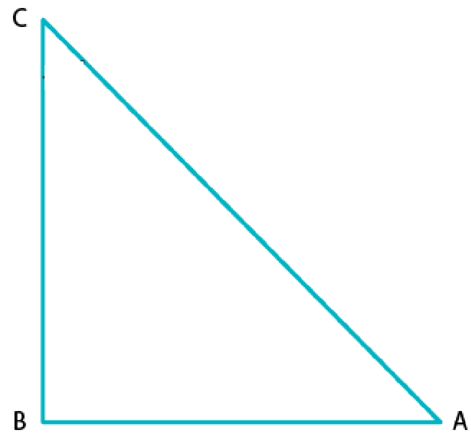
Exercise 5.1

In each of the following, one of the six trigonometric ratios s given. Find the values of the other trigonometric ratios.



(i) sin A = 2/3

Solution:

We have,

sin A = 2/3................. (1)

As we know, by sin definition;

sin A = Perpendicular/ Hypotenuse = 2/3 ….(2)

By comparing eq. (1) and (2), we have

Opposite side = 2 and Hypotenuse = 3

Now, on using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the values of perpendicular side (BC) and hypotenuse (AC) and for the base side as (AB), we get

⇒ $3^2 = AB^2 + 2^2$

$AB^2 = 3^2 - 2^2$

$AB^2 = 9 - 4$

$AB^2 = 5$

$AB = \sqrt{5}$

Hence, Base = √5

By definition,

cos A = Base/Hypotenuse

⇒ cos A = √5/3

Since, cosec A = 1/sin A = Hypotenuse/Perpendicular

⇒ cosec A = 3/2

And, sec A = Hypotenuse/Base

⇒ sec A = 3/√5

And, tan A = Perpendicular/Base

⇒ tan A = 2/√5

And, cot A = 1/ tan A = Base/Perpendicular
⇒ cot A = √5/2

(ii) cos A = 4/5
Solution:

We have,
cos A = 4/5 .... (1)
As we know, by cos defination
cos A = Base/Hypotenuse …. (2)
By comparing eq. (1) and (2), we get
Base = 4 and Hypotenuse = 5

Now, using Pythagoras theorem in Δ ABC
$AC^2 = AB^2 + BC^2$
Putting the value of base (AB) and hypotenuse (AC) and for the perpendicular (BC), we get
$5^2 = 4^2 + BC^2$
$BC^2 = 5^2 – 4^2$
$BC^2 = 25 – 16$
$BC^2 = 9$
BC= 3
Hence, Perpendicular = 3

By definition,
sin A = Perpendicular/Hypotenuse
⇒ sin A = 3/5
Then, cosec A = 1/sin A
⇒ cosec A= 1/ (3/5) = 5/3 = Hypotenuse/Perependicular
And, sec A = 1/cos A
⇒ sec A =Hypotenuse/Base
sec A = 5/4
And, tan A = Perpendicular/Base
⇒ tan A = 3/4
Next, cot A = 1/tan A = Base/Perpendicular
∴ cot A = 4/3

(iii) tan θ = 11/1
Solution:

We have, tan θ = 11….......(1)
By definition,
tan θ = Perpendicular/ Base…. (2)
On Comparing eq. (1) and (2), we get;
Base = 1 and Perpendicular = 5

Now, using Pythagoras theorem in Δ ABC.

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and perpendicular (BC) to get hypotenuse(AC), we get;

$AC^2 = 1^2 + 11^2$

$AC^2 = 1 + 121$

$AC^2 = 122$

$AC = \sqrt{122}$

Hence, hypotenuse = $\sqrt{122}$

By definition,

sin = Perpendicular/Hypotenuse

⇒ $\sin\theta = 11/\sqrt{122}$

And, $\operatorname{cosec}\theta = 1/\sin\theta$

⇒ $\operatorname{cosec}\theta = \sqrt{122}/11$

Next, $\cos\theta$ = Base/ Hypotenuse

⇒ $\cos\theta = 1/\sqrt{122}$

And, $\sec\theta = 1/\cos\theta$

⇒ $\sec\theta = \sqrt{122}/1 = \sqrt{122}$

And, $\cot\theta = 1/\tan\theta$

∴ $\cot\theta = 1/11$

(iv) sin θ = 11/15

Solution:

We have, $\sin\theta = 11/15$...............(1)

By definition,

$\sin\theta$ = Perpendicular/ Hypotenuse …. (2)

On Comparing eq. (1) and (2), we get;

Perpendicular = 11 and Hypotenuse= 15

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base (AB), we have

$15^2 = AB^2 + 11^2$

$AB^2 = 15^2 - 11^2$

$AB^2 = 225 - 121$

$AB^2 = 104$

$AB = \sqrt{104}$

$AB = \sqrt{(2\times2\times2\times13)}$

$AB = 2\sqrt{(2\times13)}$

$AB = 2\sqrt{26}$

Hence, Base = $2\sqrt{26}$

By definition,

$\cos\theta$ = Base/Hypotenuse

∴ $\cos\theta = 2\sqrt{26}/15$

And, cosec θ = 1/sin θ
∴ cosec θ = 15/11
And, secθ = Hypotenuse/Base
∴ secθ =15/ 2√26
And, tan θ = Perpendicular/Base
∴ tanθ =11/ 2√26
And, cot θ = Base/Perpendicular
∴ cotθ =2√26/ 11

