EXERCISE 4(A)

Question 1.
Find the cube of:
(i) 7
(ii) 11
(iii) 16
(iv) 23
(v) 31
(vi) 42
(vii) 54
Solution: (i) 7
$(7)^3 = 7 \times 7 \times 7 = 343$

Solution: (ii) 11
$(11)^3 = 11 \times 11 \times 11 = 1331$

Solution: (iii) 16
$(16)^3 = 16 \times 16 \times 16 = 4096$

Solution: (iv) 23
$(23)^3 = 23 \times 23 \times 23 = 12167$

Solution: (v) 31
$(31)^3 = 31 \times 31 \times 31 = 29791$

Solution: (vi) 42
$(42)^3 = 42 \times 42 \times 42 = 74088$

Solution: (vii) 54
$(54)^3 = 54 \times 54 \times 54 = 157464$

Question 2
Find which of the following perfect cubes are:
(i) 243
(ii) 588
(iii) 1331
(iv) 24000
(v) 1728
(vi) 1938
Solution: (i) 243
Taking L.C.M.

3	243
3	81
3	27
3	9
3	3
	1

∵ $243 = 3 \times 3 \times 3 \times 3 \times 3 = (3 \times 3 \times 3) \times 3 \times 3 = 3^3 \times 3 \times 3$
∴ 243 is not a perfect cube.

Solution: (ii) 588
Taking L.C.M.

2	588
2	294
7	147
7	21
3	3
	1

$588 = 2 \times 2 \times 7 \times 7 \times 3$
∴ 588 is not perfect cube.

Solution: (iii) 1331
Taking L.C.M.

11	1331
11	121
11	11
	1

∵ $1331 = 11 \times 11 \times 11 = (11)^3$
∴ 1331 is a perfect cube.

Solution: (iv) 24000
∵ $24000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5 = (2)^3 \times (2)^3 \times (5)^3 \times 3$
∴ 24000 is not a perfect cube.

Solution: (v) 1728
Taking L.C.M.

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

∵ $1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = (2)^3 \times (2)^3 \times (3)^3$
∴1728 is a perfect cube.

Solution: (vi) 1938
Taking L.C.M.

2	1938
2	936
17	323
19	19
	1

$1938 = 2 \times 3 \times 17 \times 19$
1938 is not a perfect cube.

Question 3.
Find the cubes of:
(i) 2.1
(ii) 0.4
(iii) 1.6
(iv) 2.5
(v) 0.12
(vi) 0.02
Solution: (i) 2.1

$2.1 = (2.1)^3 = (21/10)^3 = ((21 \times 21 \times 21)/(10 \times 10 \times 10))$ (Splitting the terms)
$= 9261/1000 = 9.261$

Solution: (ii) 0.4
$0.4 = (0.4)^3 = (4/10)^3 = 4 \times 4 \times (4/10) \times 10 \times 10$ (Splitting the terms)
$= (64/1000) = 0.064$

Solution: (iii) 1.6
$1.6 = (1.6)^3 = (16/10)^3 = ((16 \times 16 \times 16)/(10 \times 10 \times 10))$ (Splitting the terms)
$= (4096/1000) = 4.096$

Solution: (iv) 2.5
$2.5 = (2.5)^3 = (25/10)^3 = (25 \times 25 \times 25)/(10 \times 10 \times 10) = (15625/1000) = 15.625$

Solution: (v) 0.12
$0.12 = (0.12)^3 = (12/100)^3 = ((12 \times 12 \times 12)/(100 \times 100 \times 100)) = (1728/1000000) = 0.001728$

Solution: (vi) 0.02
$0.02 = (0.02)^3 = (2/100)^3 = (2 \times 2 \times 2)/(100 \times 100 \times 100) = 8/1000000 = 0.000008$

Solution: (vii) 0.8
$0.8 = (0.8)^3 = (8/10)^3 = (8 \times 8 \times 8)/(10 \times 10 \times 10) = 512/1000 = 0.512$

Question 4
Find the cubes of:
(i) 3/7
(ii) 8/9
(iii) 10/13
(iv) $1\frac{2}{7}$
(v) 2½
Solution: (i) 3/7
$3/7 = (3/7)^3 = (3 \times 3 \times 3)/(7 \times 7 \times 7) = 27/343$

