

NCERT Exemplar Solutions For Class 11 Maths Chapter 5- Complex Numbers And Quadratic Equations

EXERCISE

SHORT ANSWER TYPE

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1. For a positive integer n , find the value of $(1 - i)^n (1 - 1/i)^n$.

Solution:

According to the question,

We have,

$$\begin{aligned}(1 - i)^n \left(1 - \frac{1}{i}\right)^n &= (1 - i)^n \left(1 - \frac{1}{i^2}\right)^n \\ &= (1 - i)^n (1 + i)^n \\ &= (1 - i^2)^n \\ &= 2^n\end{aligned}$$

Therefore, $(1 - i)^n (1 - 1/i)^n = 2^n$

2. Evaluate

$$\sum_{n=1}^{13} (i^n + i^{n+1}),$$

where $n \in \mathbb{N}$.

Solution:

According to the question,

We have,

$$\begin{aligned}\sum_{n=1}^{13} (i^n + i^{n+1}) &= \sum_{n=1}^{13} (1 + i)i^n \\ &= (1 + i)(1 + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + i^9 + i^{10} + i^{11} + i^{12} + i^{13}) \\ &= (1 + i) \frac{i(i^{13} - 1)}{i - 1} \\ &= (1 + i) \frac{i(i - 1)}{i - 1} \\ &= (1 + i) i \\ &= i + i^2 \\ &= i - 1 \\ \therefore \sum_{n=1}^{13} (i^n + i^{n+1}) &= i - 1\end{aligned}$$

3. If

$$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy, \text{ then find } (x, y).$$

Solution:

According to the question,

We have,

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$$\begin{aligned}x + iy &= \left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 \\&= \left(\frac{(1+i)^2}{1-i^2}\right)^3 - \left(\frac{(1-i)^2}{1-i^2}\right)^3 \\&= \left(\frac{1+2i+i^2}{1+1}\right)^3 - \left(\frac{1-2i+i^2}{1+1}\right)^3 \\&= \left(\frac{2i}{2}\right)^3 - \left(\frac{-2i}{2}\right)^3 \\&= i^3 - (-i^3) \\&= 2i^3 \\&= 0 - 2i \\ \text{Thus, } (x, y) &= (0, -2)\end{aligned}$$

4. If

$$\frac{(1+i)^2}{2-i} = x + iy$$

, then find the value of $x + y$.

Solution:

According to the question,

We have,

$$\begin{aligned}x + iy &= \frac{(1+i)^2}{2-i} \\&= \frac{1+2i+i^2}{2-i} \\&= \frac{2i}{2-i}\end{aligned}$$

Rationalizing the denominator,

$$\begin{aligned}&= \frac{2i(2+i)}{(2-i)(2+i)} \\&= \frac{4i+2i^2}{(2-i)(2+i)} \\&= \frac{4-i^2}{4i-2} \\&= \frac{4+1}{-2+4i} \\&= \frac{-2}{5} + \frac{4i}{5}\end{aligned}$$

Thus,

$$x = -\frac{2}{5}, y = \frac{4}{5}$$

Hence,

$$x + y = -\frac{2}{5} + \frac{4}{5} = \frac{2}{5}$$

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5. If

$$\left(\frac{1-i}{1+i}\right)^{100} = a + ib; \quad \text{then find (a, b).}$$

Solution:

According to the question,
We have,

$$\begin{aligned} a + ib &= \left(\frac{1-i}{1+i}\right)^{100} \\ &= \left[\frac{1-i}{1+i} \cdot \frac{1-i}{1-i}\right]^{100} \\ &= \left[\frac{(1-i)^2}{1-i^2}\right]^{100} \\ &= \left(\frac{1-2i+i^2}{1+1}\right)^{100} \\ &= \left(-\frac{2i}{2}\right)^{100} \\ &= (i^4)^{25} \\ &= 1 \end{aligned}$$

Hence, $(a, b) = (1, 0)$

6. If $a = \cos \theta + i \sin \theta$, find the value of

$$\frac{1+a}{1-a}$$

Solution:

According to the question,
We have,

$$\begin{aligned} a &= \cos \theta + i \sin \theta \\ \Rightarrow \frac{1+a}{1-a} &= \frac{(1+\cos \theta) + i \sin \theta}{(1-\cos \theta) - i \sin \theta} \\ &= \frac{2 \cos^2 \frac{\theta}{2} + i 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2} - i 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \\ &= \frac{2 \cos \frac{\theta}{2} \left(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2}\right)}{2 \sin \frac{\theta}{2} \left(\sin \frac{\theta}{2} - i \cos \frac{\theta}{2}\right)} \\ &= \frac{i \cos \frac{\theta}{2} \left(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2}\right)}{\sin \frac{\theta}{2} \left(i \sin \frac{\theta}{2} - i^2 \cos \frac{\theta}{2}\right)} \\ &= \frac{i \cos \frac{\theta}{2} \left(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2}\right)}{\sin \frac{\theta}{2} \left(i \sin \frac{\theta}{2} + \cos \frac{\theta}{2}\right)} \\ &= i \cot \frac{\theta}{2} \end{aligned}$$

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7. If $(1 + i)z = (1 - i)\bar{z}$, then show that $z = i\bar{z}$.

Solution:

According to the question,

We have,

$$(1 + i)z = (1 - i)\bar{z}$$
$$\Rightarrow z = \frac{1 - i}{1 + i}\bar{z}$$

Rationalizing the denominator,

We get,

$$= \frac{(1 - i)(1 - i)}{(1 + i)(1 - i)}\bar{z}$$
$$= \frac{(1 - i)^2}{(1 - i^2)}\bar{z}$$
$$= \frac{1 - 2i + i^2}{1 - i^2}\bar{z}$$
$$= \frac{1 + 1 - 2i}{2}\bar{z}$$
$$= -i\bar{z}$$

Hence proved.

8. If $z = x + iy$, then show that $z\bar{z} + 2(z + \bar{z}) + b = 0$ where $b \in \mathbb{R}$, representing z in the complex plane is a circle.

Solution:

According to the question,

We have,

$$z = x + iy$$

$$\Rightarrow \bar{z} = x - iy$$

Now, we also have,

$$z\bar{z} + 2(z + \bar{z}) + b = 0$$

$$\Rightarrow (x + iy)(x - iy) + 2(x + iy + x - iy) + b = 0$$

$$\Rightarrow x^2 + y^2 + 4x + b = 0$$

The equation obtained represents the equation of a circle.

9. If the real part of $(\bar{z} + 2)/(\bar{z} - 1)$ is 4, then show that the locus of the point representing z in the complex plane is a circle.

Solution:

According to the question,

$$\text{Let } z = x + iy$$

Now,

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$$\begin{aligned} \frac{\bar{z}+2}{\bar{z}-1} &= \frac{x-iy+2}{x-iy-1} \\ &= \frac{[(x+2) - iy][(x-1) + iy]}{[(x-1) - iy][(x-1) + iy]} \\ &= \frac{(x-1)(x+2) + y^2 + i[(x+2)y - (x-1)y]}{(x-1)^2 + y^2} \end{aligned}$$

According to the question, we have, real part = 4.

$$\Rightarrow \frac{(x-1)(x+2) + y^2}{(x-1)^2 + y^2} = 4$$

$$\Rightarrow x^2 + x - 2 + y^2 = 4(x^2 - 2x + 1 + y^2)$$

$$\Rightarrow 3x^2 + 3y^2 - 9x + 6 = 0$$

The equation obtained represents the equation of a circle.

Hence, locus of z is a circle.

10. Show that the complex number z , satisfying the condition $\arg \left(\frac{z-1}{z+1} \right) = \pi/4$ lies on a circle.

Solution:

According to the question,

Let $z = x + iy$

$$\arg \left(\frac{z-1}{z+1} \right) = \pi/4$$

$$\Rightarrow \arg(z-1) - \arg(z+1) = \pi/4$$

$$\Rightarrow \arg(x+iy-1) - \arg(x+iy+1) = \pi/4$$

$$\Rightarrow \arg(x-1+iy) - \arg(x+1+iy) = \pi/4$$

$$\Rightarrow \tan^{-1} \frac{y}{x-1} + \tan^{-1} \frac{y}{x+1} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{y}{x-1} - \frac{y}{x+1}}{1 + \left(\frac{y}{x-1}\right)\left(\frac{y}{x+1}\right)} \right] = \frac{\pi}{4}$$

$$\Rightarrow \frac{y(x+1-x+1)}{x^2-1+y^2} = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{2y}{x^2+y^2-1} = 1$$

$$\Rightarrow x^2+y^2-1 = 2y$$

$$\Rightarrow x^2+y^2-2y-1 = 0$$

The equation obtained represents the equation of a circle.

