

# NCERT Solutions for Class-XII Maths

## Chapter-6.3 NCERT Math Class 12

1. Find the slope of the tangent to the curve  $y = 3x^4 - 4x$  at  $x = 4$ .

1. The given curve is  $y = 3x^4 - 4x$ .

Then, the slope of the tangent to the given curve at  $x = 4$  is given by,

$$\left. \frac{dy}{dx} \right]_{x=4} = 12x^3 - 4 \Big]_{x=4} = 12(4)^3 - 4 = 12(64) - 4 = 764$$

2. Find the slope of the tangent to the curve

$$y = \frac{x-1}{x-2}, x \neq 2 \text{ at } x = 10.$$

2. The given curve is  $y = \frac{x-1}{x-2}$

$$\begin{aligned} \therefore \frac{dy}{dx} &= \frac{(x-2)(1) - (x-1)(1)}{(x-2)^2} \\ &= \frac{x-2-x+1}{(x-2)^2} \\ &= \frac{-1}{(x-2)^2} \end{aligned}$$

Then, the slope of the tangent

$$\left. \frac{dy}{dx} \right]_{x=10} = \frac{-1}{(x-2)^2} \Big]_{x=10} = \frac{-1}{(10-2)^2} = \frac{-1}{64}$$

Therefore, the slope of the tangent is  $\frac{-1}{64}$ .

3. Find the slope of the tangent to curve  $y = x^3 - x + 1$  at the point whose x-coordinate is 2.

3. The given curve is  $y = x^3 - x + 1$

$$\therefore \frac{dy}{dx} = 3x^2 - 1$$

The slope of the tangent to a curve at  $(x_0, y_0)$  is  $\left. \frac{dy}{dx} \right]_{(x_0, y_0)}$

It is given that  $x_0 = 2$ .

Hence, the slope of the tangent at the point where the  $x$ -coordinate is 2 is given by,

$$\left. \frac{dy}{dx} \right|_{x=2} = 3x^2 - 1 \Big|_{x=2} = 3(2)^2 - 1 = 12 - 1 = 11$$

4. Find the slope of the tangent to the curve  $y = x^3 - 3x + 2$  at the point whose  $x$ -coordinate is 3.

4. The given curve is  $y = x^3 - 3x + 2$

$$\therefore \frac{dy}{dx} = 3x^2 - 3$$

Then, the slope of the tangent

$$\left. \frac{dy}{dx} \right|_{x=3} = 3x^2 - 3 \Big|_{x=3} = 3(3)^2 - 3 \\ = 27 - 3 = 24$$

**Therefore, the slope of the tangent 24.**

5. Find the slope of the normal to the curve  $3x = a \cos^3 \theta$ ,  $y = a \sin^3 \theta$  at  $\theta = \frac{\pi}{4}$

5. It is given that  $x = a \cos^3 \theta$  and  $y = a \sin^3 \theta$ .

$$\therefore \frac{dx}{d\theta} = 3a \cos^2 \theta (-\sin \theta) = -3a \cos^2 \theta \sin \theta$$

$$\frac{dy}{d\theta} = 3a \sin^2 \theta (\cos \theta)$$

$$\therefore \frac{dy}{dx} = \frac{\left( \frac{dy}{d\theta} \right)}{\left( \frac{dx}{d\theta} \right)} = \frac{3a \sin^2 \theta \cos \theta}{-3a \cos^2 \theta \sin \theta} = -\frac{\sin \theta}{\cos \theta} = -\tan \theta$$

Therefore, the slope of the tangent at

$$\theta = \frac{\pi}{4} \text{ is given by, } \left. \frac{dy}{dx} \right|_{\theta=\frac{\pi}{4}} = -\tan \theta \Big|_{\theta=\frac{\pi}{4}} = -\tan \frac{\pi}{4} = -1$$

Hence, the slope of the normal at  $\theta = \frac{\pi}{4}$  is given by,

$$\frac{1}{\text{slope of the tangent at } \theta = \frac{\pi}{4}} = \frac{-1}{-1} = 1$$

6. Find the slope of the normal to the curve  $2x = 1 - a\sin\theta$ ,  $y = b\cos^2\theta$  at  $\theta = \frac{\pi}{2}$

6. The given curve is  $x = 1 - a\sin\theta$  and  $y = b\cos^2\theta$

$$\therefore \frac{dx}{d\theta} = -a\cos\theta$$

$$\frac{dy}{d\theta} = -2b\sin\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2b\sin\theta\cos\theta}{-a\cos\theta} = \frac{2b}{a}\sin\theta$$

Then, the slope of the tangent  $\theta = \frac{\pi}{2}$  is given by:

$$\left. \frac{dy}{dx} \right|_{\theta=\frac{\pi}{2}} = \frac{2b}{a}\sin\theta \Big|_{\theta=\frac{\pi}{2}} = \frac{2b}{a}\sin\frac{\pi}{2} = \frac{2b}{a}$$

Then, the slope of the tangent  $\theta = \frac{\pi}{2}$  is given by:

$$\frac{1}{\text{slope of the tangent at } \theta = \frac{\pi}{2}} = \frac{-1}{\left(\frac{2b}{a}\right)} = -\frac{a}{2b}$$

Therefore, the slope of the tangent  $-\frac{a}{2b}$ .

