$ [ASA criterion]

Therefore,

AD=DC

$x + 1 = y + 2$
$\Rightarrow x = y + 1 .....(i)$

and AB = BC

$3x + 1 = 5y - 2$

Substituting the value of x from (i)

$3(y + 1) + 1 = 5y - 2$
$\Rightarrow 3y + 3 + 1 = 5y - 2$
$\Rightarrow 3y + 4 = 5y - 2$
$\Rightarrow 2y = 6$
$\Rightarrow y = 3$

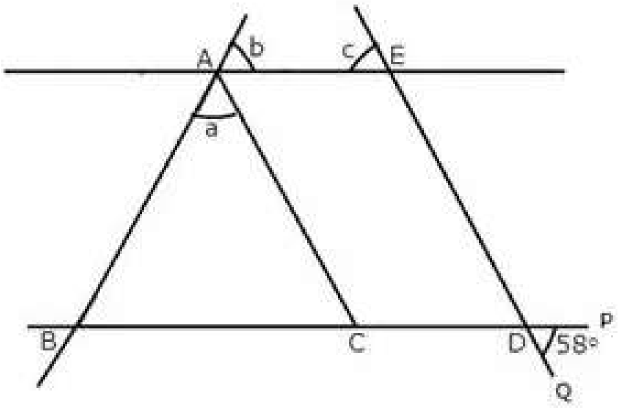
Putting y = 3 in (i)

$x = 3 + 1$

$\therefore x = 4$

Solution 10:

Let P and Q be the points as shown below:



Given: $\angle PDQ = 58°$

$\angle PDQ = \angle EDC = 58°$ [Vertically opp. angles]

$\angle EDC = \angle ACB = 58°$ [Corresponding angles $\because$ AC || ED]

In ΔABC,

AB = AC [Given]

$\therefore \angle ACB = \angle ABC = 58°$ [angles opp. to equal sides are equal]

Now,

$\angle ACB + \angle ABC + \angle BAC = 180°$

$\Rightarrow 58° + 58° + a = 180°$

$\Rightarrow a = 180° - 116°$

$\Rightarrow a = 64°$

Since AE || BD and AC is the transversal

$\angle ABC = b$ [Corresponding angles]

$\therefore b = 58°$

Also since AE || BD and ED is the transversal

$\angle EDC = c$ [Corresponding angles]

$\therefore c = 58°$

Solution 11:

In ΔACD,

$AC = CD$ [Given]

$\therefore \angle CAD = \angle CDA$

$\angle ACD = 58°$ [Given]

$\angle ACD + \angle CDA + \angle CAD = 180°$

$\Rightarrow 58° + 2\angle CAD = 180°$

$\Rightarrow 2\angle CAD = 122°$

$\Rightarrow \angle CAD = \angle CDA = 61°$..........(i)

Now,

$\angle CDA = \angle DAB + \angle DBA$ [Ext. angle is equal to sum of opp. int. angles]

But,

$\angle DAB = \angle DBA$ [Given : AD = DB]

$\therefore \angle DAB + \angle DAB = \angle CDA$

$\Rightarrow 2\angle DAB = 61°$

$\Rightarrow \angle DAB = 30.5°$..........(ii)

In ΔABC,

$\angle CAB = \angle CAD + \angle DAB$

$\therefore \angle CAB = 61° + 30.5°$

$\Rightarrow \angle AB = 91.5°$

Solution 12:

In ΔACD,

$AC = AD = CD$ [Given]

Hence, ACD is an equilateral triangle

$\therefore \angle ACD = \angle CDA = \angle CAD = 60°$

$\angle CDA = \angle DAB + \angle ABD$ [Ext. angle is equal to sum of opp. int. angles]

But,

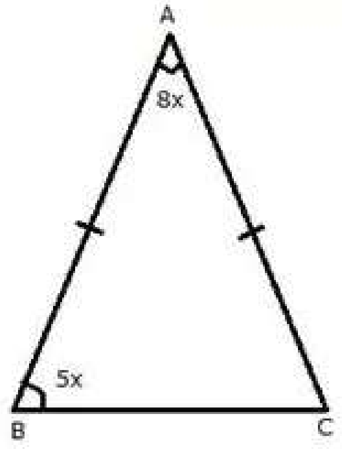
$\angle DAB = \angle ABD$ [Given : AD = DB]

$\therefore \angle ABD + \angle ABD = \angle CDA$

$\Rightarrow 2\angle ABD = 60°$

$\Rightarrow \angle ABD = \angle ABC = 30°$

Solution 13:



Let $\angle A = 8x$ and $\angle B = 5x$

Given: AB = AC

$\Rightarrow \angle B = \angle C = 5x$ [Angles opp. to equal sides are equal]

Now,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 8x + 5x + 5x = 180°$

$\Rightarrow 18x = 180°$

$\Rightarrow x = 10°$

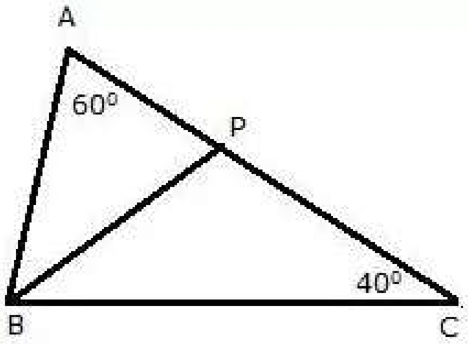
Given that :

$\angle A = 8x$

$\Rightarrow \angle A = 8 \times 10°$

$\Rightarrow \angle A = 80°$

Solution 14:



In ΔABC,

$\angle A = 60°$

$\angle C = 40°$

$\therefore \angle B = 180° - 60° - 40°$

$\Rightarrow \angle B = 80°$

Now,

BP is the bisector of $\angle ABC$

$\therefore \angle PBC = \dfrac{\angle ABC}{2}$

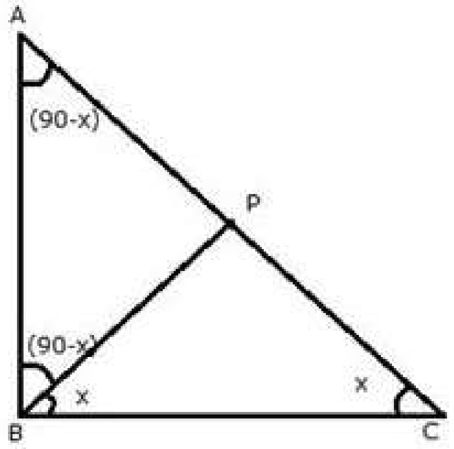
$\Rightarrow \angle PBC = 40°$

In ΔPBC,

$\angle PBC = \angle PCB = 40°$

$\therefore BP = CP$ [Sides opp. to equal angles are equal]

Solution 15:



Let $\angle PBC = \angle PCB = x$

In the right angled triangle ABC,

$\angle ABC = 90°$
$\angle ACB = x$
$\Rightarrow \angle BAC = 180° - (90° + x)$
$\Rightarrow \angle BAC = (90° - x)$.........(i)

and

$\angle ABP = \angle ABC - \angle PBC$
$\Rightarrow \angle ABP = 90° - x$........(ii)

Therefore in the triangle ABP;

$\angle BAP = \angle ABP$

Hence,

PA = PB [sides opp. to equal angles are equal]

Solution 16:

ΔABC is an equilateral triangle

$\Rightarrow$ Side AB = Side AC

$\Rightarrow \angle ABC = \angle ACB$ [If two sides of a triangle are equal, then angles opposite to them are equal]

Similarly, Side AC = Side BC

$\Rightarrow \angle CAB = \angle ABC$ [If two sides of a triangle are equal, then angles opposite to them are equal]

Hence, $\angle ABC = \angle CAB = \angle ACB = y$(say)

As the sum of all the angles of the triangle is 180°

$\angle ABC + \angle CAB + \angle ACB = 180^0$

$\Rightarrow 3y = 180°$

$\Rightarrow y = 60°$

$\angle ABC = \angle CAB = \angle ACB = 60°$

Sum of two non-adjacent interior angles of a triangle is equal to the exterior angle.