Solution: (ii) 8/9
$8/9 = (8/9)^3 = (8 \times 8 \times 8)/(9 \times 9 \times 9) = 512/729$

Solution: (iii) 10/13
$10/13 = (10/13)^3 = (10 \times 10 \times 10)/(13 \times 13 \times 13) = 1000/2197$

Solution: (iv) $1\frac{2}{7}$

$$1\frac{2}{7} = \left(1\frac{2}{7}\right)^3 = \left(\frac{1\times7+2}{7}\right)^3 = \left(\frac{9}{7}\right)^3 = \frac{9\times9\times9}{7\times7\times7} = \frac{729}{343} = 2\frac{43}{343}$$

Solution: (v) 2½

$2\frac{1}{2} = \left(2\frac{1}{2}\right)^3 = \left(\frac{5}{2}\right)^3 = \frac{5\times5\times5}{2\times2\times2} = \frac{125}{8} = 15\frac{5}{8}$

Question 5.
Find the cubes of:
(i) -3
(ii) -7
(iii) -12
(iv) -18
(v) -25
(vi) -30
(vii) -50
Solution: (i) -3
(i) $-3 = (-3)^3 = -3 \times -3 \times -3 = -(3 \times 3 \times 3) = -27$

Solution: (ii) -7
$-7 = (-7)^3 = -7 \times -7 \times -7 = -(7 \times 7 \times 7) = -343$

Solution: (iii) -12
$-12 = (-12)^3 = -12 \times -12 \times -12 = -(12 \times 12 \times 12) = -1728$

Solution: (iv) -18
$-18 = (-18)^3 = -18 \times -18 \times -18 = -(18 \times 18 \times 18) = -5832$

Solution: (v) -25
$-25 = (-25)^3 = -25 \times -25 \times -25 = -(25 \times 25 \times 25) = -15625$

Solution: (vi) -30
$-30 = (-30)^3 = -30x - 30 \times -30 = -(30 \times 30 \times 30) = -27000$

Solution: (vii) -50
$= -50 = (-50)^3 = -50 \times -50 \times -50 = -50 \times 50 \times 50 = -125000$

Question 6.
Which of the following are cubes of?
(i) An even number
(ii) An odd number
216, 729, 3375, 8000, 125, 343, 4096 and 9261
Solution:-

2	216
2	108
2	54
3	27
3	9
3	3
	1

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = (2)^3 \times (3)^3 = (6)^3$

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

3	729
3	243
3	81
3	27
3	9
3	3
	1

$= (3)^3 \times (3)^3 = (9)^3$

$3375 = 5 \times 5 \times 5 \times 3 \times 3 \times 3$

5	3375
5	675
5	135
3	27
3	9
3	3
	1

$= (5)^3 \times (3)^3 = (15)^3$

$8000 = 20 \times 20 \times 20 = (20)^3$

20	8000
20	400
20	20
	1

$125 = 5 \times 5 \times 5 = (5)^3$

5	125
5	25
5	5
	1

$343 = 7 \times 7 \times 7 = (7)^3$

7	343
7	49
7	7
	1

$4096 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$= (2)^3 \times (2)^3 \times (2)^3 \times (2)^3 = (16)^3$

(i) Cubes of an even number are 216, 8000, 4096.

(ii) Cubes of an odd number are 729, 3375, 125, 343, 9261.

Question 7.

Find the least number by which 1323 must be multiplied so that the product is a perfect cube.

Solution:

The prime factor of 1323 are

$= 3 \times 3 \times 3 \times 7 \times 7 = (3 \times 3 \times 3) \times 7 \times 7$

Clearly, 1323 must be multiplied by 7.

Question 8.

Find the smallest number by which 8768 must be divided so that the quotient is a perfect cube.

Solution:

The prime factor of 8768 are

2	8768
2	4384
2	2192
2	1096
2	548
2	274
137	137
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 137 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 137$
Clearly, 8768 must be divided by 137.

