Exercise 8(B)

Solution:

(i) Here, $f(x) = x^3 - 2x^2 - 9x + 18$
So, $x - 2 = 0 \Rightarrow x = 2$

Thus, remainder = f(2)
$= (2)^3 - 2(2)^2 - 9(2) + 18$
$= 8 - 8 - 18 + 18$
$= 0$
Therefore, (x - 2) is a factor of f(x).

Now, performing division of polynomial f(x) by (x – 2) we have

$$\begin{array}{r|l} & x^2 \quad - \quad 9 \\ \hline x - 2 & x^3 - 2x^2 - 9x + 18 \\ & x^3 - 2x^2 \\ \hline & \qquad\qquad -9x + 18 \\ & \qquad\qquad -9x + 18 \\ \hline & \qquad\qquad\quad 0 \end{array}$$

Thus, $x^3 - 2x^2 - 9x + 18 = (x - 2)(x^2 - 9) = (x - 2)(x + 3)(x - 3)$

(ii) Here, $f(x) = 2x^3 + 5x^2 - 28x - 15$
So, $x + 5 = 0 \Rightarrow x = -5$

Thus, remainder = f(-5)
$= 2(-5)^3 + 5(-5)^2 - 28(-5) - 15$
$= -250 + 125 + 140 - 15$
$= -265 + 265$
$= 0$
Therefore, (x + 5) is a factor of f(x).

Now, performing division of polynomial f(x) by (x + 5) we get

$$\begin{array}{r|l} & 2x^2 - 5x - 3 \\ \hline x + 5 & 2x^3 + 5x^2 - 28x - 15 \\ & 2x^3 + 10x^2 \\ \hline & -5x^2 - 28x - 15 \\ & -5x^2 - 25x \\ \hline & -3x - 15 \\ & -3x - 15 \\ \hline & 0 \end{array}$$

So, $2x^3 + 5x^2 - 28x - 15 = (x + 5)(2x^2 - 5x - 3)$
Further, on factorisation
$= (x + 5)[2x^2 - 6x + x - 3]$
$= (x + 5)[2x(x - 3) + 1(x - 3)] = (x + 5)(2x + 1)(x - 3)$
Thus, f(x) is factorised as $(x + 5)(2x + 1)(x - 3)$

(iii) Here, $f(x) = 3x^3 + 2x^2 - 3x - 2$
So, $3x + 2 = 0 \Rightarrow x = -2/3$

Thus, remainder = f(-2/3)
$= 3(-2/3)^3 + 2(-2/3)^2 - 3(-2/3) - 2$
$= -8/9 + 8/9 + 2 - 2$
$= 0$
Therefore, $(3x + 2)$ is a factor of f(x).

Now, performing division of polynomial f(x) by $(3x + 2)$ we get

$$\begin{array}{r|l} & x^2 - 1 \\ \hline 3x + 2 & 3x^3 + 2x^2 - 3x - 2 \\ & 3x^3 + 2x^2 \\ \hline & -3x - 2 \\ & -3x - 2 \\ \hline & 0 \end{array}$$

Thus, $3x^3 + 2x^2 - 3x - 2 = (3x + 2)(x^2 - 1) = (3x + 2)(x - 1)(x + 1)$

1. Using the Remainder Theorem, factorise each of the following completely.
(i) $3x^3 + 2x^2 - 19x + 6$
(ii) $2x^3 + x^2 - 13x + 6$
(iii) $3x^3 + 2x^2 - 23x - 30$
(iv) $4x^3 + 7x^2 - 36x - 63$
(v) $x^3 + x^2 - 4x - 4$

Solution:

(i) Let $f(x) = 3x^3 + 2x^2 - 19x + 6$

For $x = 2$, the value of $f(x)$ will be

$= 3(2)^3 + 2(2)^2 - 19(2) + 6$

$= 24 + 8 - 38 + 6 = 0$

As $f(2) = 0$, so $(x - 2)$ is a factor of $f(x)$.

Now, performing long division we have

$$\begin{array}{r|l} & 3x^2 + 8x - 3 \\ \hline x - 2 & 3x^3 + 2x^2 - 19x + 6 \\ & 3x^3 - 6x^2 \\ \hline & \quad 8x^2 - 19x + 6 \\ & \quad 8x^2 - 16x \\ \hline & \qquad -3x + 6 \\ & \qquad -3x + 6 \\ \hline & \qquad\qquad 0 \end{array}$$

Thus, $f(x) = (x - 2)(3x^2 + 8x - 3)$

$= (x - 2)(3x^2 + 9x - x - 3)$

$= (x - 2)[3x(x + 3) - 1(x + 3)]$

$= (x - 2)(x + 3)(3x - 1)$

(ii) Let $f(x) = 2x^3 + x^2 - 13x + 6$

For $x = 2$, the value of $f(x)$ will be

$f(2) = 2(2)^3 + (2)^2 - 13(2) + 6 = 16 + 4 - 26 + 6 = 0$

As $f(2) = 0$, so $(x - 2)$ is a factor of $f(x)$.