7. Find points at which the tangent to the curve  $y = x^3 - 3x^2 - 9x + 7$  is parallel to the x-axis.

7. The equation of the given curve is  $y = x^3 - 3x^2 - 9x + 7$

$$\therefore \frac{dy}{dx} = 3x^2 - 6x - 9$$

Now, the tangent is parallel to the x-axis if the slope of the tangent is zero.

$$\therefore 3x^2 - 6x - 9 = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x-3)(x+1) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -1$$

$$\text{When } x = 3, y = (3)^3 - 3(3)^2 - 9(3) + 7 = 27 - 27 - 27 + 7 = -20$$

When  $x = -1, y = (-1)^3 - 3(-1)^2 - 9(-1) + 7 = -1 - 3 + 9 + 7 = 12$ . Hence, the points at which the tangent is parallel to the x-axis are  $(3, -20)$  and  $(-1, 12)$

8. Find a point on the curve  $y = (x - 2)^2$  at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

8. We know that if a tangent is parallel to the chord joining the points (2,0) and (4,4) , then Slope of the tangent = slope of the curve.....(1)

$$\text{And, the slope of the curve} = \frac{4-0}{4-2} = 2$$

Now, slope of the tangent to the given curve at a point (x,y) is:

$$\frac{dy}{dx} = 2(x - 2)$$

Now, from (1) we have,

$$2(x - 2) = 2$$

$$\Rightarrow x - 2 = 1$$

$$\Rightarrow x = 3$$

So, when  $x = 3$  then  $y = (3-2)^2 = 1$

**Therefore, required points are (3,1).**



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9. Find the point on the curve  $y = x^3 - 11x + 5$  at which the tangent is  $y = x - 11$ .

9. The equation of the given curve is  $y = x^3 - 11x + 5$ .

The equation of the tangent to the given curve is given as  $y = x - 11$

(which is of the form  $y = mx + c$ )

$\therefore$  Slope of the tangent = 1

Now, the slope of the tangent to the given curve at the point  $(x, y)$  is given by,

$$\frac{dy}{dx} = 3x^2 - 11$$

Then, we have:

$$3x^2 - 11 = 1$$

$$\Rightarrow 3x^2 = 12$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$$\text{When } x = 2, y = (2)^3 - 11(2) + 5 = 8 - 22 + 5 = -9$$

$$\text{When } x = -2, y = (-2)^3 - 11(-2) + 5 = -8 + 22 + 5 = 19$$

Hence, the required points are  $(2, -9)$  and  $(-2, 19)$ .

10. Find the equation of all lines having slope  $-1$  that are tangents to the curve  $y = \frac{1}{x-1}, x \neq 1$

10. It is given that equation of the curve  $y = \frac{1}{x-1}, x \neq 1$

Now, slope of the tangent to the given curve at a point  $(x, y)$  is:

$$\frac{dy}{dx} = \frac{-1}{(x-1)^2}$$

Now, if the slope of the tangent is  $-1$ , then we get,

$$\frac{-1}{(x-1)^2} = -1$$

$$\Rightarrow (x-1)^2 = 1$$

$$\Rightarrow (x-1) = \pm 1$$

$$\Rightarrow x = 2, 0$$

So, when  $x = 2$  then  $y = 1$

And when  $x = 0$  then  $y = 1$

**Therefore, required points are  $(0, -1)$  and  $(2, 1)$ .**

Now, the equation of the tangent  $(0, 1)$  is given by:

$$y - (-1) = -1(x-0)$$

$$\Rightarrow y + 1 = -x$$

$$\Rightarrow y + x + 1 = 0$$

And the equation of the tangent (2,1) is given by:

$$y - 1 = -1(x-2)$$

$$\Rightarrow y - 1 = -x + 2$$

$$\Rightarrow y + x - 3 = 0$$

**Therefore, the equations of the required lines are  $y + x + 1 = 0$  and  $y + x - 3 = 0$ .**

11. Find the equation of all lines having slope 2 which are tangents to the curve  $y = \frac{1}{x-3}, x \neq 3$
11. The equation of the given curve is  $y = \frac{1}{x-3}, x \neq 3$ .