11. Solve that equation $|z| = z + 1 + 2i$.

Solution:

According to the question,

We have,

$$|z| = z + 1 + 2i$$

Substituting $z = x + iy$, we get,

$$\Rightarrow |x + iy| = x + iy + 1 + 2i$$

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We know that,

$$|z| = \sqrt{(x^2 + y^2)}$$

$$\sqrt{(x^2 + y^2)} = (x + 1) + i(y + 2)$$

Comparing real and imaginary parts,

We get,

$$\sqrt{(x^2 + y^2)} = (x + 1)$$

$$\text{And } 0 = y + 2$$

$$\Rightarrow y = -2$$

Substituting the value of y in $\sqrt{(x^2 + y^2)} = (x + 1)$,

We get,

$$\Rightarrow x^2 + (-2)^2 = (x + 1)^2$$

$$\Rightarrow x^2 + 4 = x^2 + 2x + 1$$

$$\text{Hence, } x = 3/2$$

$$\text{Hence, } z = x + iy$$

$$= 3/2 - 2i$$

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12. If $|z + 1| = z + 2(1 + i)$, then find z .

Solution:

According to the question,

We have,

$$|z + 1| = z + 2(1 + i)$$

Substituting $z = x + iy$, we get,

$$\Rightarrow |x + iy + 1| = x + iy + 2(1 + i)$$

We know,

$$|z| = \sqrt{(x^2 + y^2)}$$

$$\sqrt{((x + 1)^2 + y^2)} = (x + 2) + i(y + 1)$$

Comparing real and imaginary parts,

$$\Rightarrow \sqrt{((x + 1)^2 + y^2)} = x + 2$$

$$\text{And } 0 = y + 2$$

$$\Rightarrow y = -2$$

Substituting the value of y in $\sqrt{((x + 1)^2 + y^2)} = x + 2$,

$$\Rightarrow (x + 1)^2 + (-2)^2 = (x + 2)^2$$

$$\Rightarrow x^2 + 2x + 1 + 4 = x^2 + 4x + 4$$

$$\Rightarrow 2x = 1$$

$$\text{Hence, } x = 1/2$$

$$\text{Hence, } z = x + iy$$

$$= 1/2 - 2i$$

13. If $\arg(z - 1) = \arg(z + 3i)$, then find $x - 1 : y$. where $z = x + iy$

Solution:

According to the question,

Let $z = x + iy$

Given that,

$$\arg(z - 1) = \arg(z + 3i)$$

$$\Rightarrow \arg(x + iy - 1) = \arg(x + iy + 3i)$$

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$$\Rightarrow \arg(x - 1 + iy) = \arg(x + I(y)) = \pi/4$$

$$\Rightarrow \tan^{-1} \frac{y}{x-1} = \tan^{-1} \frac{y+3}{x}$$

$$\Rightarrow \frac{y}{x-1} = \frac{y+3}{x}$$

$$\Rightarrow xy = xy - y + 3x - 3$$

$$\Rightarrow 3x - 3 = y$$

$$\Rightarrow (x - 1)/y = 1/3$$

$$\text{Hence, } (x - 1): y = 1: 3$$

14. Show that $|(z - 2) / (z - 3)| = 2$ represents a circle. Find its centre and radius.

Solution:

According to the question,

We have,

$$|(z - 2) / (z - 3)| = 2$$

Substituting $z = x + iy$, we get

$$\Rightarrow |(x + iy - 2) / (x + iy - 3)| = 2$$

$$\Rightarrow |x - 2 + iy| = 2 |x - 3 + iy|$$

$$\Rightarrow \sqrt{(x - 2)^2 + y^2} = 2\sqrt{(x - 3)^2 + y^2}$$

$$\Rightarrow x^2 - 4x + 4 + y^2 = 4(x^2 - 6x + 9 + y^2)$$

$$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$$

$$\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$$

$$\Rightarrow \left(x - \frac{10}{3}\right)^2 + y^2 + \frac{32}{3} - \frac{100}{9} = 0$$

$$\Rightarrow \left(x - \frac{10}{3}\right)^2 + (y - 0)^2 = \frac{4}{9}$$

Therefore, centre of circle is $(10/3, 0)$ and radius is $4/9$ or $2/3$.

