EXERCISE 14.1

Solve the following quadratic equations by factorization method only:

1. $x^2 + 1 = 0$
Solution:
Given: $x^2 + 1 = 0$
We know, $i^2 = -1 \Rightarrow 1 = -i^2$
By substituting $1 = -i^2$ in the above equation, we get
$x^2 - i^2 = 0$
[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]
$(x + i)(x - i) = 0$
$x + i = 0$ or $x - i = 0$
$x = -i$ or $x = i$
∴ The roots of the given equation are i, -i

2. $9x^2 + 4 = 0$
Solution:
Given: $9x^2 + 4 = 0$
$9x^2 + 4 \times 1 = 0$
We know, $i^2 = -1 \Rightarrow 1 = -i^2$
By substituting $1 = -i^2$ in the above equation, we get
So,
$9x^2 + 4(-i^2) = 0$
$9x^2 - 4i^2 = 0$
$(3x)^2 - (2i)^2 = 0$
[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]
$(3x + 2i)(3x - 2i) = 0$
$3x + 2i = 0$ or $3x - 2i = 0$
$3x = -2i$ or $3x = 2i$
$x = -2i/3$ or $x = 2i/3$
∴ The roots of the given equation are 2i/3, -2i/3

3. $x^2 + 2x + 5 = 0$
Solution:
Given: $x^2 + 2x + 5 = 0$
$x^2 + 2x + 1 + 4 = 0$
$x^2 + 2(x)(1) + 1^2 + 4 = 0$
$(x + 1)^2 + 4 = 0$ [since, $(a + b)^2 = a^2 + 2ab + b^2$]

$(x + 1)^2 + 4 \times 1 = 0$
We know, $i^2 = -1 \Rightarrow 1 = -i^2$
By substituting $1 = -i^2$ in the above equation, we get
$(x + 1)^2 + 4(-i^2) = 0$
$(x + 1)^2 - 4i^2 = 0$
$(x + 1)^2 - (2i)^2 = 0$
[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]
$(x + 1 + 2i)(x + 1 - 2i) = 0$
$x + 1 + 2i = 0$ or $x + 1 - 2i = 0$
$x = -1 - 2i$ or $x = -1 + 2i$
∴ The roots of the given equation are -1+2i, -1-2i

4. $4x^2 - 12x + 25 = 0$
Solution:
Given: $4x^2 - 12x + 25 = 0$
$4x^2 - 12x + 9 + 16 = 0$
$(2x)^2 - 2(2x)(3) + 3^2 + 16 = 0$
$(2x - 3)^2 + 16 = 0$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]
$(2x - 3)^2 + 16 \times 1 = 0$
We know, $i^2 = -1 \Rightarrow 1 = -i^2$
By substituting $1 = -i^2$ in the above equation, we get
$(2x - 3)^2 + 16(-i^2) = 0$
$(2x - 3)^2 - 16i^2 = 0$
$(2x - 3)^2 - (4i)^2 = 0$
[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]
$(2x - 3 + 4i)(2x - 3 - 4i) = 0$
$2x - 3 + 4i = 0$ or $2x - 3 - 4i = 0$
$2x = 3 - 4i$ or $2x = 3 + 4i$
$x = 3/2 - 2i$ or $x = 3/2 + 2i$
∴ The roots of the given equation are $3/2 + 2i$, $3/2 - 2i$

5. $x^2 + x + 1 = 0$
Solution:
Given: $x^2 + x + 1 = 0$
$x^2 + x + ¼ + ¾ = 0$
$x^2 + 2(x)(1/2) + (1/2)^2 + ¾ = 0$
$(x + 1/2)^2 + ¾ = 0$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]
$(x + 1/2)^2 + ¾ \times 1 = 0$
We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(x + ½)^2 + ¾ (-1)^2 = 0$

$(x + ½)^2 + ¾ i^2 = 0$

$(x + ½)^2 - (\sqrt{3}i/2)^2 = 0$

[By using the formula, $a^2 - b^2 = (a + b) (a - b)$]