(v) tan α = 5/12

Solution:

We have, tan α = 5/12 …. (1)
By definition,
tan α = Perpendicular/Base…. (2)
On Comparing eq. (1) and (2), we get
Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in Δ ABC
$AC^2 = AB^2 + BC^2$
Putting the value of base (AB) and the perpendicular (BC) to get hypotenuse (AC), we have
$AC^2 = 12^2 + 5^2$
$AC^2 = 144 + 25$
$AC^2 = 169$
AC = 13 [After taking sq root on both sides]
Hence, Hypotenuse = 13

By definition,
sin α = Perpendicular/Hypotenuse
∴ sin α = 5/13
And, cosec α = Hypotenuse/Perpendicular
∴ cosec α = 13/5
And, cos α = Base/Hypotenuse
∴ cos α = 12/13
And, sec α =1/cos α
∴ sec α = 13/12
And, tan α = sin α/cos α
∴ tan α=5/12
Since, cot α = 1/tan α
∴ cot α =12/5

(vi) sin θ = √3/2

Solution:

We have, sin θ = √3/2..................(1)

By definition,
sin θ = Perpendicular/ Hypotenuse….(2)
On Comparing eq. (1) and (2), we get;
Perpendicular = √3 and Hypotenuse = 2

Now, using Pythagoras theorem in Δ ABC
$AC^2 = AB^2 + BC^2$
Putting the value of perpendicular (BC) and hypotenuse (AC) and get the base (AB), we get;
$2^2 = AB^2 + (\sqrt{3})^2$
$AB^2 = 2^2 - (\sqrt{3})^2$
$AB^2 = 4 - 3$
$AB^2 = 1$
AB = 1
Thus, Base = 1

By definition,
cos θ = Base/Hypotenuse
∴ cos θ = 1/2
And, cosec θ = 1/sin θ
Or cosec θ= Hypotenuse/Perpendicualar
∴ cosec θ =2/√3
And, sec θ = Hypotenuse/Base
∴ sec θ = 2/1
And, tan θ = Perpendicula/Base
∴ tan θ = √3/1
And, cot θ = Base/Perpendicular
∴ cot θ = 1/√3

(vii) cos θ = 7/25
Solution:

We have, cos θ = 7/25 .... (1)
By definition,
cos θ = Base/Hypotenuse
On Comparing eq. (1) and (2), we get;
Base = 7 and Hypotenuse = 25

Now, using Pythagoras theorem in Δ ABC
$AC^2= AB^2 + BC^2$
Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC)
$25^2 = 7^2 + BC^2$
$BC^2 = 25^2 - 7^2$
$BC^2 = 625 - 49$
$BC^2 = 576$
$BC= \sqrt{576}$
BC= 24

Hence, Perpendicular side = 24

By definition,

sin θ = perpendicular/Hypotenuse

∴ sin θ = 24/25

Since, cosec θ = 1/sin θ

Also, cosec θ= Hypotenuse/Perpendicualar

∴ cosec θ = 25/24

Since, sec θ = 1/cosec θ

Also, sec θ = Hypotenuse/Base

∴ sec θ = 25/7

Since, tan θ = Perpendicular/Base

∴ tan θ = 24/7

Now, cot = 1/tan θ

So, cot θ = Base/Perpendicular

∴ cot θ = 7/24

(viii) tan θ = 8/15

Solution:

We have, tan θ = 8/15..................(1)

By definition,

tan θ = Perpendicular/Base …. (2)

On Comparing eq. (1) and (2), we get;

Base = 15 and Perpendicular = 8

Now, using Pythagoras theorem in Δ ABC

$AC^2 = 15^2 + 8^2$

$AC^2 = 225 + 64$

$AC^2 = 289$

$AC = \sqrt{289}$

AC = 17

Hence, Hypotenuse = 17

By definition,

Since, sin θ = perpendicular/Hypotenuse

∴ sin θ = 8/17

Since, cosec θ = 1/sin θ

Also, cosec θ = Hypotenuse/Perpendicualar

∴ cosec θ = 17/8

Since, cos θ = Base/Hypotenuse

∴ cos θ = 15/17

Since, sec θ = 1/cos θ

Also, sec θ = Hypotenuse/Base

∴ sec θ = 17/15

Since, cot θ = 1/tan θ

Also, cot θ = Base/Perpendicular
∴ cot θ = 15/8

(ix) cot θ = 12/5
Solution:

We have, cot θ = 12/5..................(1)
By definition,
cot θ = 1/tan θ
cot θ = Base/Perpendicular...........(2)
On Comparing eq. (1) and (2), we have
Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in Δ ABC
$AC^2 = AB^2 + BC^2$
Putting the value of base (AB) and perpendicular (BC) to get the hypotenuse (AC);
$AC^2 = 12^2 + 5^2$
$AC^2 = 144 + 25$
$AC^2 = 169$
AC = √169
AC = 13
Hence, Hypotenuse = 13

By definition,
Since, sin θ = perpendicular/Hypotenuse
∴ sin θ= 5/13
Since, cosec θ = 1/sin θ
Also, cosec θ= Hypotenuse/Perpendicualar
∴ cosec θ = 13/5
Since, cos θ = Base/Hypotenuse
∴ cos θ = 12/13
Since, sec θ = 1/cosθ
Also, sec θ = Hypotenuse/Base
∴ sec θ = 13/12
Since, tanθ = 1/cot θ
Also, tan θ = Perpendicular/Base
∴ tan θ = 5/12