15. If $(z - 1)/(z + 1)$ is a purely imaginary number ($z \neq -1$), then find the value of $|z|$.

Solution:

According to the question,

Let $z = x + iy$

Now,

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$$\begin{aligned}\frac{z-1}{z+1} &= \frac{x+iy-1}{x+iy+1} \\ &= \frac{[(x-1) + iy][(x+1) - iy]}{[(x+1) + iy][(x+1) - iy]} \\ &= \frac{(x^2 - 1) + y^2 + i[(x+1)y - (x-1)y]}{(x+1)^2 + y^2}\end{aligned}$$

According to the question, we have,

$$\frac{z-1}{z+1} \text{ is purely imaginary.}$$

$$\begin{aligned}\Rightarrow \frac{(x^2 - 1) + y^2}{(x+1)^2 + y^2} &= 0 \\ \Rightarrow x^2 - 1 + y^2 &= 0 \\ \Rightarrow x^2 + y^2 &= 1 \\ \Rightarrow \sqrt{x^2 + y^2} &= 1 \\ \text{Hence, } |z| &= 1\end{aligned}$$

16. z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then show that $z_1 = -\bar{z}_2$.

Solution:

According to the question,

$$\text{Let } z_1 = |z_1| (\cos \theta_1 + i \sin \theta_1) \text{ and } z_2 = |z_2| (\cos \theta_2 + i \sin \theta_2)$$

$$\text{Given that } |z_1| = |z_2|$$

$$\text{And } \arg(z_1) + \arg(z_2) = \pi$$

$$\Rightarrow \theta_1 + \theta_2 = \pi$$

$$\Rightarrow \theta_1 = \pi - \theta_2$$

$$\text{Now, } z_1 = |z_2| (\cos(\pi - \theta_2) + i \sin(\pi - \theta_2))$$

$$\Rightarrow z_1 = |z_2| (-\cos \theta_2 + i \sin \theta_2)$$

$$\Rightarrow z_1 = -|z_2| (\cos \theta_2 - i \sin \theta_2)$$

$$\Rightarrow z_1 = -[|z_2| (\cos \theta_2 - i \sin \theta_2)]$$

$$\text{Hence, } z_1 = -\bar{z}_2$$

Hence proved.

17. If $|z_1| = 1$ ($z_1 \neq -1$) and $z_2 = (z_1 - 1) / (z_1 + 1)$, then show that the real part of z_2 is zero.

Solution:

According to the question,

$$\text{Let } z_1 = x + iy$$

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$$\begin{aligned} \Rightarrow |z_1| &= \sqrt{x^2 + y^2} = 1 \\ z_2 &= \frac{z_1 - 1}{z_1 + 1} = \frac{x + iy - 1}{x + iy + 1} \\ &= \frac{[(x - 1) + iy][(x + 1) - iy]}{[(x + 1) + iy][(x + 1) - iy]} \\ &= \frac{(x^2 - 1) + y^2 + i[(x + 1)y - (x - 1)y]}{(x + 1)^2 + y^2} \\ &= \frac{x^2 + y^2 - 1 + 2iy}{(x + 1)^2 + y^2} \end{aligned}$$

Since $x^2 + y^2 = 1$

$$\begin{aligned} &= \frac{1 - 1 + 2iy}{(x + 1)^2 + y^2} \\ &= 0 + \frac{2iy}{(x + 1)^2 + y^2} \end{aligned}$$

Therefore, the real part of z_2 is zero.

18. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then find $\arg(z_1/z_4) + \arg(z_2/z_3)$.

Solution:

According to the question,

We have,

z_1 and z_2 are conjugate complex numbers.

