Chapter 8. Logarithms

Exercise 8(A)

Solution 1:

(i)

$5^3 = 125$

$\Rightarrow \log_5 125 = 3 \quad [a^b = c \Rightarrow \log_a c = b]$

(ii)

$3^{-2} = \frac{1}{9}$

$\Rightarrow \log_3 \frac{1}{9} = -2 \quad [a^b = c \Rightarrow \log_a c = b]$

(iii)

$10^{-3} = 0.001$

$\Rightarrow \log_{10} 0.001 = -3 \quad [a^b = c \Rightarrow \log_a c = b]$

(iv)

$(81)^{\frac{3}{4}} = 27$

$\Rightarrow \log_{81} 27 = \frac{3}{4}$ [By definition of logarithm, $a^b = c \Rightarrow \log_a c = b$]

Solution 2:

(i)

$\log_8 0.125 = -1$

$\Rightarrow 8^{-1} = 0.125 \quad [\log_a c = b \Rightarrow a^b = c]$

(ii)

$\log_{10} 0.01 = -2$

$\Rightarrow 10^{-2} = 0.01 \quad [\log_a c = b \Rightarrow a^b = c]$

(iii)

$\log_a A = x$

$\Rightarrow a^x = A \quad [\log_a c = b \Rightarrow a^b = c]$

(iv)

$\log_{10} 1 = 0$

$\Rightarrow 10^0 = 1 \quad [\log_a c = b \Rightarrow a^b = c]$

Solution 3:

$\log_{10} x = -2$

$\Rightarrow 10^{-2} = x \quad [\log_a c = b \Rightarrow a^b = c]$

$\Rightarrow x = 10^{-2}$

$\Rightarrow x = \frac{1}{10^2}$

$\Rightarrow x = \frac{1}{100}$

$\Rightarrow x = 0.01$

Solution 4:

(i)

Let $\log_{10}100 = x$

$\therefore 10^x = 100$

$\Rightarrow 10^x = 10 \times 10$

$\Rightarrow 10^x = 10^2$

$\Rightarrow x = 2 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\therefore \log_{10}100 = 2$

(ii)

Let $\log_{10}0.1 = x$

$\therefore 10^x = 0.1$

$\Rightarrow 10^x = \frac{1}{10}$

$\Rightarrow 10^x = 10^{-1}$

$\Rightarrow x = -1 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\therefore \log_{10}0.1 = -1$

(iii)

Let $\log_{10}0.001 = x$

$\therefore 10^x = 0.001$

$\Rightarrow 10^x = \frac{1}{1000}$

$\Rightarrow 10^x = \frac{1}{10^3}$

$\Rightarrow 10^x = 10^{-3}$

$\Rightarrow x = -3 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\therefore \log_{10}0.001 = -3$

(iv)

Let $\log_4 32 = x$

$\therefore 4^x = 32$

$\Rightarrow (2^2)^x = 2 \times 2 \times 2 \times 2 \times 2$

$\Rightarrow 2^{2x} = 2^5$

$\Rightarrow 2x = 5 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\Rightarrow x = \frac{5}{2}$

$\therefore \log_4 32 = \frac{5}{2}$

(v)

Let $\log_2 0.125 = x$

$\therefore 2^x = 0.125$

$\Rightarrow 2^x = \frac{125}{1000}$

$\Rightarrow 2^x = \frac{1}{8}$

$\Rightarrow 2^x = 8^{-1}$

$\Rightarrow 2^x = (2 \times 2 \times 2)^{-1}$

$\Rightarrow 2^x = (2^3)^{-1}$

$\Rightarrow 2^x = 2^{-3}$

$\Rightarrow x = -3$ [if $a^m = a^n$; then m=n]

$\therefore \log_2 0.125 = -3$

(vi)

Let $\log_4 \frac{1}{16} = x$

$\therefore 4^x = \frac{1}{16}$

$\Rightarrow 4^x = \frac{1}{4 \times 4}$

$\Rightarrow 4^x = (4 \times 4)^{-1}$

$\Rightarrow 4^x = (4^2)^{-1}$

$\Rightarrow 4^x = 4^{-2}$

$\Rightarrow x = -2$ [if $a^m = a^n$; then m=n]

$\therefore \log_4 \frac{1}{16} = -2$

(vii)