The slope of the tangent to the given curve at any point  $(x, y)$  is given by,  $\frac{dy}{dx} = \frac{-1}{(x-3)^2}$

If the slope of the tangent is 2, then we have:

$$\frac{-1}{(x-3)^2} = 2$$

$$\Rightarrow 2(x-3)^2 = -1$$

$$\Rightarrow (x-3)^2 = \frac{-1}{2}$$

This is not possible since the L.H.S, is positive while the R.H.S. is negative. Hence, there is no tangent to the given curve having slope 2.

12. Find the equations of all lines having slope 0 which are tangent to the curve  $y = \frac{1}{x^2 - 2x + 3}$
12. It is given that equation of the curve  $y = \frac{1}{x^2 - 2x + 3}$ ,

Now, slope of the tangent to the given curve at a point  $(x, y)$  is:

$$\frac{dy}{dx} = \frac{-(2x-2)}{(x^2-2x+3)^2} = \frac{-2(x-1)}{(x^2-2x+3)^2}$$

Now, if the slope of the tangent is 0, then we get,

$$\frac{-2(x-1)}{(x^2-2x+3)^2} = 0$$

$$\Rightarrow -2(x-1) = 0$$

$$\Rightarrow x = 1$$

So, when  $x = 1$  then  $y = \frac{1}{1-2+3} = \frac{1}{2}$

Now, the equation of the tangent  $(0, \frac{1}{2})$  is given by:

$$y - \frac{1}{2} = 0(x-1)$$

$$\Rightarrow y - \frac{1}{2} = 0$$

$$\Rightarrow y = \frac{1}{2}$$

Therefore, the equations of the required line is  $y = \frac{1}{2}$ .

13. Find points on the curve  $\frac{x^2}{9} + \frac{y^2}{16} = 1$  at which the tangents are

- (i) parallel to x-axis
- (ii) parallel to y-axis.

13. The equation of the given curve is  $\frac{x^2}{9} + \frac{y^2}{16} = 1$ .

On differentiating both sides with respect to  $x$ , we have:

$$\frac{2x}{9} + \frac{2y}{16} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-16x}{9y}$$

(i). The tangent is parallel to the  $x$ -axis if the slope of the tangent  $\frac{-16x}{9y} = 0$ , is 0

which is possible if  $x = 0$ .

for  $x = 0$

Then,  $\frac{x^2}{9} + \frac{y^2}{16} = 1$

$$\Rightarrow y^2 = 16 \Rightarrow y = \pm 4$$

Hence, the points at which the tangents are parallel to the  $x$ -axis are  $(0, 4)$  and  $(0, -4)$ .

(ii). The tangent is parallel to the  $y$ -axis if the slope of the normal is 0 ,

which gives  $\frac{-1}{\left(\frac{-16x}{9y}\right)} = \frac{9y}{16x} = 0 \Rightarrow y = 0.$

Then,  $\frac{x^2}{9} + \frac{y^2}{16} = 1$

$\Rightarrow x = \pm 3$  for  $y = 0$

Hence, the points at which the tangents are parallel to the  $y$ -axis are  $(3,0)$  and  $(-3, 0).$

**14.** Find the equations of the tangent and normal to the given curves at the indicated points:

(i)  $y = x^4 - 6x^3 + 13x^2 - 10x + 5$  at  $(0, 5)$

(ii)  $y = x^4 - 6x^3 + 13x^2 - 10x + 5$  at  $(1, 3)$

(iii)  $y = x^3$  at  $(1, 1)$

(iv)  $y = x^2$  at  $(0, 0)$

(v)  $x = \cos t, y = \sin t$  at  $t = \frac{\pi}{4}$

**14. (i)** It is given that equation of curve is  $y = x^4 - 6x^3 + 13x^2 - 10x + 5$

On differentiating with respect to  $x$ , we get

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$\left. \frac{dy}{dx} \right|_{(0,5)} = 4(0)^3 - 18(0)^2 + 26(0) - 10 = -10$$

Therefore, the slope of the tangent at  $(0, 5)$  is  $-10.$

Then, the equation of the tangent is

$$y - 5 = -10(x - 0)$$

$$\Rightarrow y - 5 = -10x$$

$$\Rightarrow 10x + y = 5$$

Then, slope of normal at  $(0,5)$

$$= \frac{-1}{\text{Slope of the tangent at } (0,5)} = \frac{1}{10}$$

Now, equation of the normal at  $(0,5)$

$$y - 5 = \frac{1}{10}(x - 0)$$

$$\Rightarrow 10y - 50 = x$$

$$\Rightarrow x - 10y + 50 = 0$$

**(ii)** It is given that equation of curve is  $y = x^4 - 6x^3 + 13x^2 - 10x + 5$

On differentiating with respect to  $x$ , we get

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$\left. \frac{dy}{dx} \right|_{(1,3)} = 4 - 18 + 26 - 10 = 2$$

Therefore, the slope of the tangent at (1, 3) is 2.