The negative side of the real axis

$$= r_1 (\cos \theta_1 - i \sin \theta_1)$$

$$= r_1 [\cos (-\theta_1) + i \sin (-\theta_1)]$$

Similarly, $z_3 = r_2 (\cos \theta_2 - i \sin \theta_2)$

$$\Rightarrow z_4 = r_2 [\cos (-\theta_2) + i \sin (-\theta_2)]$$

$$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3)$$

$$= \theta_1 - (-\theta_2) + (-\theta_1) - \theta_2$$

$$= \theta_1 + \theta_2 - \theta_1 - \theta_2$$

$$= 0$$

$$\Rightarrow \arg(z_1/z_4) + \arg(z_2/z_3) = 0$$

19. If $|z_1| = |z_2| = \dots = |z_n| = 1$, then show that $|z_1 + z_2 + z_3 + \dots + z_n| = |1/z_1 + 1/z_2 + 1/z_3 + \dots + 1/z_n|$

Solution:

According to the question,

We have,

$$|z_1| = |z_2| = \dots = |z_n| = 1$$

$$\Rightarrow |z_1|^2 = |z_2|^2 = \dots = |z_n|^2 = 1$$

$$\Rightarrow z_1 \bar{z}_1 = z_2 \bar{z}_2 = z_3 \bar{z}_3 = \dots = z_n \bar{z}_n = 1$$

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$$\Rightarrow z_1 = \frac{1}{\bar{z}_1}, z_2 = \frac{1}{\bar{z}_2}, \dots, z_n = \frac{1}{\bar{z}_n}$$

Now,

$$\begin{aligned} \Rightarrow |z_1 + z_2 + z_3 + z_4 + \dots + z_n| &= \left| \frac{z_1 \bar{z}_1}{\bar{z}_1} + \frac{z_2 \bar{z}_2}{\bar{z}_2} + \frac{z_3 \bar{z}_3}{\bar{z}_3} + \dots + \frac{z_n \bar{z}_n}{\bar{z}_n} \right| \\ &= \left| \frac{1}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3} + \dots + \frac{1}{\bar{z}_n} \right| \\ &= \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots + \frac{1}{z_n} \right| \\ &= \left| \frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n} \right| \end{aligned}$$

Hence proved.

20. If for complex numbers z_1 and z_2 , $\arg(z_1) - \arg(z_2) = 0$, then show that $|z_1 - z_2| = |z_1| - |z_2|$.

Solution:

According to the question,

$$\text{Let } z_1 = |z_1| (\cos \theta_1 + i \sin \theta_1) \text{ and } z_2 = |z_2| (\cos \theta_2 + i \sin \theta_2)$$

We have,

$$\arg(z_1) - \arg(z_2) = 0$$

$$\Rightarrow \theta_1 - \theta_2 = 0$$

$$\Rightarrow \theta_1 = \theta_2$$

We also have,

$$z_2 = |z_2| (\cos \theta_1 + i \sin \theta_1)$$

$$\Rightarrow z_1 - z_2 = (|z_1| \cos \theta_1 - |z_2| \cos \theta_1) + i (|z_1| \sin \theta_1 - |z_2| \sin \theta_1)$$

$$\Rightarrow |z_1 - z_2| = \sqrt{(|z_1| \cos \theta_1 - |z_2| \cos \theta_1)^2 + (|z_1| \sin \theta_1 - |z_2| \sin \theta_1)^2}$$

$$= \sqrt{|z_1|^2 + |z_2|^2 - 2|z_1||z_2| \cos^2 \theta_1 - 2|z_1||z_2| \sin^2 \theta_1}$$

$$= \sqrt{|z_1|^2 + |z_2|^2 - 2|z_1||z_2| [\cos^2 \theta_1 + \sin^2 \theta_1]}$$

We know that $\cos^2 \theta + \sin^2 \theta = 1$

$$= \sqrt{|z_1|^2 + |z_2|^2 - 2|z_1||z_2|}$$

$$= \sqrt{(|z_1| - |z_2|)^2}$$

$$\text{Hence, } |z_1 - z_2| = |z_1| - |z_2|$$

Hence proved.