$(x + ½ + \sqrt{3}i/2) (x + ½ - \sqrt{3}i/2) = 0$

$(x + ½ + \sqrt{3}i/2) = 0$ or $(x + ½ - \sqrt{3}i/2) = 0$

$x = -1/2 - \sqrt{3}i/2$ or $x = -1/2 + \sqrt{3}i/2$

∴ The roots of the given equation are $-1/2 + \sqrt{3}i/2$, $-1/2 - \sqrt{3}i/2$

6. $4x^2 + 1 = 0$

Solution:

Given: $4x^2 + 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$4x^2 - i^2 = 0$

$(2x)^2 - i^2 = 0$

[By using the formula, $a^2 - b^2 = (a + b) (a - b)$]

$(2x + i) (2x - i) = 0$

$2x + i = 0$ or $2x - i = 0$

$2x = -i$ or $2x = i$

$x = -i/2$ or $x = i/2$

∴ The roots of the given equation are $i/2$, $-i/2$

7. $x^2 - 4x + 7 = 0$

Solution:

Given: $x^2 - 4x + 7 = 0$

$x^2 - 4x + 4 + 3 = 0$

$x^2 - 2(x) (2) + 2^2 + 3 = 0$

$(x - 2)^2 + 3 = 0$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$(x - 2)^2 + 3 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(x - 2)^2 + 3(-i^2) = 0$

$(x - 2)^2 - 3i^2 = 0$

$(x - 2)^2 - (\sqrt{3}i)^2 = 0$

[By using the formula, $a^2 - b^2 = (a + b) (a - b)$]

$(x - 2 + \sqrt{3}i) (x - 2 - \sqrt{3}i) = 0$

$(x - 2 + \sqrt{3}i) = 0$ or $(x - 2 - \sqrt{3}i) = 0$

$x = 2 - \sqrt{3}i$ or $x = 2 + \sqrt{3}i$
$x = 2 \pm \sqrt{3}i$
∴ The roots of the given equation are $2 \pm \sqrt{3}i$

8. $x^2 + 2x + 2 = 0$
Solution:
Given: $x^2 + 2x + 2 = 0$
$x^2 + 2x + 1 + 1 = 0$
$x^2 + 2(x)(1) + 1^2 + 1 = 0$
$(x + 1)^2 + 1 = 0$ [∵ $(a + b)^2 = a^2 + 2ab + b^2$]
We know, $i^2 = -1 \Rightarrow 1 = -i^2$
By substituting $1 = -i^2$ in the above equation, we get
$(x + 1)^2 + (-i^2) = 0$
$(x + 1)^2 - i^2 = 0$
$(x + 1)^2 - (i)^2 = 0$
[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]
$(x + 1 + i)(x + 1 - i) = 0$
$x + 1 + i = 0$ or $x + 1 - i = 0$
$x = -1 - i$ or $x = -1 + i$
$x = -1 \pm i$
∴ The roots of the given equation are $-1 \pm i$

9. $5x^2 - 6x + 2 = 0$
Solution:
Given: $5x^2 - 6x + 2 = 0$
We shall apply discriminant rule,
Where, $x = (-b \pm\sqrt{(b^2 - 4ac)})/2a$
Here, $a = 5, b = -6, c = 2$
So,
$x = (-(-6) \pm\sqrt{(-6^2 - 4(5)(2))})/ 2(5)$
$= (6 \pm \sqrt{(36-40)})/10$
$= (6 \pm \sqrt{(-4)})/10$
$= (6 \pm \sqrt{4(-1)})/10$
We have $i^2 = -1$
By substituting $-1 = i^2$ in the above equation, we get
$x = (6 \pm \sqrt{4i^2})/10$
$= (6 \pm 2i)/10$
$= 2(3\pm i)/10$
$= (3\pm i)/5$