Now, performing long division we have

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ \hline x - 2 & 2x^3 + x^2 - 13x + 6 \\ & 2x^3 - 4x^2 \\ \hline & \quad 5x^2 - 13x + 6 \\ & \quad 5x^2 - 10x \\ \hline & \qquad -3x + 6 \\ & \qquad -3x + 6 \\ \hline & \qquad\qquad 0 \end{array}$$

Thus, $f(x) = (x - 2)(2x^2 + 5x - 3)$

$= (x - 2)[2x^2 + 6x - x - 3]$

$= (x - 2)[2x(x + 3) - 1(x + 3)]$

$= (x - 2)\,[2x(x + 3) - 1(x + 3)]$
$= (x - 2)\,(2x - 1)\,(x + 3)$

(iii) Let $f(x) = 3x^3 + 2x^2 - 23x - 30$
For $x = -2$, the value of f(x) will be
$f(-2) = 3(-2)^3 + 2(-2)^2 - 23(-2) - 30$
$= -24 + 8 + 46 - 30 = -54 + 54 = 0$
As $f(-2) = 0$, so $(x + 2)$ is a factor of f(x).
Now, performing long division we have

$$
\begin{array}{r|l}
 & 3x^2 - 4x - 15 \\
\hline
x + 2 & 3x^3 + 2x^2 - 23x - 30 \\
 & 3x^3 + 6x^2 \\
\hline
 & \quad -4x^2 - 23x - 30 \\
 & \quad -4x^2 - 8x \\
\hline
 & \qquad -15x - 30 \\
 & \qquad -15x - 30 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Thus, $f(x) = (x + 2)\,(3x^2 - 4x - 15)$
$= (x + 2)\,(3x^2 - 9x + 5x - 15)$
$= (x + 2)\,[3x(x - 3) + 5(x - 3)]$
$= (x + 2)\,(3x + 5)\,(x - 3)$

(iv) Let $f(x) = 4x^3 + 7x^2 - 36x - 63$
For $x = 3$, the value of f(x) will be
$f(3) = 4(3)^3 + 7(3)^2 - 36(3) - 63$
$= 108 + 63 - 108 - 63 = 0$
As $f(3) = 0$, $(x + 3)$ is a factor of f(x).
Now, performing long division we have

$$
\begin{array}{r|l}
 & 4x^2 + 19x + 21 \\
\hline
x - 3 & 4x^3 + 7x^2 - 36x - 63 \\
 & 4x^3 - 12x^2 \\
\hline
 & \quad 19x^2 - 36x - 63 \\
 & \quad 19x^2 - 57x \\
\hline
 & \qquad 21x - 63 \\
 & \qquad 21x - 63 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Thus, $f(x) = (x + 3)(4x^2 - 5x - 21)$
$= (x + 3)(4x^2 - 12x + 7x - 21)$
$= (x + 3)[4x(x - 3) + 7(x - 3)]$
$= (x + 3)(4x + 7)(x - 3)$

(v) Let $f(x) = x^3 + x^2 - 4x - 4$
For $x = -1$, the value of f(x) will be
$f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$
$= -1 + 1 + 4 - 4 = 0$
As, $f(-1) = 0$ so $(x + 1)$ is a factor of f(x).
Now, performing long division we have

$$\begin{array}{r|l} & x^2 - 4 \\ \hline x + 1 & x^3 + x^2 - 4x - 4 \\ & x^3 + x^2 \\ \hline & \quad\quad -4x - 4 \\ & \quad\quad -4x - 4 \\ \hline & \quad\quad\quad\quad 0 \end{array}$$

Thus, $f(x) = (x + 1)(x^2 - 4)$
$= (x + 1)(x - 2)(x + 2)$

2. Using the Remainder Theorem, factorise the expression $3x^3 + 10x^2 + x - 6$. Hence, solve the equation $3x^3 + 10x^2 + x - 6 = 0$.
Solution:

Let's take $f(x) = 3x^3 + 10x^2 + x - 6$
For $x = -1$, the value of f(x) will be
$f(-1) = 3(-1)^3 + 10(-1)^2 + (-1) - 6 = -3 + 10 - 1 - 6 = 0$
As, $f(-1) = 0$ so $(x + 1)$ is a factor of f(x).
Now, performing long division we have

$$\begin{array}{r|l} & 3x^2 + 7x - 6 \\ \hline x + 1 & 3x^3 + 10x^2 + x - 6 \\ & 3x^3 + 3x^2 \\ \hline & \quad\quad 7x^2 + x - 6 \\ & \quad\quad 7x^2 + 7x \\ \hline & \quad\quad\quad\quad -6x - 6 \\ & \quad\quad\quad\quad -6x - 6 \\ \hline & \quad\quad\quad\quad\quad\quad 0 \end{array}$$

Thus, $f(x) = (x + 1)(3x^2 + 7x - 6)$
$= (x + 1)(3x^2 + 9x - 2x - 6)$
$= (x + 1)[3x(x + 3) - 2(x + 3)]$
$= (x + 1)(x + 3)(3x - 2)$

Now, $3x^3 + 10x^2 + x - 6 = 0$
$(x + 1)(x + 3)(3x - 2) = 0$
Therefore,
$x = -1, -3$ or $2/3$

3. Factorise the expression $f(x) = 2x^3 - 7x^2 - 3x + 18$. Hence, find all possible values of x for which $f(x) = 0$.
Solution:

Let $f(x) = 2x^3 - 7x^2 - 3x + 18$
For $x = 2$, the value of f(x) will be
$f(2) = 2(2)^3 - 7(2)^2 - 3(2) + 18$
$= 16 - 28 - 6 + 18 = 0$
As $f(2) = 0$, $(x - 2)$ is a factor of f(x).
Now, performing long division we have

$$
\begin{array}{r|l}
 & 2x^2 - 3x - 9 \\
x - 2 & 2x^3 - 7x^2 - 3x + 18 \\
 & 2x^3 - 4x^2 \\
\hline
 & -3x^2 - 3x + 18 \\
 & -3x^2 + 6x \\
\hline
 & -9x + 18 \\
 & -9x + 18 \\
\hline
 & 0
\end{array}
$$

Thus, $f(x) = (x - 2)(2x^2 - 3x - 9)$
$= (x - 2)(2x^2 - 6x + 3x - 9)$
$= (x - 2)[2x(x - 3) + 3(x - 3)]$
$= (x - 2)(x - 3)(2x + 3)$
Now, for $f(x) = 0$
$(x - 2)(x - 3)(2x + 3) = 0$
Hence $x = 2, 3$ or $-3/2$

4. Given that $x - 2$ and $x + 1$ are factors of $f(x) = x^3 + 3x^2 + ax + b$; calculate the values of a and b. Hence, find all the factors of f(x).
Solution:

Let $f(x) = x^3 + 3x^2 + ax + b$

As, $(x - 2)$ is a factor of $f(x)$, so $f(2) = 0$
$(2)^3 + 3(2)^2 + a(2) + b = 0$
$8 + 12 + 2a + b = 0$
$2a + b + 20 = 0$ ... (1)

And as, $(x + 1)$ is a factor of $f(x)$, so $f(-1) = 0$
$(-1)^3 + 3(-1)^2 + a(-1) + b = 0$
$-1 + 3 - a + b = 0$
$-a + b + 2 = 0$ ... (2)

Subtracting (2) from (1), we have
$3a + 18 = 0$
$a = -6$
On substituting the value of a in (ii), we have
$b = a - 2 = -6 – 2 = -8$

Thus, $f(x) = x^3 + 3x^2 - 6x - 8$
Now, for $x = -1$
$f(-1) = (-1)^3 + 3(-1)^2 - 6(-1) - 8 = -1 + 3 + 6 - 8 = 0$
Therefore, $(x + 1)$ is a factor of $f(x)$.
Now, performing long division we have

$$
\begin{array}{r|l}
 & x^2 + 2x - 8 \\
\hline
x + 1 & x^3 + 3x^2 - 6x - 8 \\
 & x^3 + x^2 \\
\hline
 & 2x^2 - 6x - 8 \\
 & 2x^2 + 2x \\
\hline
 & -8x - 8 \\
 & -8x - 8 \\
\hline
 & 0
\end{array}
$$

Hence, $f(x) = (x + 1)(x^2 + 2x - 8)$
$= (x + 1)(x^2 + 4x – 2x - 8)$
$= (x + 1)[x(x + 4) – 2(x + 4)]$
$= (x + 1)(x + 4)(x - 2)$