

EXERCISE 23.17

1. Prove that the area of the parallelogram formed by the lines

$$a_1x + b_1y + c_1 = 0, a_1x + b_1y + d_1 = 0, a_2x + b_2y + c_2 = 0, a_2x + b_2y + d_2 = 0$$

is $\left| \frac{(d_1 - c_1)(d_2 - c_2)}{a_1b_2 - a_2b_1} \right|$ sq. units.

Deduce the condition for these lines to form a rhombus.

Solution:

Given:

The given lines are

$$a_1x + b_1y + c_1 = 0 \dots (1)$$

$$a_1x + b_1y + d_1 = 0 \dots (2)$$

$$a_2x + b_2y + c_2 = 0 \dots (3)$$

$$a_2x + b_2y + d_2 = 0 \dots (4)$$

Let us prove, the area of the parallelogram formed by the lines $a_1x + b_1y + c_1 = 0, a_1x +$

$$b_1y + d_1 = 0, a_2x + b_2y + c_2 = 0, a_2x + b_2y + d_2 = 0 \text{ is } \left| \frac{(d_1 - c_1)(d_2 - c_2)}{a_1b_2 - a_2b_1} \right| \text{ sq. units.}$$

The area of the parallelogram formed by the lines $a_1x + b_1y + c_1 = 0, a_1x + b_1y + d_1 = 0,$
 $a_2x + b_2y + c_2 = 0$ and $a_2x + b_2y + d_2 = 0$ is given below:

$$\text{Area} = \left| \frac{(c_1 - d_1)(c_2 - d_2)}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}} \right|$$

$$\text{Since, } \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$\therefore \text{Area} = \left| \frac{(c_1 - d_1)(c_2 - d_2)}{a_1b_2 - a_2b_1} \right| = \left| \frac{(d_1 - c_1)(d_2 - c_2)}{a_1b_2 - a_2b_1} \right|$$

If the given parallelogram is a rhombus, then the distance between the pair of parallel lines is equal.

$$\therefore \left| \frac{c_1 - d_1}{\sqrt{a_1^2 + b_1^2}} \right| = \left| \frac{c_2 - d_2}{\sqrt{a_2^2 + b_2^2}} \right|$$

Hence proved.

2. Prove that the area of the parallelogram formed by the lines $3x - 4y + a = 0$, $3x - 4y + 3a = 0$, $4x - 3y - a = 0$ and $4x - 3y - 2a = 0$ is $2a^2/7$ sq. units.

Solution:

Given:

The given lines are

$$3x - 4y + a = 0 \dots (1)$$

$$3x - 4y + 3a = 0 \dots (2)$$

$$4x - 3y - a = 0 \dots (3)$$

$$4x - 3y - 2a = 0 \dots (4)$$

Let us prove, the area of the parallelogram formed by the lines $3x - 4y + a = 0$, $3x - 4y + 3a = 0$, $4x - 3y - a = 0$ and $4x - 3y - 2a = 0$ is $2a^2/7$ sq. units.

From above solution, we know that

$$\text{Area of the parallelogram} = \left| \frac{(c_1 - d_1)(c_2 - d_2)}{(a_1 b_2 - a_2 b_1)} \right|$$

$$\text{Area of the parallelogram} = \left| \frac{(a - 3a)(2a - a)}{(-9 + 16)} \right| = \frac{2a^2}{7} \text{ square units}$$

Hence proved.

3. Show that the diagonals of the parallelogram whose sides are $lx + my + n = 0$, $lx + my + n' = 0$, $mx + ly + n = 0$ and $mx + ly + n' = 0$ include an angle $\pi/2$.

Solution:

Given:

The given lines are

$$lx + my + n = 0 \dots (1)$$

$$mx + ly + n' = 0 \dots (2)$$

$$lx + my + n' = 0 \dots (3)$$

$$mx + ly + n = 0 \dots (4)$$

Let us prove, the diagonals of the parallelogram whose sides are $lx + my + n = 0$, $lx + my + n' = 0$, $mx + ly + n = 0$ and $mx + ly + n' = 0$ include an angle $\pi/2$.

By solving (1) and (2), we get

$$B = \left(\frac{mn' - ln}{l^2 - m^2}, \frac{mn - ln'}{l^2 - m^2} \right)$$

Solving (2) and (3), we get,

$$C = \left(-\frac{n'}{m+l}, -\frac{n}{m+l} \right)$$

Solving (3) and (4), we get,

$$D = \left(\frac{mn - ln'}{l^2 - m^2}, \frac{mn' - ln}{l^2 - m^2} \right)$$

Solving (1) and (4), we get,

$$A = \left(\frac{-n}{m+1}, \frac{-n}{m+1} \right)$$

Let m_1 and m_2 be the slope of AC and BD.

Now,

$$m_1 = \frac{\frac{-n'}{m+1} + \frac{n}{m+1}}{\frac{-n'}{m+1} + \frac{n}{m+1}} = 1$$

$$m_2 = \frac{\frac{mn' - ln}{l^2 - m^2} - \frac{mn - ln'}{l^2 - m^2}}{\frac{mn - ln'}{l^2 - m^2} - \frac{mn' - ln}{l^2 - m^2}} = -1$$

$$\therefore m_1 m_2 = -1$$

Hence proved.

mc²⁴

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