(x) sec θ = 13/5
Solution:

We have, sec θ = 13/5… ....(1)
By definition,
sec θ = Hypotenuse/Base…..............(2)
On Comparing eq. (1) and (2), we get
Base = 5 and Hypotenuse = 13

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

And. putting the value of base side (AB) and hypotenuse (AC) to get the perpendicular side (BC)

$13^2 = 5^2 + BC^2$

$BC^2 = 13^2 – 5^2$

$BC^2 = 169 – 25$

$BC^2 = 144$

$BC = \sqrt{144}$

$BC = 12$

Hence, Perpendicular = 12

By definition,

Since, sin θ = perpendicular/Hypotenuse

∴ sin θ= 12/13

Since, cosec θ= 1/ sin θ

Also, cosec θ= Hypotenuse/Perpendicualar

∴ cosec θ = 13/12

Since, cos θ= 1/sec θ

Also, cos θ = Base/Hypotenuse

∴ cos θ = 5/13

Since, tan θ = Perpendicular/Base

∴ tan θ = 12/5

Since, cot θ = 1/tan θ

Also, cot θ = Base/Perpendicular

∴ cot θ = 5/12

(xi) cosec θ = √10

Solution:

We have, cosec θ = √10/1 ....(1)

By definition,

cosec θ = Hypotenuse/ Perpendicualar...............(2)

And, cosecθ = 1/sin θ

On comparing eq.(1) and(2), we get

Perpendicular side = 1 and Hypotenuse = √10

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base side (AB)

$(\sqrt{10})^2 = AB^2 + 1^2$

$AB^2 = (\sqrt{10})^2 – 1^2$

$AB^2 = 10 – 1$

$AB = \sqrt{9}$

$AB = 3$

So, Base side = 3

By definition,
Since, sin θ = Perpendicular/Hypotenuse
∴ sin θ = 1/√10
Since, cos θ = Base/Hypotenuse
∴ cos θ = 3/√10
Since, sec θ = 1/cos θ
Also, sec θ = Hypotenuse/Base
∴ sec θ = √10/3
Since, tan θ = Perpendicular/Base
∴ tan θ = 1/3
Since, cot θ = 1/tan θ
∴ cot θ = 3/1

(xii) cos θ =12/15
Solution:

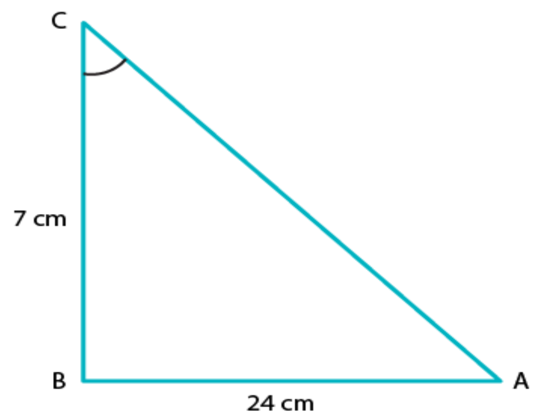
We have; cos θ = 12/15.............. (1)
By definition,
cos θ = Base/Hypotenuse…..........(2)
By comparing eq. (1) and (2), we get;
Base =12 and Hypotenuse = 15

Now, using Pythagoras theorem in Δ ABC, we get
$AC^2 = AB^2 + BC^2$
Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC);
$15^2 = 12^2 + BC^2$
$BC^2 = 15^2 – 12^2$
$BC^2 = 225 – 144$
$BC^2 = 81$
BC = √81
BC = 9
So, Perpendicular = 9

By definition,
Since, sin θ = perpendicular/Hypotenuse
∴ sin θ = 9/15 = 3/5
Since, cosec θ = 1/sin θ
Also, cosec θ = Hypotenuse/Perpendicualar
∴ cosec θ= 15/9 = 5/3
Since, sec θ = 1/cos θ
Also, sec θ = Hypotenuse/Base
∴ sec θ = 15/12 = 5/4
Since, tan θ = Perpendicular/Base
∴ tan θ = 9/12 = 3/4
Since, cot θ = 1/tan θ
Also, cot θ = Base/Perpendicular

∴ $\cot \theta = 12/9 = 4/3$

2. In a △ ABC, right angled at B, AB = 24 cm , BC = 7 cm. Determine
(i) sin A , cos A **(ii) sin C, cos C**
Solution:



(i) Given: In △ABC, AB = 24 cm, BC = 7cm and ∠ABC = 90º
To find: sin A, cos A

By using Pythagoras theorem in △ABC we have

$AC^2 = AB^2 + BC^2$
$AC^2 = 24^2 + 7^2$
$AC^2 = 576 + 49$
$AC^2 = 625$
$AC = \sqrt{625}$
AC= 25

Hence, Hypotenuse = 25

By definition,

sin A = Perpendicular side opposite to angle A/ Hypotenuse
sin A = BC/ AC
sin A = 7/ 25

And,

cos A = Base side adjacent to angle A/Hypotenuse
cos A = AB/ AC
cos A = 24/ 25

(ii) Given: In △ABC , AB = 24 cm and BC = 7cm and ∠ABC = 90º
To find: sin C, cos C

