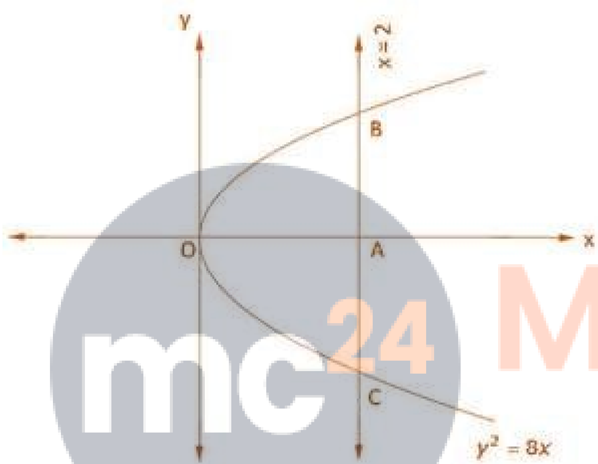
EXERCISE 21.1

Question. 1
Solution:
From the question it is given that two equations,
x = 2 … [equation (i)]
$y^2 = 8x$ … [equation (ii)]
So, equation (i) represents a line parallel to y – axis and equation (ii) represents a parabola with vertex at origin and x – axis as it axis, is as shown in the rough sketch below,



Now, we have to find the area of OCBO,
Then, the area can be found by taking a small slice in each region of width Δx,
And length = (y – 0) = y
The area of sliced part will be as it is a rectangle = y Δx
So, this rectangle can move horizontal from x = 0 to x = 2
The required area of the region bounded between the lines = Region OCBO
= 2 (region OABO)
$= 2\int_0^2 y\,dx$

Given, $y^2 = 8x$
$y = \sqrt{(8x)}$
$= 2\int_0^2 \sqrt{8x}\,dx$
$= 2.2\sqrt{2}\int_0^2 \sqrt{x}\,dx$

On integrating we get,
$= 4\sqrt{2}\left[\frac{2}{3}x\sqrt{x}\right]_0^2$

Now, applying limits we get,

$= 4\sqrt{2}[((2/3)2\sqrt{2}) - ((2/3)0\sqrt{0})]$

$= 4\sqrt{2}(4\sqrt{2}/3)$

Therefore, the required area = 32/3 square units.

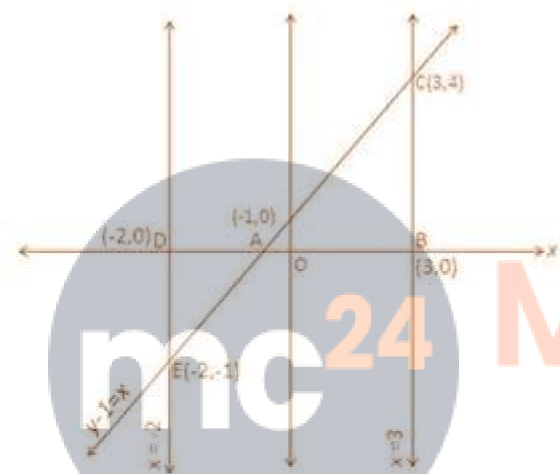
Question. 2

Solution:

From the question it is given that equation,

$y – 1 = x$ … [equation (i)]

So, equation (i) represents a line that meets at (0, 1) and (-1, 0), is as shown in the rough sketch below,



Now, we have to find the area of the region bounded by the line $y – 1 = x$,

So, Required area = Region ABCA + Region ADEA

$$A = \int_{-1}^{3} y\,dx + \left|\int_{-2}^{-1} y\,dx\right|$$

$$= \int_{-1}^{3} (x+1)\,dx + \left|\int_{-2}^{-1} (x+1)\,dx\right|$$

On intigrating we get,

$$= \left(\frac{x^2}{2} + x\right)_{-1}^{3} + \left|\left(\frac{x^2}{2} + x\right)_{-2}^{-1}\right|$$

Now applying limits we get,

$= [((9/2) + 3) – ((½) - 1)] + [((½) - 1) – (2 - 2)]$

$= [(15/2) + (½)] + [-½]$

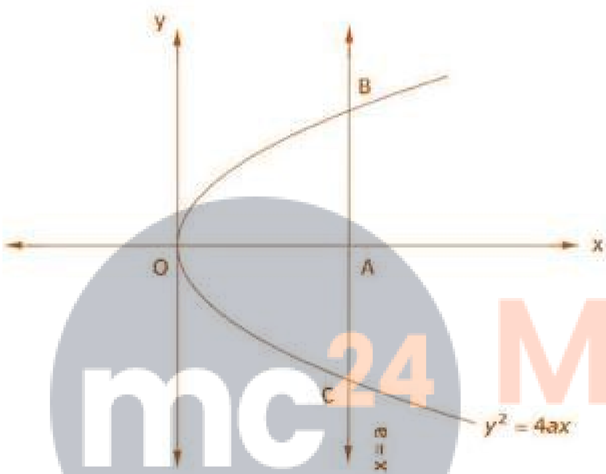
On simplification we get,

$= 8 + ½$

$A = 17/2$ square units

Therefore, the area of the region bounded by the line y – 1 = x is 17/2 square units.