Then, the equation of the tangent is

$$y - 3 = 2(x - 1)$$

$$\Rightarrow y - 3 = 2x - 2$$

$$\Rightarrow y = 2x + 1$$

Then, slope of normal at (1,3)

$$= \frac{-1}{\text{Slope of the tangent at (1,3)}} = \frac{-1}{2}$$

Now, equation of the normal at (1,3)

$$y - 3 = -\frac{1}{2}(x - 1)$$

$$\Rightarrow 2y - 6 = -x + 1$$

$$\Rightarrow x + 2y - 7 = 0$$

(iii) It is given that equation of curve is  $y = x^3$   
On differentiating with respect to x, we get

$$\frac{dy}{dx} = 3x^2$$

$$\left. \frac{dy}{dx} \right|_{(1,1)} = 3$$

Therefore, the slope of the tangent at (1, 1) is 3.

Then, the equation of the tangent is

$$y - 1 = 3(x - 1)$$

$$\Rightarrow y = 3x - 2$$

Then, slope of normal at (1,1)

$$= \frac{-1}{\text{Slope of the tangent at (1,1)}} = \frac{-1}{3}$$

Now, equation of the normal at (1,1)

$$y - 1 = -\frac{1}{3}(x - 1)$$

$$\Rightarrow 3y - 3 = -x + 1$$

$$\Rightarrow x + 3y - 4 = 0$$

(iv) It is given that equation of curve is  $y = x^2$

On differentiating with respect to x, we get

$$\frac{dy}{dx} = 2x$$

$$\left. \frac{dy}{dx} \right|_{(0,0)} = 0$$

Therefore, the slope of the tangent at (0, 0) is 0.

Then, the equation of the tangent is

$$y - 0 = 0(x - 0)$$

$$\Rightarrow y = 0$$

Then, slope of normal at (0,0)

$$= \frac{-1}{\text{Slope of the tangent at (0,0)}} = \frac{-1}{0}, \text{ which is not defined}$$

Now, equation of the normal at (0,0)

$$x = 0$$

(v) It is given that equation of curve is  $x = \cos t$ ,  $y = \sin t$

$$\frac{dx}{dt} = -\sin t, \frac{dy}{dt} = \cos t$$

On differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t} = -\cot t$$

$$\left. \frac{dy}{dx} \right|_{t=\frac{\pi}{4}} = -\cot t = -1$$

Therefore, the slope of the tangent at  $t = \frac{\pi}{4}$  is -1.

$$\text{When } t = \frac{\pi}{4}, x = \frac{1}{\sqrt{2}} \text{ and } y = \frac{1}{\sqrt{2}}$$

Then, the equation of the tangent is  $t = \frac{\pi}{4}$  is

$$y - \frac{1}{\sqrt{2}} = -1(x - \frac{1}{\sqrt{2}})$$

$$\Rightarrow x + y - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow x + y - \sqrt{2} = 0$$

Then, slope of normal at  $t = \frac{\pi}{4}$

$$= \frac{-1}{\text{Slope of the tangent at } t = \frac{\pi}{4}} = 1$$

Now, equation of the normal at  $t = \frac{\pi}{4}$

$$y - \frac{1}{\sqrt{2}} = 1\left(x - \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow x = y$$

**15.** Find the equation of the tangent line to the curve  $y = x^2 - 2x + 7$  which is

(a) parallel to the line  $2x - y + 9 = 0$

(b) perpendicular to the line  $5y - 15x = 13$ .

**15.** The equation of the given curve is  $y = x^2 - 2x + 7$

On differentiating with respect to  $x$ , we get:

$$\frac{dy}{dx} = 2x - 2$$

(a) The equation of the line is  $2x - y + 9 = 0$

$$2x - y + 9 = 0 \therefore y = 2x + 9$$

This is of the form  $y = mx + c$ .

$\therefore$  Slope of the line = 2

If a tangent is parallel to the line  $2x - y + 9 = 0$ , then the slope of the tangent is equal to the slope of the line.