By using Pythagoras theorem in △ABC we have

$AC^2 = AB^2 + BC^2$
$AC^2 = 24^2 + 7^2$
$AC^2 = 576 + 49$
$AC^2 = 625$

AC = √625

AC= 25

Hence, Hypotenuse = 25

By definition,

sin C = Perpendicular side opposite to angle C/Hypotenuse

sin C = AB/ AC

sin C = 24/ 25

And,

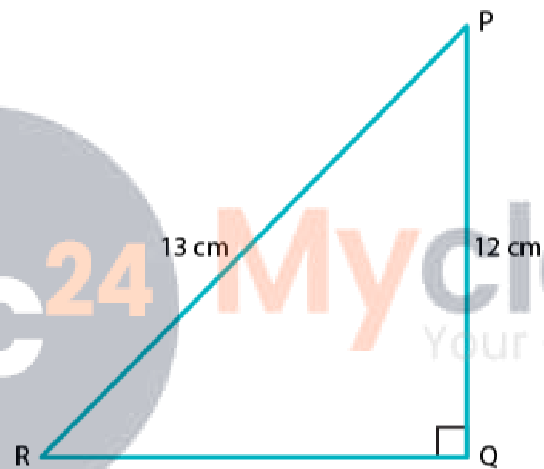
cos C = Base side adjacent to angle C/Hypotenuse

cos A = BC/AC

cos A = 7/25

3. In fig. 5.37, find tan P and cot R. Is tan P = cot R?

Solution:



By using Pythagoras theorem in △PQR, we have

$PR^2 = PQ^2 + QR^2$

Putting the length of given side PR and PQ in the above equation

$13^2 = 12^2 + QR^2$

$QR^2 = 13^2 – 12^2$

$QR^2 = 169 – 144$

$QR^2 = 25$

QR = √25 = 5

By definition,

tan P = Perpendicular side opposite to P/ Base side adjacent to angle P

tan P = QR/PQ

tan P = 5/12 ....(1)

And,

cot R= Base/Perpendicular

cot R= QR/PQ

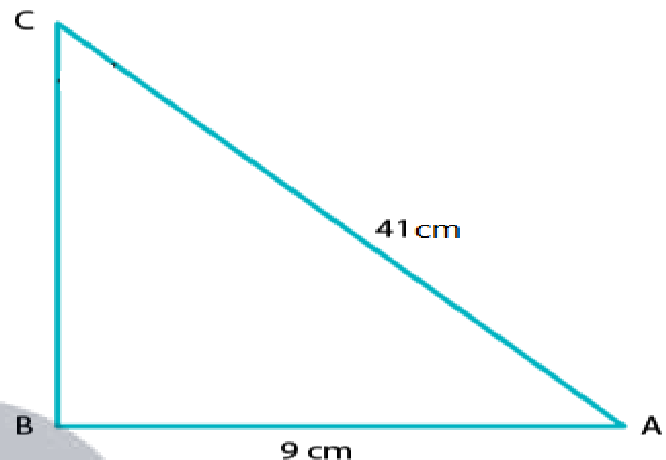
cot R= 5/12 …. (2)

When comparing equation (1) and (2), we can see that R.H.S of both the equation is equal.

Therefore, L.H.S of both equations should also be equal.
∴ tan P = cot R

Yes, tan P = cot R = 5/12

4. If sin A = 9/41, compute cos A and tan A.
Solution:



Given that, sin A = 9/41 .... (1)
Required to find: cos A, tan A
By definition, we know that
sin A = Perpendicular/ Hypotenuse…................(2)
On Comparing eq. (1) and (2), we get;
Perpendicular side = 9 and Hypotenuse = 41

Let's construct △ABC as shown below,
And, here the length of base AB is unknown.
Thus, by using Pythagoras theorem in △ABC, we get;

$AC^2 = AB^2 + BC^2$
$41^2 = AB^2 + 9^2$
$AB^2 = 41^2 – 9^2$
$AB^2 = 168 – 81$
AB= 1600
AB = √1600
AB = 40

⇒ Base of triangle ABC, AB = 40

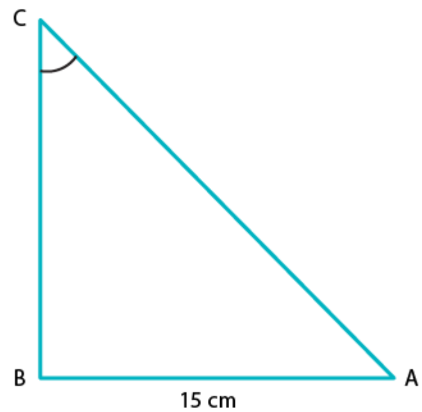
We know that,
cos A = Base/ Hypotenuse
cos A =AB/AC
cos A =40/41