Let $\log_9 27 = x$

$\therefore 9^x = 27$

$\Rightarrow (3 \times 3)^x = 3 \times 3 \times 3$

$\Rightarrow \quad (3^2)^x = (3^3)$

$\Rightarrow \quad 3^{2x} = (3^3)$

$\Rightarrow \quad 2x = 3 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\Rightarrow \quad x = \dfrac{3}{2}$

$\therefore \log_9 27 = \dfrac{3}{2}$

(viii)

Let $\log_{27} \dfrac{1}{81} = x$

$\therefore 27^x = \dfrac{1}{81}$

$\Rightarrow (3 \times 3 \times 3)^x = \dfrac{1}{3 \times 3 \times 3 \times 3}$

$\Rightarrow \quad (3^3)^x = \dfrac{1}{3^4}$

$\Rightarrow \quad (3^3)^x = (3^4)^{-1}$

$\Rightarrow \quad 3^{3x} = (3^{-4})$

$\Rightarrow \quad 3x = -4 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\Rightarrow \quad x = \dfrac{-4}{3}$

$\therefore \log_{27} \dfrac{1}{81} = \dfrac{-4}{3}$

Solution 5:

(i)

Consider the equation

$\log_{10} x = a$

$\Rightarrow 10^a = x$

Thus the statement, $10^x = a$ is false

(ii)

Consider the equation

$x^y = z$

$\Rightarrow \log_x z = y$

Thus the statement, $\log_z x = y$ is false

(iii)

Consider the equation

$\log_2 8 = 3$

$\Rightarrow 2^3 = 8....(1)$

Now consider the equation

$\log_8 2 = \frac{1}{3}$

$\Rightarrow 8^{\frac{1}{3}} = 2$

$\Rightarrow (2^3)^{\frac{1}{3}} = 2....(2)$

Both the equations (1) and (2) are correct

Thus the given statements, $\log_2 8 = 3$ and $\log_8 2 = \frac{1}{3}$ are true

Solution 6:

(i)

Consider the equation

$\log_3 x = 0$

$\Rightarrow 3^0 = x$

$\Rightarrow 1 = x$ or $x = 1$

(ii)

Consider the equation

$\log_x 2 = -1$

$\Rightarrow x^{-1} = 2$

$\Rightarrow \frac{1}{x} = 2$

$\Rightarrow \; x = \frac{1}{2}$

(iii)

Consider the equation

$\log_9 243 = x$

$\Rightarrow 9^x = 243$

$\Rightarrow \left(3^2\right)^x = 3^5$

$\Rightarrow 3^{2x} = 3^5$

$\Rightarrow \; 2x = 5$

$\Rightarrow x = \frac{5}{2}$

$\Rightarrow x = 2\frac{1}{2}$

(iv)

Consider the equation

$\log_5(x-7)=1$

$\Rightarrow 5^1 = x-7$

$\Rightarrow 5 = x-7$

$\Rightarrow x = 5+7$

$\Rightarrow x=12$

(v)

Consider the equation

$\log_4 32 = x-4$

$\Rightarrow 4^{x-4} = 32$

$\Rightarrow (2^2)^{x-4} = 2^5$

$\Rightarrow 2^{2(x-4)} = 2^5$

$\Rightarrow 2x-8=5$

$\Rightarrow 2x = 5+8$

$\Rightarrow 2x = 13$

$\Rightarrow x = \frac{13}{2}$

$\Rightarrow x = 6\frac{1}{2}$

(vi)

Consider the equation

$\log_7(2x^2-1) = 2$

$\Rightarrow 7^2 = 2x^2-1$

$\Rightarrow 7\times 7 = 2x^2-1$

$\Rightarrow 2x^2-1-49=0$

$\Rightarrow 2x^2-50=0$

$\Rightarrow 2x^2 = 50$

$\Rightarrow x^2 = \frac{50}{2}$

$\Rightarrow x^2 = 25$

$\Rightarrow x = \pm\sqrt{25}$

$\Rightarrow x = 5$ [neglecting the negative value]

Solution 7:

(i)

Let $\log_{10} 0.01 = x$

$\Rightarrow 10^x = 0.01$

$\Rightarrow 10^x = \frac{1}{100}$

$\Rightarrow 10^x = \frac{1}{10 \times 10}$

$\Rightarrow 10^x = \frac{1}{10^2}$

$\Rightarrow 10^x = 10^{-2}$

$\Rightarrow x = -2$

Thus, $\log_{10} 0.01 = -2$

(ii)