Therefore, we have:

$$2 = 2x - 2$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

$$\text{Now, } x = 2 \Rightarrow y = 4$$

$$-4 + 7 = 7$$

Thus, the equation of the tangent passing through  $(2, 7)$  is given by,

$$y - 7 = 2(x - 2)$$

$$\Rightarrow y - 2x - 3 = 0$$

Hence, the equation of the tangent line to the given curve (which is parallel to line  $2x - y + 9 = 0$ ) is  $y - 2x - 3 = 0$

(b) The equation of the line is  $5y - 15x = 13$ .

$$5y - 15x = 13 \therefore y = 3x + \frac{13}{5}$$

This is of the form  $y = mx + c$ .

$\therefore$  Slope of the line = 3

If a tangent is perpendicular to the line  $5y - 15x = 13$ , then the slope of the tangent is

$$\frac{-1}{\text{slope of the line}} = \frac{-1}{3}$$

$$\Rightarrow 2x - 2 = \frac{-1}{3}$$

$$\Rightarrow 2x = \frac{-1}{3} + 2$$

$$\Rightarrow 2x = \frac{5}{3}$$

$$\Rightarrow x = \frac{5}{6}$$

$$\text{Now, } x = \frac{5}{6}$$

$$\Rightarrow y = \frac{25}{36} - \frac{10}{6} + 7 = \frac{25 - 60 + 252}{36} = \frac{217}{36}$$

Thus, the equation of the tangent passing through  $\left(\frac{5}{6}, \frac{217}{36}\right)$  is given by,

$$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$$

$$\Rightarrow \frac{36y - 217}{36} = \frac{-1}{18}(6x - 5)$$

$$\Rightarrow 36y - 217 = -2(6x - 5)$$

$$\Rightarrow 36y - 217 = -12x + 10$$

$$\Rightarrow 36y + 12x - 227 = 0$$

Hence, the equation of the tangent line to the given curve (which is perpendicular to line  $5y - 15x = 13$ ) is  $36y + 12x - 227 = 0$ .

**16.** Show that the tangents to the curve  $y = 7x^3 + 11$  at the points where  $x = 2$  and  $x = -2$  are parallel.

**16.** The given curve  $y = 7x^3 + 11$

Then, the slope of the tangent to the given curve at  $x = 4$  is given by,

$$\left. \frac{dy}{dx} \right|_{x=2} = 21x^2 \Big|_{x=2} = 21(2)^2 = 84$$

It is cleared that the slopes of the tangents at the points where  $x = 2$  and  $x = -2$  are equal.  
**Therefore, the two tangents are parallel.**

17. Find the points on the curve  $y = x^3$  at which the slope of the tangent is equal to the y-coordinate of the point.
17. The equation of the given curve is  $y = x^3$ .

$$\therefore \frac{dy}{dx} = 3x^2$$

The slope of the tangent at the point  $(x, y)$  is given by,

$$\left. \frac{dy}{dx} \right|_{(x,y)} = 3x^2$$

When the slope of the tangent is equal to the y-coordinate of the point, then  $y = 3x^2$ .

Also, we have  $y = x^3$ .

$$\therefore 3x^2 = x^3$$

$$\therefore x^2(x - 3) = 0$$

$$\therefore x = 0, x = 3$$

When  $x = 0$ , then  $y = 0$  and when  $x = 3$ , then  $y = 3(3)^2 = 27$ .

Hence, the required points are  $(0, 0)$  and  $(3, 27)$ .

18. For the curve  $y = 4x^3 - 2x^5$ , find all the points at which the tangent passes through the origin.
18. The given curve  $y = 4x^3 - 2x^5$

$$\frac{dy}{dx} = 12x^2 - 10x^4$$

Then, the slope of the tangent at the point  $(x, y)$  is  $12x^2 - 10x^4$

The equation of the tangent at  $(x, y)$  is given by,

$$Y - y = (12x^2 - 10x^4)(X - x) \dots\dots\dots(1)$$

When the tangent passes through the origin  $(0, 0)$ , then  $X = Y = 0$

Then equation (1) becomes,

$$-y = (12x^2 - 10x^4)(-x)$$

$$\Rightarrow y = (12x^3 - 10x^5)$$

Also, we have  $y = 4x^3 - 2x^5$

$$\Rightarrow (12x^3 - 10x^5) = 4x^3 - 2x^5$$

$$\Rightarrow 8x^3 - 8x^5 = 0$$

$$\Rightarrow x^3 - 2x^5 = 0$$

$$\Rightarrow x^3(x^2 - 1) = 0$$

$$\Rightarrow x = 0, \pm 1$$

When  $x = 0$  then  $y = 0$

When  $x = 1$  then  $y = 2$

And when  $x = -1$  then  $y = -2$

19. Find the points on the curve  $x^2 + y^2 - 2x - 3 = 0$  at which the tangents are parallel to the  $x$ -axis.

19. The equation of the given curve is  $x^2 + y^2 - 2x - 3 = 0$ .

On differentiating with respect to  $x$ , we have:

$$2x + 2y \frac{dy}{dx} - 2 = 0$$

$$\Rightarrow y \frac{dy}{dx} = 1 - x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-x}{y}$$