And,
tan A = Perpendicular/ Base
tan A = BC/AB

tan A = 9/40

5. Given 15cot A= 8, find sin A and sec A.

Solution

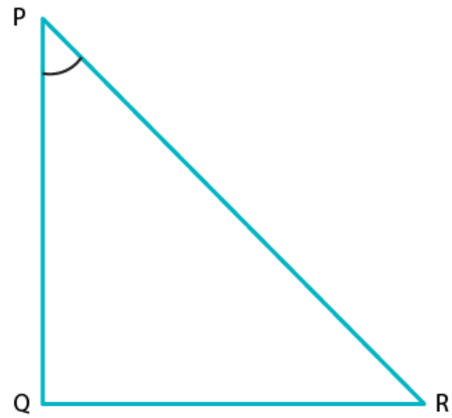


We have, 15cot A = 8
Required to find: sin A and sec A

As, 15 cot A = 8
⇒ cot A = 8/15 …….(1)
And we know,
cot A = 1/tan A
Also by definition,
Cot A = Base side adjacent to ∠A/ Perpendicular side opposite to ∠A …. (2)
On comparing equation (1) and (2), we get;
Base side adjacent to ∠A = 8
Perpendicular side opposite to ∠A = 15

So, by using Pythagoras theorem to △ABC, we have
$AC^2 = AB^2 + BC^2$
Substituting values for sides from the figure
$AC^2 = 8^2 + 15^2$
$AC^2 = 64 + 225$
$AC^2 = 289$
AC = √289
AC = 17
Therefore, hypotenuse =17
By definition,
sin A = Perpendicular/Hypotenuse
⇒ sin A= BC/AC
sin A= 15/17 (using values from the above)
Also,
sec A= 1/ cos A
⇒ secA = Hypotenuse/ Base side adjacent to ∠A
∴ sec A= 17/8

6. In △PQR, right-angled at Q, PQ = 4cm and RQ = 3 cm. Find the value of sin P, sin R, sec P and sec R.
Solution:



Given:

△PQR is right-angled at Q.
PQ = 4cm
RQ = 3cm

Required to find: sin P, sin R, sec P, sec R

Given △PQR,
By using Pythagoras theorem to △PQR, we get

$PR^2 = PQ^2 + RQ^2$
Substituting the respective values,
$PR^2 = 4^2 + 3^2$
$PR^2 = 16 + 9$
$PR^2 = 25$
$PR = \sqrt{25}$
PR = 5

⇒ Hypotenuse =5

By definition,

sin P = Perpendicular side opposite to angle P/ Hypotenuse
sin P = RQ/ PR

⇒ sin P = 3/5

And,

sin R = Perpendicular side opposite to angle R/ Hypotenuse
sin R = PQ/ PR

⇒ sin R = 4/5

And,

sec P=1/cos P
secP = Hypotenuse/ Base side adjacent to ∠P
sec P = PR/ PQ

⇒ sec P = 5/4

Now,

sec R = 1/cos R

secR = Hypotenuse/ Base side adjacent to ∠R

sec R = PR/ RQ

⇒ sec R = 5/3

7. If cot θ = 7/8, evaluate

(i) (1+sin θ)(1–sin θ)/ (1+cos θ)(1–cos θ)

(ii) $\cot^2 \theta$

Solution:

(i) Required to evaluate: $\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$, given = cot θ = 7/8

Taking the numerator, we have

$(1+\sin\theta)(1-\sin\theta) = 1 - \sin^2\theta$ [Since, $(a+b)(a-b) = a^2 - b^2$]

Similarly,

$(1+\cos\theta)(1-\cos\theta) = 1 - \cos^2\theta$

We know that,

$\sin^2\theta + \cos^2\theta = 1$

⇒ $1 - \cos^2\theta = \sin^2\theta$

And,

$1 - \sin^2\theta = \cos^2\theta$

Thus,

$(1+\sin\theta)(1-\sin\theta) = 1 - \sin^2\theta = \cos^2\theta$

$(1+\cos\theta)(1-\cos\theta) = 1 - \cos^2\theta = \sin^2\theta$

⇒ $\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$

$= \cos^2\theta/ \sin^2\theta$

$= (\cos\theta/\sin\theta)^2$

And, we know that (cos θ/sin θ) = cot θ

⇒ $\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$

$= (\cot\theta)^2$

$= (7/8)^2$

= 49/ 64

(ii) Given,

cot θ = 7/8

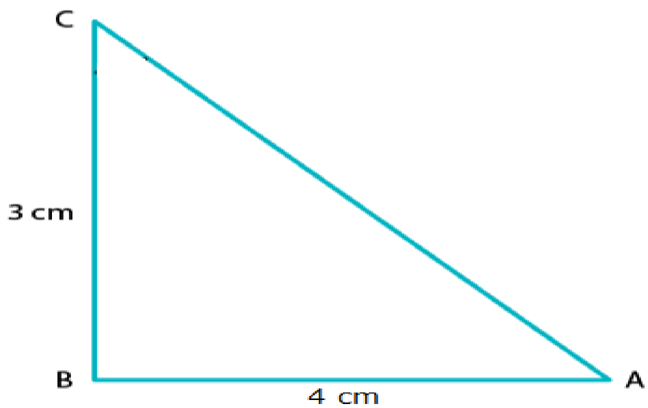
So, by squaring on both sides we get

$(\cot \theta)^2 = (7/8)^2$

$\therefore \cot \theta^2 = 49/64$

8. If 3cot A = 4, check whether $(1-\tan^2A)/(1+\tan^2A) = (\cos^2A - \sin^2A)$ or not.

Solution:



Given,

3cot A = 4

⇒ cot A = 4/3

By definition,

tan A = 1/ Cot A = 1/ (4/3)