Let $\log_2 \frac{1}{8} = x$

$\Rightarrow 2^x = \frac{1}{8}$

$\Rightarrow 2^x = \frac{1}{2 \times 2 \times 2}$

$\Rightarrow 2^x = \frac{1}{2^3}$

$\Rightarrow 2^x = 2^{-3}$

$\Rightarrow x = -3$

Thus, $\log_2 \frac{1}{8} = -3$

(iii)

Let $\log_5 1 = x$

$\Rightarrow 5^x = 1$

$\Rightarrow 5^x = 5^0$

$\Rightarrow x = 0$

Thus, $\log_5 1 = 0$

(iv)

Let $\log_5 125 = x$

$\Rightarrow 5^x = 125$

$\Rightarrow 5^x = 5 \times 5 \times 5$

$\Rightarrow 5^x = 5^3$

$\Rightarrow x = 3$

Thus, $\log_5 125 = 3$

(v)

Let $\log_{16} 8 = x$

$\Rightarrow 16^x = 8$

$\Rightarrow (2 \times 2 \times 2 \times 2)^x = 2 \times 2 \times 2$

$\Rightarrow (2^4)^x = 2^3$

$\Rightarrow 2^{4x} = 2^3$

$\Rightarrow 4x = 3$

$\Rightarrow x = \frac{3}{4}$

Thus, $\log_{16} 8 = \frac{3}{4}$

(vi)

Let $\log_{0.5} 16 = x$

$\Rightarrow 0.5^x = 16$

$\Rightarrow \left(\frac{5}{10}\right)^x = 2 \times 2 \times 2 \times 2$

$\Rightarrow \left(\frac{1}{2}\right)^x = 2^4$

$\Rightarrow \frac{1}{2^x} = 2^4$

$\Rightarrow 2^{-x} = 2^4$

$\Rightarrow -x = 4$

$\Rightarrow x = -4$

Thus, $\log_{0.5} 16 = -4$

Solution 8:

$\log_a m = n$

$\Rightarrow a^n = m$

$\Rightarrow \frac{a^n}{a} = \frac{m}{a}$

$\Rightarrow a^{n-1} = \frac{m}{a}$

Solution 9:

$\log_2 x = m$ and $\log_5 y = n$

$\Rightarrow 2^m = x$ and $5^n = y$

(i) Consider $2^m = x$

$\Rightarrow \frac{2^m}{2^3} = \frac{x}{2^3}$

$\Rightarrow 2^{m-3} = \frac{x}{8}$

(ii) Consider $5^n = y$

$\Rightarrow (5^n)^3 = y^3$

$\Rightarrow 5^{3n} = y^3$

$\Rightarrow 5^{3n} \times 5^2 = y^3 \times 5^2$

$\Rightarrow 5^{3n+2} = 25y^3$

Solution 10:

Given that :

$\log_2^x = a$ and $\log_3^y = a$

$\Rightarrow 2^a = x$ and $3^a = y$ $\left[\because \log_a^m = n \Rightarrow a^n = m\right]$

Now prime factorization of 72 is

$72 = 2 \times 2 \times 2 \times 3 \times 3$

Hence,

$(72)^a = (2 \times 2 \times 2 \times 3 \times 3)^a$

$= (2^3 \times 3^2)^a$

$= 2^{3a} \times 3^{2a}$

$= (2^a)^3 \times (3^a)^2$ $\left[\text{as } 2^a = x,\ 3^a = y\right]$

$= x^3 y^2$

Solution 11:

$\log(x-1)+\log(x+1)=\log_2 1$

$\Rightarrow \log(x-1)+\log(x+1)=0$

$\Rightarrow \log[(x-1)(x+1)]=0$

$\Rightarrow (x-1)(x+1)=1 \ldots (\text{Since} \log 1 = 0)$

$\Rightarrow x^2-1=1$

$\Rightarrow x^2=2$

$\Rightarrow x=\pm\sqrt{2}$

$-\sqrt{2}$ cannot be possible, since log of a negative number is not defined.

So, $x=\sqrt{2}$.

Solution 12:

$\log(x^2-21)=2$

$\Rightarrow x^2-21=10^2$

$\Rightarrow x^2-21=100$

$\Rightarrow x^2=121$

$\Rightarrow x=\pm 11$