Question. 3
Solution:
From the question it is given that two equations,
x = a … [equation (i)]
$y^2 = 4ax$ … [equation (ii)]
So, equation (i) represents a line parallel to y – axis and equation (ii) represents a parabola with vertex at origin and x – axis as it axis, is as shown in the rough sketch below,



Now, we have to find the area of OCBO,
Then, the area can be found by taking a small slice in each region of width Δx,
And length = (y – 0) = y
The area of sliced part will be as it is a rectangle = y Δx
So, this rectangle can move horizontal from x = 0 to x = a
The required area of the region bounded between the lines = Region OCBO
= 2(Region OABO)

$= 2\int_0^a y\,dx$

Given, $y^2 = 4ax$
$y = \sqrt{(4ax)}$

$= 2\int_0^a \sqrt{4ax}\,dx$

$= 2.2\sqrt{a}\int_0^a \sqrt{x}\,dx$

On integrating we get,

$$= 4\sqrt{a}.\left(\frac{2}{3}x\sqrt{x}\right)_0^a$$

Now, applying limits we get,

$= 4\sqrt{a}[((2/3)a\sqrt{a})$

Therefore, the required area = $(8/3)a^2$ square units.

Question. 4

Solution:

From the question it is given that equation,

$y = 4x – x^2$

Adding 4 on both side,

$y + 4 = 4x – x^2 + 4$

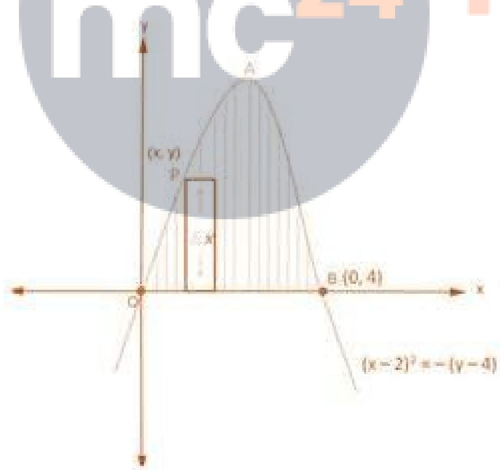
Transposing we get,

$x^2 – 4x + 4 = - y + 4$

We know that, $(a - b)^2 = a^2 – 2ab + b^2$

$(x – 2)^2 = - (y - 4)$ … [equation (i)]

So, equation (i) represents a downward parabola with vertex (2, 4) and passing through (0, 0) and (0, 4), is as shown in the rough sketch below,



Then, the area can be found by taking a small slice in each region of width Δx,

And length = (y – 0) = y

The area of sliced part will be as it is a rectangle = y Δx

So, this rectangle can move horizontal from x = 0 to x = a

The required area of the region bounded between the lines = Region OABO

Required area = region OABO

$$= \int_0^4 \left(4x - x^2\right) dx$$

On integrating we get,

$$= \left(4\frac{x^2}{2} - \frac{x^3}{3}\right)_0^4$$

Now we have to apply limits,

= [((4 × 16)/2) – (64/3)] – [0 – 0]

On simplification we get,

= 64/6

Divide both numerator and denominator by 2 we get,

= 32/3

Therefore, the required area is 32/3 square units.

Question. 5

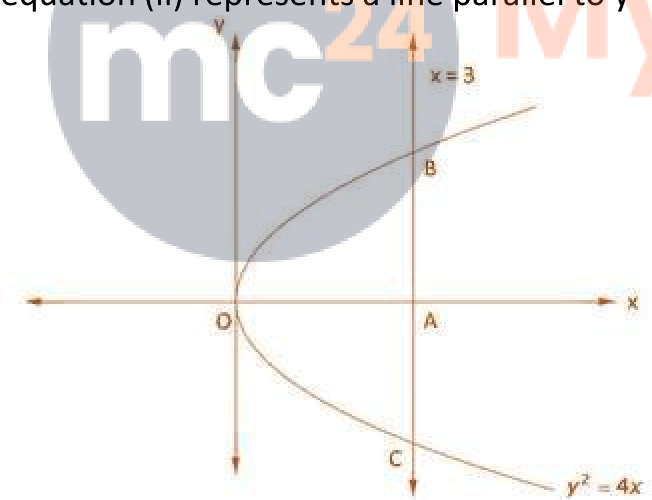
Solution:

From the question it is given that two equations,

$y^2 = 4x$ … [equation (i)]

$x = 3$ … [equation (ii)]

So, equation (i) represents a parabola with vertex at origin and x – axis as it axis and equation (ii) represents a line parallel to y – axis, is as shown in the rough sketch below,



Now, we have to find the area of OCBO,

Then, the area can be found by taking a small slice in each region of width Δx,

And length = (y – 0) = y

The area of sliced part will be as it is a rectangle = y Δx

So, this rectangle can move horizontal from x = 0 to x = 3

The required area of the region bounded between the lines = Region OCBO

= 2 (region OABO)

$= 2\int_0^3 y\,dx$

Given, $y^2 = 4x$

$y = \sqrt{(4x)}$

$= 2\int_0^3 \sqrt{4x}\,dx$

$= 4\int_0^3 \sqrt{x}\,dx$

On integrating we get,

$= 4\left(\frac{2}{3}x\sqrt{x}\right)_0^3$

Now, applying limits we get,

$= 8\sqrt{3}$

Therefore, the required area is $8\sqrt{3}$ square units.