⇒ tan A = 3/4

Thus,

Base side adjacent to ∠A = 4

Perpendicular side opposite to ∠A = 3

In ΔABC, Hypotenuse is unknown

Thus, by applying Pythagoras theorem in ΔABC

We get

$AC^2 = AB^2 + BC^2$

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9$

$AC^2 = 25$

$AC = \sqrt{25}$

$AC = 5$

Hence, hypotenuse = 5

Now, we can find that

sin A = opposite side to ∠A/ Hypotenuse = 3/5

And,

cos A = adjacent side to ∠A/ Hypotenuse = 4/5

Taking the LHS,

L.H.S = $\frac{1-\tan^2 A}{1+\tan^2 A}$

Putting value of tan A

We get,

L.H.S= $\frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2}$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2}$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

Take L.C.M of both numerator and denominator;

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{7}{25}$

Thus, LHS = 7/25

Now, taking RHS

R.H.S = $\cos^2 A - \sin^2 A$

Putting value of sin A and cos A

R.H.S= $(\frac{4}{5})^2 - (\frac{3}{5}^2)$

$\cos^2 A - \sin^2 A = (\frac{4}{5})^2 - (\frac{3}{5}^2)$

$\cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$

$\cos^2 A - \sin^2 A = \frac{16-9}{25}$

$\cos^2 A - \sin^2 A = \frac{7}{25}$

Therefore,

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

Hence Proved

9. If tan θ = a/b, find the value of (cos θ + sin θ)/ (cos θ - sin θ)
Solution:

Given,

tan θ = a/b

And, we know by definition that

tan θ = opposite side/ adjacent side

Thus, by comparison

Opposite side = a and adjacent side = b

To find the hypotenuse, we know that by Pythagoras theorem that

Hypotenuse^2 = opposite side^2 + adjacent side^2

⇒ Hypotenuse = $\sqrt{(a^2 + b^2)}$

So, by definition

sin θ = opposite side/ Hypotenuse

sin θ = a/ $\sqrt{(a^2 + b^2)}$

And,

cos θ = adjacent side/ Hypotenuse

cos θ = b/ $\sqrt{(a^2 + b^2)}$

Now,

After substituting for cos θ and sin θ, we have

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{(a+b)/\sqrt{(a^2+b^2)}}{(a-b)/\sqrt{(a^2+b^2)}}$$

$$\therefore \quad \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{(a+b)}{(a-b)}$$

Hence Proved.

10. If 3 tan θ = 4, find the value of $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$
Solution:

Given, 3 tan θ = 4
⇒ tan θ = 4/3

From, $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$ let's divide the numerator and denominator by cos θ.

We get,
(4 – tan θ) / (2 + tan θ)
⇒ (4 – (4/3)) / (2 + (4/3)) [using the value of tan θ]
⇒ (12 – 4) / (6 + 4) [After taking LCM and cancelling it]
⇒ 8/10 = 4/5

$$\therefore \quad \frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = 4/5$$

11. If 3 cot θ = 2, find the value of $\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$

Solution:

Given, 3 cot θ = 2
⇒ cot θ = 2/3

From, $\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$ let's divide the numerator and denominator by sin θ.

We get,
(4 –3 cot θ) / (2 + 6 cot θ)
⇒ (4 – 3(2/3)) / (2 + 6(2/3)) [using the value of tan θ]
⇒ (4 – 2) / (2 + 4) [After taking LCM and simplifying it]
⇒ 2/6 = 1/3

$\therefore \quad \dfrac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta} = 1/3$

12. If tan θ = a/b, prove that $\dfrac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \dfrac{a^2 - b^2}{a^2 + b^2}$

Solution:

Given, tan θ = a/b

From LHS, $\dfrac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ let's divide the numerator and denominator by cos θ.

And we get,

(a tan θ – b) / (a tan θ + b)

⇒ (a(a/b) – b) / (a(a/b) + b) [using the value of tan θ]

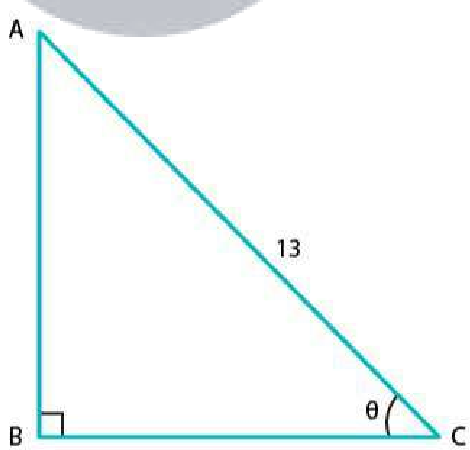
⇒ $(a^2 - b^2)/b^2 / (a^2 + b^2)/b^2$ [After taking LCM and simplifying it]

⇒ $(a^2 - b^2)/ (a^2 + b^2)$

= RHS

- Hence Proved

13. If sec θ = 13/5, show that $\dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$ **Solution:**



Given,

sec θ = 13/5

We know that,

sec θ = 1/ cos θ

⇒ cos θ = 1/ sec θ = 1/ (13/5)

∴ $\cos\theta = 5/13$ ……. (1)

By definition,
$\cos\theta$ = adjacent side/ hypotenuse ….. (2)