$x = 3/5 \pm i/5$
∴ The roots of the given equation are $3/5 \pm i/5$

10. $21x^2 + 9x + 1 = 0$
Solution:
Given: $21x^2 + 9x + 1 = 0$
We shall apply discriminant rule,
Where, $x = (-b \pm \sqrt{(b^2 - 4ac)})/2a$
Here, a = 21, b = 9, c = 1
So,
$x = (-9 \pm \sqrt{(9^2 - 4 (21)(1))})/ 2(21)$
$= (-9 \pm \sqrt{(81-84)})/42$
$= (-9 \pm \sqrt{(-3)})/42$
$= (-9 \pm \sqrt{3(-1)})/42$
We have $i^2 = -1$
By substituting $-1 = i^2$ in the above equation, we get
$x = (-9 \pm \sqrt{3i^2})/42$
$= (-9 \pm \sqrt{(\sqrt{3}i)^2}/42$
$= (-9 \pm \sqrt{3}i)/42$
$= -9/42 \pm \sqrt{3}i/42$
$= -3/14 \pm \sqrt{3}i/42$
∴ The roots of the given equation are $-3/14 \pm \sqrt{3}i/42$

11. $x^2 - x + 1 = 0$
Solution:
Given: $x^2 - x + 1 = 0$
$x^2 - x + ¼ + ¾ = 0$
$x^2 - 2 (x) (1/2) + (1/2)^2 + ¾ = 0$
$(x - 1/2)^2 + ¾ = 0$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]
$(x - 1/2)^2 + ¾ \times 1 = 0$
We know, $i^2 = -1 \Rightarrow 1 = -i^2$
By substituting $1 = -i^2$ in the above equation, we get
$(x - ½)^2 + ¾ (-1)^2 = 0$
$(x - ½)^2 + ¾ (-i)^2 = 0$
$(x - ½)^2 - (\sqrt{3}i/2)^2 = 0$
[By using the formula, $a^2 - b^2 = (a + b) (a - b)$]
$(x - ½ + \sqrt{3}i/2) (x - ½ - \sqrt{3}i/2) = 0$
$(x - ½ + \sqrt{3}i/2) = 0$ or $(x - ½ - \sqrt{3}i/2) = 0$
$x = 1/2 - \sqrt{3}i/2$ or $x = 1/2 + \sqrt{3}i/2$

∴ The roots of the given equation are $1/2 + \sqrt{3}i/2$, $1/2 - \sqrt{3}i/2$

12. $x^2 + x + 1 = 0$

Solution:

Given: $x^2 + x + 1 = 0$

$x^2 + x + ¼ + ¾ = 0$

$x^2 + 2 (x) (1/2) + (1/2)^2 + ¾ = 0$

$(x + 1/2)^2 + ¾ = 0$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$(x + 1/2)^2 + ¾ \times 1 = 0$

We know, $i^2 = –1 \Rightarrow 1 = –i^2$

By substituting $1 = –i^2$ in the above equation, we get

$(x + ½)^2 + ¾ (-1)^2 = 0$

$(x + ½)^2 + ¾ i^2 = 0$

$(x + ½)^2 – (\sqrt{3}i/2)^2 = 0$

[By using the formula, $a^2 – b^2 = (a + b) (a – b)$]

$(x + ½ + \sqrt{3}i/2) (x + ½ - \sqrt{3}i/2) = 0$

$(x + ½ + \sqrt{3}i/2) = 0$ or $(x + ½ - \sqrt{3}i/2) = 0$

$x = -1/2 - \sqrt{3}i/2$ or $x = -1/2 + \sqrt{3}i/2$

∴ The roots of the given equation are $-1/2 + \sqrt{3}i/2$, $-1/2 - \sqrt{3}i/2$

13. $17x^2 – 8x + 1 = 0$

Solution:

Given: $17x^2 – 8x + 1 = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm\sqrt{(b^2 – 4ac)})/2a$

Here, $a = 17$, $b = -8$, $c = 1$

So,

$x = (-(-8) \pm\sqrt{(-8^2 – 4 (17)(1))})/ 2(17)$

$= (8 \pm \sqrt{(64-68)})/34$

$= (8 \pm \sqrt{(-4)})/34$

$= (8 \pm \sqrt{4(-1)})/34$

We have $i^2 = –1$

By substituting $–1 = i^2$ in the above equation, we get

$x = (8 \pm \sqrt{(2i)^2})/34$

$= (8 \pm 2i)/34$

$= 2(4\pm i)/34$

$= (4\pm i)/17$

$x = 4/17 \pm i/17$

∴ The roots of the given equation are $4/17 \pm i/17$