Now, the tangents are parallel to the  $x$ -axis if the slope of the tangent is 0 .

$$\therefore \frac{1-x}{y} = 0 \Rightarrow 1-x = 0 \Rightarrow x = 1$$

But,  $x^2 + y^2 - 2x - 3 = 0$  for  $x = 1$

$$y^2 = 4 \Rightarrow y = \pm 2$$

Hence, the points at which the tangents are parallel to the  $x$ -axis are  $(1, 2)$  and  $(1, -2)$ .

20. Find the equation of the normal at the point  $(am^2, am^3)$  for the curve  $ay^2 = x^3$ .

20. It is given that  $ay^2 = x^3$

Now, differentiating both sides with respect to  $x$ , we get

$$2ay \cdot \frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

Then, the slope of the tangent to the given curve at  $(am^2, am^3)$  is

$$\left. \frac{dy}{dx} \right|_{(am^2, am^3)} = \frac{3(am^2)^2}{2a(am^3)} = \frac{3a^2m^4}{2a^2m^3} = \frac{3m}{2}$$

Then, slope of normal at  $(am^2, am^3)$

$$= \frac{-1}{\text{Slope of the tangent at } (am^2, am^3)} = \frac{-2}{3m}$$

Therefore, equation of the normal at  $(am^2, am^3)$  is given by:

$$y - am^3 = \frac{-2}{3m}(x - am^2)$$

$$\Rightarrow 3my - 3am^4 = -2x + 2am^2$$

$$\Rightarrow 2x + 3my - am^2(2 + 3m^2) = 0$$

Therefore, equation of the normal at  $(am^2, am^3)$  is  $2x + 3my - am^2(2 + 3m^2) = 0$

21. Find the equation of the normals to the curve  $y = x^3 + 2x + 6$  which are parallel to the line  $x + 14y + 4 = 0$ .
21. The equation of the given curve is  $y = x^3 + 2x + 6$ .

The slope of the tangent to the given curve at any point  $(x, y)$  is given by,  $\frac{dy}{dx} = 3x^2 + 2$

$\therefore$  Slope of the normal to the given curve at any point  $(x, y)$

$$= \frac{-1}{\text{Slope of the tangent at the point } (x, y)} = \frac{-1}{3x^2 + 2}$$

The equation of the given line is  $x + 14y + 4 = 0$ .

$x + 14y + 4 = 0 \therefore$  (which is of the form  $y = mx + c$ )

$$y = -\frac{1}{14}x - \frac{4}{14}$$

$\therefore$  Slope of the given line  $= \frac{-1}{14}$

If the normal is parallel to the line, then we must have the slope of the normal being equal to the slope of the line.

$$\therefore \frac{-1}{3x^2 + 2} = \frac{-1}{14}$$

$$\Rightarrow 3x^2 + 2 = 14$$

$$\Rightarrow 3x^2 = 12$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

When  $x = 2, y = 8 + 4 + 6 = 18$

When  $x = -2, y = -8 - 4 + 6 = -6$

Therefore, there are two normals to the given curve with slope  $\frac{-1}{14}$  and passing through the points  $(2, 18)$  and  $(-2, -6)$ .

Thus, the equation of the normal through  $(2, 18)$  is given by,

$$y - 18 = \frac{-1}{14}(x - 2)$$

$$\Rightarrow 14y - 252 = -x + 2$$

$$\Rightarrow x + 14y - 254 = 0$$

And, the equation of the normal through  $(-2, -6)$  is given by,

$$y - (-6) = \frac{-1}{14}[x - (-2)]$$

$$\Rightarrow y + 6 = \frac{-1}{14}(x + 2)$$

$$\Rightarrow 14y + 84 = -x - 2$$

$$\Rightarrow x + 14y + 86 = 0$$

Hence, the equations of the normals to the given curve (which are parallel to the given line) are  $x + 14y - 254 = 0$  and  $x + 14y + 86 = 0$

22. Find the equations of the tangent and normal to the parabola  $y^2 = 4ax$  at the point  $(at^2, 2at)$ .

22. The equation of a parabola is  $y^2 = 4ax$ , then,  
On differentiating it with respect to  $x$ , we get

$$2y \frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{2a}{y}$$

Then, the slope of the tangent at  $(at^2, 2at)$  is  $\left. \frac{dy}{dx} \right|_{(at^2, 2at)} = \frac{2a}{2at} = \frac{1}{t}$ .