Comparing (1) and (2), we have
Adjacent side = 5 and hypotenuse = 13

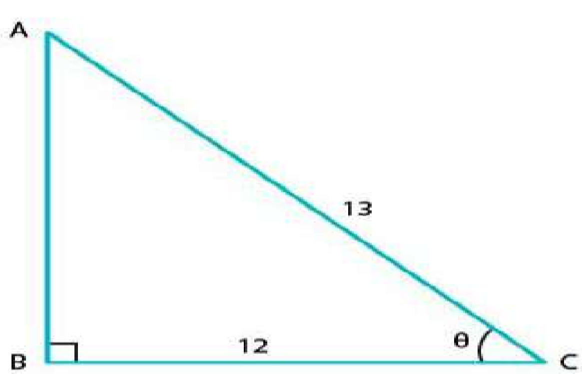
By Pythagoras theorem,
Opposite side = $\sqrt{(\text{hypotenuse})^2 - (\text{adjacent side})^2}$
$= \sqrt{(13^2 - 5^2)}$
$= \sqrt{(169 - 25)}$
$= \sqrt{(144)}$
$= 12$
Thus, opposite side = 12
By definition,
$\tan\theta$ = opposite side/ adjacent side
∴ $\tan\theta = 12/5$

From, $\dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$ let's divide the numerator and denominator by $\cos\theta$.

We get,
$(2\tan\theta - 3) / (4\tan\theta - 9)$
⇒ $(2(12/5) - 3) / (4(12/5) - 9)$ [using the value of tan θ]
⇒ $(24 - 15) / (48 - 45)$ [After taking LCM and cancelling it]
⇒ $9/3 = 3$

∴ $\dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$

14. If cos θ = 12/13, show that sin θ(1 – tan θ) = 35/156
Solution:



Given, cos θ = 12/13…… (1)
By definition we know that,
cos θ = Base side adjacent to ∠θ / Hypotenuse……. (2)

When comparing equation (1) and (2), we get
Base side adjacent to ∠θ = 12 and Hypotenuse = 13
From the figure,
Base side BC = 12
Hypotenuse AC = 13
Side AB is unknown here and it can be found by using Pythagoras theorem
Thus by applying Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$13^2 = AB^2 + 12^2$
Therefore,
$AB^2 = 13^2 – 12^2$
$AB^2 = 169 – 144$
$AB^2 = 25$
AB = √25
AB = 5 …. (3)

Now, we know that
sin θ = Perpendicular side opposite to ∠θ / Hypotenuse
Thus, sin θ = AB/AC [from figure]
⇒ sin θ = 5/13… (4)
And, tan θ = sin θ / cos θ = (5/13) / (12/13)
⇒ tan θ = 12/13… (5)

Taking L.H.S we have
L.H.S = sin θ (1 – tan θ)

Substituting the value of sin θ and tan θ from equation (4) and (5)
We get,

$\Rightarrow$

$$L.H.S = \frac{5}{13}\left(1 - \frac{5}{12}\right)$$

$$L.H.S = \frac{5}{13}\left(\frac{1 \times 12}{1 \times 12} - \frac{5}{12}\right) \quad \text{[Taking LCM]}$$

$$L.H.S = \frac{5}{13}\left(\frac{12-5}{12}\right)$$

$$L.H.S = \frac{5}{13}\left(\frac{7}{12}\right)$$

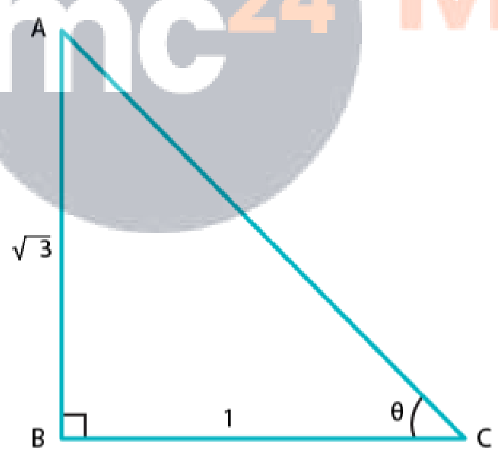
$$L.H.S = \frac{5 \times 7}{13 \times 12}$$

L.H.S = 35/156

Therefore it's shown that sin θ(1 - tan θ)= 35/156

15. If $\cot\theta = \frac{1}{\sqrt{3}}$, show that $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

Solution:



Given, cot θ = 1/√3........(1)
By definition we know that,
cot θ = 1/ tan θ
And, since tan θ = perpendicular side opposite to ∠θ / Base side adjacent to ∠θ

$\Rightarrow$ cot θ = Base side adjacent to ∠θ / perpendicular side opposite to ∠θ …… (2)
[Since they are reciprocal to each other]

On comparing equation (1) and (2), we get
Base side adjacent to ∠θ = 1 and Perpendicular side opposite to ∠θ = √3

Therefore, the triangle formed is,

On substituting the values of known sides as AB = √3 and BC = 1

$AC^2 = (\sqrt{3}) + 1$

$AC^2 = 3 + 1$

$AC^2 = 4$

$AC = \sqrt{4}$

Therefore, AC = 2 … (3)

Now, by definition

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ $\sin\theta = \sqrt{3}/2$ ……(4)

And, cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ $\cos\theta = 1/2$ ….. (5)