21. Solve the system of equations $\operatorname{Re}(z^2) = 0$, $|z| = 2$.

Solution:

According to the question,

We have,

$$\operatorname{Re}(z^2) = 0, |z| = 2$$

Let $z = x + iy$.

$$\text{Then, } |z| = \sqrt{x^2 + y^2}$$

Given in the question,

$$\sqrt{x^2 + y^2} = 2$$

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$$\begin{aligned} \Rightarrow x^2 + y^2 &= 4 \dots (i) \\ z^2 &= x^2 + 2ixy - y^2 \\ &= (x^2 - y^2) + 2ixy \\ \text{Now, Re}(z^2) &= 0 \\ \Rightarrow x^2 - y^2 &= 0 \dots (ii) \\ \text{Equating (i) and (ii), we get} \\ \Rightarrow x^2 &= y^2 = 2 \\ \Rightarrow x &= y = \pm\sqrt{2} \\ \text{Hence, } z &= x + iy \\ &= \pm\sqrt{2} \pm i\sqrt{2} \\ &= \sqrt{2} + i\sqrt{2}, \sqrt{2} - i\sqrt{2}, -\sqrt{2} + i\sqrt{2} \text{ and } -\sqrt{2} - i\sqrt{2} \\ \text{Hence, we have four complex numbers.} \end{aligned}$$

22. Find the complex number satisfying the equation $z + \sqrt{2}|(z + 1)| + i = 0$.

Solution:

According to the question,

We have,

$$z + \sqrt{2}|(z + 1)| + i = 0 \dots (1)$$

Substituting $z = x + iy$, we get

$$\Rightarrow x + iy + \sqrt{2}|x + iy + 1| + i = 0$$

$$\Rightarrow x + i(1 + y) + \sqrt{2}[\sqrt{(x + 1)^2 + y^2}] = 0$$

$$\Rightarrow x + i(1 + y) + \sqrt{2}\sqrt{(x^2 + 2x + 1 + y^2)} = 0$$

Comparing real and imaginary parts to zero, we get

$$\Rightarrow x + \sqrt{2}\sqrt{x^2 + 2x + 1 + y^2} = 0 \dots (2)$$

And,

$$y + 1 = 0$$

$$\Rightarrow y = -1$$

Substituting $y = -1$ into equation (2), we get

$$\Rightarrow x + \sqrt{2}\sqrt{x^2 + 2x + 1 + 1} = 0$$

$$\Rightarrow \sqrt{2}\sqrt{x^2 + 2x + 2} = -x$$

$$\Rightarrow 2x^2 + 4x + 4 = x^2$$

$$\Rightarrow x^2 + 4x + 4 = 0$$

$$\Rightarrow (x + 2)^2 = 0$$

$$\Rightarrow x = -2$$

Hence, $z = x + iy$

$$= -2 - i$$

23. Write the complex number

$$z = \frac{1 - i}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}} \text{ in polar form.}$$

Solution:

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According to the question,
We have,

$$\begin{aligned}
 z &= \frac{1-i}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}} \\
 &= \frac{\sqrt{2} \left[\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right]}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}} \\
 &= \frac{\sqrt{2} \left[\cos \frac{\pi}{4} - i \sin \frac{\pi}{4} \right]}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}} \\
 &= \sqrt{2} \left[\cos \left(-\frac{\pi}{4} - \frac{\pi}{3} \right) + i \sin \left(-\frac{\pi}{4} - \frac{\pi}{3} \right) \right] \\
 &= \sqrt{2} \left[\cos \left(-\frac{7\pi}{12} \right) + i \sin \left(-\frac{7\pi}{12} \right) \right] \\
 &= \sqrt{2} \left[\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right]
 \end{aligned}$$

24. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \pi/2$, then show that $\bar{z}w = -i$.

Solution:

Let $z = |z| (\cos \theta_1 + i \sin \theta_1)$ and $w = |w| (\cos \theta_2 + i \sin \theta_2)$

Given $|zw| = |z| |w| = 1$

Also $\arg(z) - \arg(w) = \pi/2$

$\Rightarrow \theta_1 - \theta_2 = \pi/2$

Now, $\bar{z}w = |z| (\cos \theta_1 - i \sin \theta_1) |w| (\cos \theta_2 + i \sin \theta_2) = 1$

$= |z| |w| (\cos(-\theta_1) + i \sin(-\theta_1)) (\cos \theta_2 + i \sin \theta_2)$

$= 1 [\cos(\theta_2 - \theta_1) + i \sin(\theta_2 - \theta_1)]$

$= [\cos(-\pi/2) + i \sin(-\pi/2)]$

$= 1 [0 - i]$

$= -i$

Hence proved.