$\Rightarrow \angle CAB + \angle CBA = \angle ACE$

$\Rightarrow 60° + 60^0 = \angle ACE$

$\Rightarrow \angle ACE = 120°$

Now ΔACE is an isosceles triangle with AC = CF

$\Rightarrow \angle EAC = \angle AEC$

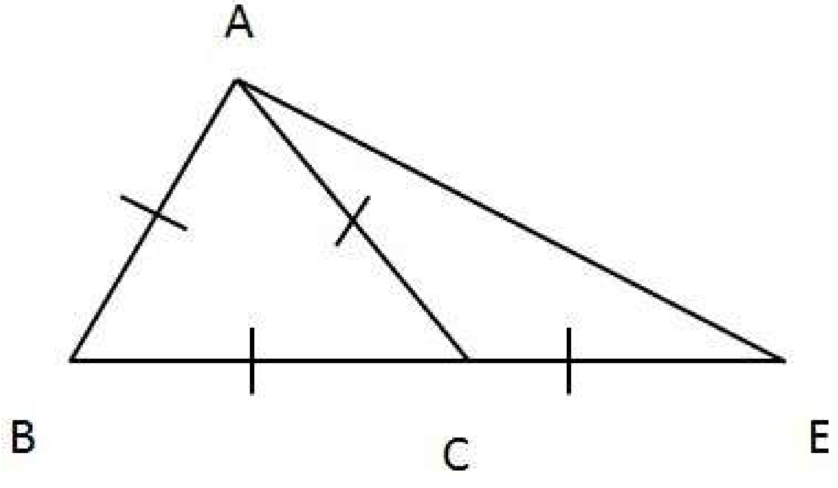
Sum of all the angles of a triangle is 180°

$\angle EAC + \angle AEC + \angle ACE = 180°$

$\Rightarrow 2\angle AEC + 120° = 180°$

$\Rightarrow 2\angle AEC = 180° - 120°$

$\Rightarrow \angle AEC = 30°$



Solution 17:

ΔDBC is an isosceles triangle

As, Side CD = Side DB

$\Rightarrow \angle DBC = \angle DCB$ [If two sides of a triangle are equal, then angles opposite to them are equal]

And $\angle B = \angle DBC = \angle DCB = 28°$

As the sum of all the angles of the triangle is 180°

$\angle DCB + \angle DBC + \angle BCD = 180^0$

$\Rightarrow 28° + 28° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 56°$

$\Rightarrow \angle BCD = 124°$

Sum of two non-adjacent interior angles of a triangle is equal to the exterior angle.

$\Rightarrow \angle DBC + \angle DCB = \angle DAC$

$\Rightarrow 28° + 28° = 56°$

$\Rightarrow \angle DAC = 56°$

Now ΔACD is an isosceles triangle with AC = DC

$\Rightarrow \angle ADC = \angle DAC = 56°$

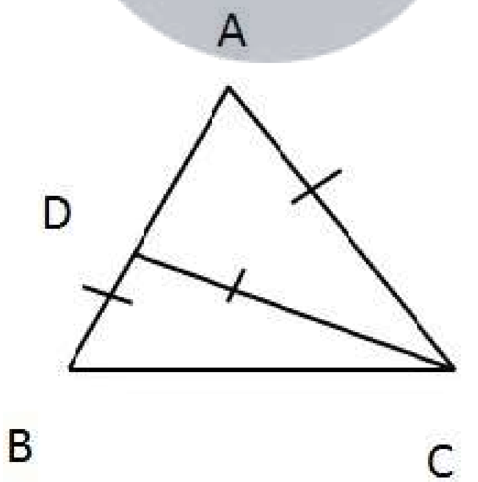
Sum of all the angles of a triangle is 180°

$\angle ADC + \angle DAC + \angle DCA = 180°$

$\Rightarrow 56° + 56° + \angle DCA = 180°$

$\Rightarrow \angle DCA = 180° - 112°$

$\Rightarrow \angle DCA = 64° = \angle ACD$



Solution 18:

We can see that the ΔABC is an isosceles triangle with Side AB = Side AC.

$\Rightarrow \angle ACB = \angle ABC$

As $\angle ACB = 65^\circ$

hence $\angle ABC = 65^\circ$

Sum of all the angles of a triangle is 180°

$\angle ACB + \angle CAB + \angle ABC = 180^\circ$

$65^\circ + 65^\circ + \angle CAB = 180^\circ$

$\angle CAB = 180^\circ - 130^\circ$

$\angle CAB = 50^\circ$

As BD is parallel to CA

Therefore, $\angle CAB = \angle DBA$ since they are alternate angles.

$\angle CAB = \angle DBA = 50^\circ$

We see that ΔADB is an isosceles triangle with Side AD = Side AB.

$\Rightarrow \angle ADB = \angle DBA = 50^\circ$

Sum of all the angles of a trianlge is 180°

$\angle ADB + \angle DAB + \angle DBA = 180^\circ$

$50^\circ + \angle DAB + 50^\circ = 180^\circ$

$\angle DAB = 180^0 - 100^\circ = 80^\circ$

$\angle DAB = 80^\circ$

The angle DAC is sum of angle DAB and CAB.

$\angle DAC = \angle CAB + \angle DAB$

$\angle DAC = 50^\circ + 80^\circ$

$\angle DAC = 130^\circ$