Question 9.
Find the smallest number by which 27783 be multiplied to get a perfect square number.
Solution:

3	27783
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$= 3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7 = (3 \times 3 \times 3) \times (7 \times 7 \times 7) \times 3$
Clearly, 27783 must be multiplied by
$3 \times 3 = 9$

Question 10.
With what least number must 8640 be divided so that the quotient is a perfect cube?
Solution:
The prime factors of 8640 are

2	8640
2	4320
2	2160
2	540
2	270
3	135
3	45
3	15
5	5
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times 5$

Clearly, 8640 must be divided by 5.

Question 11.

Which is the smallest number that must be multiplied to 77175 to make it a perfect cube?

Solution:

The prime factors of 77175 are

3	77175
3	25725
5	8575
5	1715
7	343
7	49
7	7
	1

$= 3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 7 = (7 \times 7 \times 7) \times 3 \times 3 \times 5 \times 5$

Clearly, 77175 must be multiplied by

$3 \times 5 = 15$

EXERCISE 4(B)

Question 1.
Find the cube-roots of:
(i) 64
(ii) 343
(iii) 729
(iv) 1728
(v) 9261
(vi) 4096
(vii) 8000
(viii) 3375
Solution: (i) 64

2	64
2	32
2	16
2	8
2	4
2	2
	1

$64 = \sqrt[3]{64} = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2 \times 2 = 4$

Solution: (ii) 343

7	343
7	49
7	7
	1

$\sqrt[3]{343} = 7 \times 7 \times 7 = 7$

Solution: (iii) 729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$729 = \sqrt[3]{729} = (3 \times 3 \times 3) \times (3 \times 3 \times 3) = 3 \times 3 = 9$

Solution: (iv) 1728

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$1728 = \sqrt[3]{1728} = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) = 2 \times 2 \times 3 = 12$

Solution: (v) 9261

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$9261 = \sqrt[3]{9261} = (3 \times 3 \times 3) \times (7 \times 7 \times 7) = 3 \times 7 = 21$

Solution: (vi) 4096

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$4096 = \sqrt[3]{4096} = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2 \times 2 \times 2 \times 2 = 16$

Solution: (vii) 8000

4	8000
4	2000
4	500
5	125
5	25
5	5
	1

$8000 = \sqrt[3]{8000} = (4 \times 4 \times 4) \times (5 \times 5 \times 5) = 4 \times 5 = 20$

Solution: (viii) 3375

5	3375
5	675
5	135
3	27
3	9
3	3
	1

$3375 = \sqrt[3]{3375} = (5 \times 5 \times 5) \times (3 \times 3 \times 3) = 5 \times 3 = 15$

Question 2:
Find the cube-roots of:
(i) 27/64
(ii) 125/216
(iii) 343/512
(iv) 64 × 729
(v) 64 × 27
(vi) 729 × 8000
(vii) 3375 × 512

Solution: (i) $\frac{27}{64}\ \frac{27}{64} = \sqrt[3]{\frac{27}{64}} = \frac{\sqrt{3\times3\times3}}{\sqrt{4\times4\times4}} = \frac{3}{4}$

Solution: (ii) $\frac{125}{216}\ \frac{125}{216} = \sqrt[3]{\frac{125}{216}} = \frac{\sqrt{5\times5\times5}}{\sqrt{6\times6\times6}} = \frac{5}{6}$

Solution: (iii) $\frac{343}{512}\ \frac{343}{512} = \sqrt[3]{\frac{343}{512}} = \frac{\sqrt{7\times7\times7}}{\sqrt{8\times8\times8}} = \frac{7}{8}$

Solution: (iv) $64 \times 729\ 64 \times 729 = \sqrt[3]{64 \times 729} = \sqrt{4 \times 4 \times 4 \times 9 \times 9 \times 9} = 4 \times 9 = 36$

Solution: (v) $64 \times 27\ 64 \times 27 = \sqrt[3]{64 \times 27} = \sqrt{4 \times 4 \times 4 \times 3 \times 3 \times 3} = 4 \times 3 = 12$

Solution: (vi) $729 \times 8000\ 729 \times 8000 = \sqrt[3]{729 \times 8000} = \sqrt{9 \times 9 \times 9 \times 20 \times 20 \times 20}$
$= 9 \times 20 = 180$

Solution: (vii) $3375 \times 512\ 3375 \times 512 = \sqrt[3]{3375 \times 512} = \sqrt{15 \times 15 \times 15 \times 8 \times 8 \times 8}$
$= 15 \times 8 = 120$