Then, the equation of the tangent at  $(at^2, 2at)$  is given by,

$$y - 2at = \frac{1}{t}(x - at^2)$$

$$\Rightarrow ty - 2at^2 = x - at^2$$

$$\Rightarrow ty = x + at^2$$

Now, Then, slope of normal at  $(at^2, 2at)$

$$= \frac{-1}{\text{Slope of the tangent at } (at^2, 2at)} = -t$$

Then, the equation of the normal at  $(at^2, 2at)$  is given by:

$$y - 2at = -t(x - at^2)$$

$$\Rightarrow y - 2at = -tx + at^3$$

$$\Rightarrow y = -tx + 2at + at^3$$

**Therefore, the equation of the normal at  $(at^2, 2at)$  is  $y = -tx + 2at + at^3$ .**

23. Prove that the curves  $x = y^2$  and  $xy = k$  cut at right angles\* if  $8k^2 = 1$ .

23. The equations of the given curves are given as  $x = y^2$  and  $xy = k$ .

Putting  $x = y^2$  in  $xy = k$ , we get:

$$y^3 = k \Rightarrow y = k^{\frac{1}{3}}$$

$$\therefore x = k^{\frac{2}{3}}$$

Thus, the point of intersection of the given curves is  $\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)$ .

Differentiating  $x = y^2$  with respect to  $x$ , we have:

$$1 = 2y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

Therefore, the slope of the tangent to the curve  $x = y^2$  at  $\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)$

$$\left. \frac{dy}{dx} \right|_{\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)} = \frac{1}{2k^{\frac{1}{3}}}$$

On differentiating  $xy = k$  with respect to  $x$ , we have:

$$x \frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

$\therefore$  Slope of the tangent to the curve  $xy = k$  at  $\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)$  is given by,

$$\left. \frac{dy}{dx} \right|_{\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)} = \frac{-y}{x} \left. \right|_{\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)} = -\frac{k^{\frac{1}{3}}}{k^{\frac{2}{3}}} = \frac{-1}{k^{\frac{1}{3}}}$$

We know that two curves intersect at right angles if the tangents to the curves at the point of intersection i.e., at  $\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)$  are perpendicular to each other.

This implies that we should have the product of the tangents as  $-1$ .

Thus, the given two curves cut at right angles if the product of the slopes of their respective tangents at  $\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)$  is  $-1$ .

$$\text{i.e., } \left(\frac{1}{2k^{\frac{1}{3}}}\right) \left(\frac{-1}{k^{\frac{1}{3}}}\right) = -1$$

$$\Rightarrow 2k^{\frac{2}{3}} = 1$$

$$\Rightarrow \left(2k^{\frac{2}{3}}\right)^3 = (1)^3$$

$$\Rightarrow 8k^2 = 1$$

Hence, the given two curves cut at right angles if  $8k^2 = 1$ .

24. Find the equations of the tangent and normal to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  at the point  $(x_0, y_0)$ .

24. It is given that the equations of the tangent and normal to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , then,

On differentiating it with respect to  $x$ , we get,

$$\frac{2x}{a^2} - \frac{2y}{b^2} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2y}{b^2} \cdot \frac{dy}{dx} = \frac{2x}{a^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b^2 x}{a^2 y}$$

Therefore, the slope of the tangent at  $(x_0, y_0)$  is

$$\left. \frac{dy}{dx} \right|_{(x_0, y_0)} = \frac{b^2 x_0}{a^2 y_0}$$

Then, the equation of the tangent at  $(x_0, y_0)$  is given by:

$$y - y_0 = \frac{b^2 x_0}{a^2 y_0} (x - x_0)$$

$$\Rightarrow a^2 y y_0 - a^2 y_0^2 = b^2 x x_0 - b^2 x_0^2$$

$$\Rightarrow b^2 x x_0 - a^2 y y_0 - b^2 x_0^2 + a^2 y_0^2 = 0$$

$$\Rightarrow \frac{x x_0}{a^2} - \frac{y y_0}{b^2} - \left( \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} \right) = 0$$

$$\Rightarrow \frac{x x_0}{a^2} - \frac{y y_0}{b^2} - 1 = 0$$

$$\Rightarrow \frac{x x_0}{a^2} - \frac{y y_0}{b^2} = 1$$

Then, slope of normal at  $(x_0, y_0)$

$$= \frac{-1}{\text{Slope of the tangent at } (x_0, y_0)} = \frac{-a^2 y_0}{b^2 x_0}$$

Therefore, the equation of the normal at  $(x_0, y_0)$  is

$$y - y_0 = \frac{-a^2 y_0}{b^2 x_0} (x - x_0)$$

$$\Rightarrow \frac{y - y_0}{a^2 y_0} = \frac{-(x - x_0)}{bx_0}$$

$$\Rightarrow \frac{y - y_0}{a^2 y_0} + \frac{(x - x_0)}{bx_0} = 0$$

Therefore, the equation of the normal at  $(x_0, y_0)$  is  $\frac{y - y_0}{a^2 y_0} + \frac{(x - x_0)}{bx_0} = 0$ .