Now, taking L.H.S we have

$$\text{L.H.S} = \frac{1-\cos^2\theta}{2-\sin^2\theta}$$

Substituting the values from equation (4) and (5), we have

$$\text{L.H.S} = \frac{\frac{[illegible]}{4}}{\frac{(4\times 2)-3}{4}}$$

$$\text{L.H.S} = \frac{\frac{4-1}{4}}{\frac{8-3}{4}}$$

$$\text{L.H.S} = \frac{3}{4}\times\frac{4}{5}$$

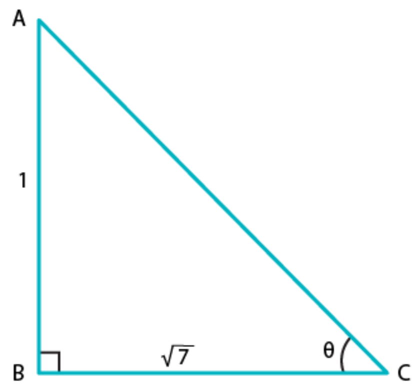
$$\text{L.H.S} = \frac{3}{5} = \text{R.H.S}$$

Therefore, $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

16. If $\tan\theta = \dfrac{1}{\sqrt{7}}$, then show that $\dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \dfrac{3}{4}$

Solution:

Given, tan θ = 1/ √7 …..(1)



By definition, we know that

tan θ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ ……(2)

On comparing equation (1) and (2), we have

Perpendicular side opposite to ∠θ = 1

Base side adjacent to ∠θ = √7

Thus, the triangle representing ∠ θ is,

Hypotenuse AC is unknown and it can be found by using Pythagoras theorem

By applying Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$AC^2 = 1^2 + (\sqrt{7})^2$

$AC^2 = 1 + 7$

$AC^2 = 8$

$AC = \sqrt{8}$

⇒ $AC = 2\sqrt{2}$

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ sin θ = 1/ 2√2

And, since cosec θ = 1/sin θ

⇒ cosec θ = 2√2 ....(3)

Now,

cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ cos θ = √7/ 2√2

And, since sec θ = 1/ sin θ

⇒ sec θ = 2√2/ √7 ……. (4)

Taking the L.H.S of the equation,

$$\text{L.H.S} = \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$$

Substituting the value of cosec θ and sec θ from equation (3) and (4), we get

$$\text{L.H.S} = \frac{[(2\sqrt{2})]^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{[(2\sqrt{2})]^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$\text{L.H.S} = \frac{(8) - \left(\frac{8}{7}\right)}{(8) + \left(\frac{8}{7}\right)} = \frac{\frac{56-8}{7}}{\frac{56+8}{7}}$$

$$\text{L.H.S} = \frac{\frac{48}{7}}{\frac{64}{7}}$$

Therefore,

L.H.S = 48/64 = 3/4 = R.H.S

Hence proved that $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$

17. If sec θ = 5/4, find the value of $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

Solution:

Given,

sec θ = 5/4

We know that,

sec θ = 1/ cos θ

⇒ cos θ = 1/ (5/4) = 4/5 …… (1)

By definition,

cos θ = Base side adjacent to ∠θ / Hypotenuse …. (2)

On comparing equation (1) and (2), we have

Hypotenuse = 5

Base side adjacent to ∠θ = 4

Thus, the triangle representing ∠ θ is ABC.

Perpendicular side opposite to ∠θ, AB is unknown and it can be found by using Pythagoras theorem

By applying Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 + BC^2$

$AB^2 = 5^2 – 4^2$

$AB^2 = 25 – 16$

$AB = \sqrt{9}$

⇒ $AB = 3$

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ sin θ = 3/ 5 …..(3)

Now, tan θ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ

⇒ tan θ = 3/ 4 ……(4)

And, since cot θ = 1/ tan θ

⇒ cot θ = 4/ 3 ……(5)

Now,

Substituting the value of sin θ, cos θ, cot θ and tan θ from the equations (1), (3), (4) and (5) we have,

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{\frac{3}{5} - 2\left(\frac{4}{5}\right)}{\frac{3}{4} - \frac{4}{3}}$$

= 12/7

Therefore,

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{12}{7}$$

18. If tan θ = 12/13, find the value of $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

Solution:

Given,

tan θ = 12/13............(1)

We know that by definition,

tan θ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ …… (2)

On comparing equation (1) and (2), we have

Perpendicular side opposite to ∠θ = 12

Base side adjacent to ∠θ = 13

Thus, in the triangle representing ∠ θ we have,
Hypotenuse AC is the unknown and it can be found by using Pythagoras theorem
So by applying Pythagoras theorem, we have

$AC^2 = 12^2 + 13^2$

$AC^2 = 144 + 169$

$AC^2 = 313$

⇒ $AC = \sqrt{313}$

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ sin θ = 12/ √313…..(3)

And, cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ cos θ = 13/ √313 …..(4)

Now, substituting the value of sin θ and cos θ from equation (3) and (4) respectively in the equation below

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{2 \times \frac{13}{\sqrt{313}} \times \frac{12}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2 - \left(\frac{12}{\sqrt{313}}\right)^2}$$

$$= \frac{\frac{312}{313}}{\frac{25}{313}}$$

$$= \frac{312}{25}$$

Therefore,

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{312}{25}$$