Question 3.
Find the cube-roots of:
(i) -216
(ii) -512
(iii) -1331
(iv) −27/125
(v) −64/343
(vi) −512/343
(vii) -2197
(viii) -5832
(ix) -2744000

Solution: (i)-216

$-216 = \sqrt[3]{-216} = \sqrt{-6x - 6x - 6} = -6$

Solution: (ii) -512

$-512 = \sqrt[3]{-512} = \sqrt{-8x - 8x - 8} = -8$

Solution: (iii) -1331

$-1331 = \sqrt[3]{-1331} = \sqrt{-11x - 11x - 11} = -11$

Solution: (iv) $\frac{-27}{125}\ -\frac{27}{125} = -\frac{\sqrt{27}}{\sqrt{125}} = -\sqrt{\frac{3\times3\times3}{5\times5\times5}} = -\frac{3}{5}$

Solution: (v) $\frac{-64}{343}\ \frac{-64}{343} = \frac{\sqrt[3]{-64}}{\sqrt[3]{343}} = \frac{\sqrt[3]{-4\times-4\times-4}}{\sqrt[3]{7\times7\times7}} = \frac{-4}{7}$

Solution: (vi) $\frac{-512}{343}\ -\frac{512}{343} = -\sqrt[3]{\frac{512}{343}} = -\sqrt[3]{\frac{8\times8\times8}{7\times7\times7}} = -\frac{8}{7}$

Solution: (vii) -2197

$-2197 = \sqrt[3]{-2197}$

13	2197
13	167
13	13
	1

= $\sqrt[3]{}$(-13x – 13x - 13) = -13

Solution: (viii) – 5832

2	5832
2	2916
2	1458
2	729
3	243
3	81
3	27
3	9
3	3
	1

$= \sqrt{-2x - 2x - 2x - 3x - 3x - 3x - 3x - 3x - 3} = -2 \times -3 \times -3 = -18$ - 5832 = $\sqrt[3]{}$(-5832)

Solution: (ix) -2744000

2	2744000
2	1372000
2	68600007
7	343000
7	49000
7	7000
10	1000
10	100
	1

$= \sqrt{-2 \times -2 \times -2 \times -7 \times -7 \times -7 \times -10 \times -10 \times -10}$

$= -2 \times -7 \times -10 = -140$

Question 4.
Find the cube-roots of:
(i) 2.744
(ii) 9.261
(iii) 0.000027
(iv) -0.512
(v) -15.625
(vi) −125 × 1000
Solution: (i) 2.744

$2.744 = \sqrt[3]{(2744/1000)}$

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$= \sqrt[3]{\frac{2\times2\times2\times7\times7\times7}{10\times10\times10}} = \frac{2\times7}{10} = \frac{14}{10} = 1.4$

Solution: (ii) 9.261

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$9.261 = \sqrt[3]{\frac{9261}{1000}} = \sqrt{\frac{3\times3\times3\times7\times7\times7}{10\times10\times10}} = \frac{3\times7}{10} = \frac{21}{10} = 2.1$

Solution: (iii) 0.000027

$0.000027 = \sqrt[3]{\frac{27}{1000000}} = \sqrt[3]{\frac{3\times3\times3}{100\times100\times100}} = \frac{3}{100} = 0.03$

Solution: (iv) -0.512

$-0.512 = \sqrt[3]{\frac{-512}{1000}} = \sqrt{\frac{-8x-8\times-8}{10\times10\times10}} = \frac{-8}{10} = -0.8$

Solution: (v)= 15.625

$-15.625 = \sqrt[3]{\frac{-15625}{1000}}$

5	15625
3	3125
5	625
5	125
5	255
5	5
	1

$\sqrt{\frac{-(5\times5\times5)\times(5\times5\times5)}{10\times10\times10}} = \frac{-5\times5}{10} = \frac{-25}{10} = -2.5$

Solution: (vi) -125×1000 $-125 \times 1000 = \sqrt{-125 \times 100} = \sqrt{-(5 \times 5 \times 5) \times (10 \times 10 \times 10)}$
$= -5 \times 10 = -50$