EXERCISE 23.7

Find the equation of a line for which
(i) p = 5, α = 60°
(ii) p = 4, α = 150°
Solution:
(i) p = 5, α = 60°
Given:
p = 5, α = 60°
The equation of the line in normal form is given by
Using the formula,
x cos α + y sin α = p
Now, substitute the values, we get
x cos 60° + y sin 60° = 5
$x/2 + \sqrt{3}y/2 = 5$
$x + \sqrt{3}y = 10$
∴ The equation of line in normal form is $x + \sqrt{3}y = 10$.

(ii) p = 4, α = 150°
Given:
p = 4, α = 150°
The equation of the line in normal form is given by
Using the formula,
x cos α + y sin α = p
Now, substitute the values, we get
x cos 150° + y sin 150° = 4
cos (180° – θ) = – cos θ , sin (180° – θ) = sin θ
x cos(180° – 30°) + y sin(180° – 30°) = 4
– x cos 30° + y sin 30° = 4
$-\sqrt{3}x/2 + y/2 = 4$
$-\sqrt{3}x + y = 8$
∴ The equation of line in normal form is $-\sqrt{3}x + y = 8$.

2. Find the equation of the line on which the length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of x–axis is 30°.
Solution:
Given:
p = 4, α = 30°

The equation of the line in normal form is given by
Using the formula,
x cos α + y sin α = p
Now, substitute the values, we get
x cos 30° + y sin 30° = 4
x√3/2 + y1/2 = 4
√3x + y = 8
∴ The equation of line in normal form is √3x + y = 8.

3. Find the equation of the line whose perpendicular distance from the origin is 4 units and the angle which the normal makes with the positive direction of x–axis is 15°.
Solution:
Given:
p = 4, α = 15°
The equation of the line in normal form is given by
We know that, cos 15° = cos (45° – 30°) = cos45°cos30° + sin45°sin30°
Cos (A – B) = cos A cos B + sin A sin B
So,

$$\cos 15 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

And sin 15 = sin (45° – 30°) = sin 45° cos 30° – cos 45° sin 3°
Sin (A – B) = sin A cos B – cos A sin B
So,

$$\sin 15 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Now, by using the formula,
x cos α + y sin α = p
Now, substitute the values, we get

$$\frac{\sqrt{3}+1}{2\sqrt{2}}x + \frac{\sqrt{3}-1}{2\sqrt{2}}y = 4$$

(√3+1)x +(√3-1) y = 8√2
∴ The equation of line in normal form is (√3+1)x +(√3-1) y = 8√2.

4. Find the equation of the straight line at a distance of 3 units from the origin such that the perpendicular from the origin to the line makes an angle α given by tan α =

5/12 with the positive direction of x–axis.

Solution:

Given:

$p = 3$, $\alpha = \tan^{-1}(5/12)$

So, $\tan \alpha = 5/12$

$\sin \alpha = 5/13$

$\cos \alpha = 12/13$

The equation of the line in normal form is given by

By using the formula,

$x \cos \alpha + y \sin \alpha = p$

Now, substitute the values, we get

$12x/13 + 5y/13 = 3$

$12x + 5y = 39$

∴ The equation of line in normal form is $12x + 5y = 39$.

5. Find the equation of the straight line on which the length of the perpendicular from the origin is 2 and the perpendicular makes an angle α with x–axis such that sin α = 1/3.

Solution:

Given:

$p = 2$, $\sin \alpha = 1/3$

We know that $\cos \alpha = \sqrt{(1 - \sin^2 \alpha)}$

$= \sqrt{(1 - 1/9)}$

$= 2\sqrt{2}/3$

The equation of the line in normal form is given by

By using the formula,

$x \cos \alpha + y \sin \alpha = p$

Now, substitute the values, we get

$x2\sqrt{2}/3 + y/3 = 2$

$2\sqrt{2}x + y = 6$

∴ The equation of line in normal form is $2\sqrt{2}x + y = 6$.