25. Find the equation of the tangent to the curve  $y = \sqrt{3x - 2}$  which is parallel to the line  $4x - 2y + 5 = 0$ .
25. The equation of the given curve is  $y = \sqrt{3x - 2}$ .

The slope of the tangent to the given curve at any point  $(x, y)$  is given by,  $\frac{dy}{dx} = \frac{3}{2\sqrt{3x - 2}}$

The equation of the given line is  $4x - 2y + 5 = 0$ .

$$4x - 2y + 5 = 0 \therefore y = 2x + \frac{5}{2} \text{ (which is of the form } y = mx + c \text{)}$$

$\therefore$  Slope of the line = 2

Now, the tangent to the given curve is parallel to the line  $4x - 2y - 5 = 0$  if the slope of the tangent is equal to the slope of the line.

$$\frac{3}{2\sqrt{3x - 2}} = 2 \Rightarrow \sqrt{3x - 2} = \frac{3}{4} \Rightarrow 3x - 2 = \frac{9}{16} \Rightarrow 3x = \frac{9}{16} + 2 = \frac{41}{16}$$

$$\Rightarrow x = \frac{41}{48}$$

$$\text{When } x = \frac{41}{48}, y = \sqrt{3\left(\frac{41}{48}\right) - 2} = \sqrt{\frac{41}{16} - 2} = \sqrt{\frac{41 - 32}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

Equation of tangent at point  $\left(\frac{41}{48}, \frac{3}{4}\right)$  is given by

$$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$$

$$\Rightarrow \frac{4y - 3}{4} = 2\left(\frac{48x - 41}{48}\right)$$

$$\Rightarrow 4y - 3 = \frac{48x - 41}{6}$$

$$\Rightarrow 24y - 18 = 48x - 41$$

$$\Rightarrow 48x - 24y = 23$$

Hence, the equation of the required tangent is  $48x - 24y = 23$ .

Choose the correct answer in Exercises 26 and 27.

26. The slope of the normal to the curve  $y = 2x^2 + 3 \sin x$  at  $x = 0$  is

- (A) 3 (B)  $\frac{1}{3}$   
(C) -3 (D)  $-\frac{1}{3}$

26. **Explanation:** It is given that the slope of the normal to the curve  $y = 2x^2 + 3 \sin x$ ,  
Then, the slope of the tangent at  $x = 0$  is

$$\left. \frac{dy}{dx} \right|_{x=0} = 4x + 3\cos x \Big|_{x=0} = 0 + 3\cos 0 = 3$$

Therefore, the slope of the normal to the curve at  $x = 0$  is

$$= \frac{-1}{\text{Slope of the tangent at } x=0} = \frac{-1}{3}$$

Therefore, the slope of the normal to the curve  $y = 2x^2 + 3 \sin x$  at  $x = 0$  is  $-\frac{1}{3}$ .

27. The line  $y = x + 1$  is a tangent to the curve  $y^2 = 4x$  at the point

- (A) (1, 2) (B) (2, 1)  
(C) (1, -2) (D) (-1, 2)

27. The equation of the given curve is  $y^2 = 4x$ .

Differentiating with respect to  $x$ , we have:

$$2y \frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

Therefore, the slope of the tangent to the given curve at any point  $(x, y)$  is given by,

$$\frac{dy}{dx} = \frac{2}{y}$$

The given line is  $y = x + 1$  (which is of the form  $y = mx + c$ )

$\therefore$  Slope of the line = 1

The line  $y = x + 1$  is a tangent to the given curve if the slope of the line is equal to the slope of the tangent.

Also, the line must intersect the curve.

Thus, we must have:

$$\frac{2}{y} = 1$$

$$\Rightarrow y = 2$$

$$\text{Now, } y = x + 1 \Rightarrow x = y - 1 \Rightarrow x = 2 - 1 = 1$$

Hence, the line  $y = x + 1$  is a tangent to the given curve at the point  $(1, 2)$ .

The correct answer is A.



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