NCERT Solutions for Class-XII Maths

Chapter-4.5

NCERT Math Class 12

Find adjoint of each of the matrices in exercises 1 and 2.

1. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

1. Hence, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, therefore, $A_{11} = 4$ $A_{12} = -3$ $A_{21} = -2$ $A_{22} = 1$

Adjoint of matrix $A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

2. Adjoint of the matrix $A = [a_{ij}]_{n\times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n\times n}$ where A_{ij} is the co-factor of the element a_{ij}.
Let's find the cofactors for all the positions first-
Here, $A_{11} = 1\{(3\times1-0\times5)\}=3$
Similarly,
$A_{12} = -12$, $A_{13}=6$, $A_{21}=1$, $A_{22}=5$, $A_{23}=2$, $A_{31}=-11$, $A_{32}=-1$, $A_{33} = 5$.

$\therefore$ Adj $A= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

$= \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

3. Here, $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$, therefore,

$A_{11} = -6$ $A_{12} = 4$ $A_{21} = -3$ $A_{22} = 2$

$|A| = -12 + 12 = 0$

$$adjA = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$A\,(adj\;A) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$(adj\;A).A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} = \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$|A|.I = 0.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, A((adj A) = (adj A). A = |a|. I = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

4. $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

4. Adjoint of the matrix $A = [a_{ij}]_{n\times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n\times n}$ where A_{ij} is the co-factor of the element a_{ij}.

Let's find the cofactors for all the positions first-

Here, $A_{11} = 0$, $A_{12} = -11$, $A_{13}=0$, $A_{21}=3$, $A_{22} = 1$, $A_{23} = -1$, $A_{31}=2$, $A_{32} = 8$, $A_{33} = 3$.

$$\therefore \text{Adj A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\text{So,LHS} = \text{A(AdjA)} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$\text{Also AdjA(A)} = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Determinant of A = |A|=11

So RHS= $|A|I = 11\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}11 & 0 & 0\\0 & 11 & 0\\0 & 0 & 11\end{bmatrix}$

Hence A(AdjA)=AdjA(A)= $|A|I = \begin{bmatrix}11 & 0 & 0\\0 & 11 & 0\\0 & 0 & 11\end{bmatrix}$ {hence proved}

Find the inverse of each of the matrices (if it exists) given in Exercises 5 to 11.

5. $\begin{bmatrix}2 & -2\\4 & 3\end{bmatrix}$

5. Here, $A = \begin{bmatrix}2 & -2\\4 & 3\end{bmatrix}$,

Therefore, $A_{11} = 3 \quad A_{12} = -4 \quad A_{21} = 2 A_{22} = 2$

$|A| = 6 + 8 = 14 \neq 0 \Rightarrow A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix}A_{11} & A_{21}\\A_{12} & A_{22}\end{bmatrix} = \frac{1}{14}\begin{bmatrix}3 & 2\\-4 & 2\end{bmatrix}$$

6. $\begin{bmatrix}-1 & 5\\-3 & 2\end{bmatrix}$

6. We know that $A^{-1} = \frac{1}{|A|}\text{AdjA}$

Adjoint of the matrix $A = [a_{ij}]_{n\times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n\times n}$ where A_{ij} is the co-factor of the element a_{ij}.

Let's find the cofactors for all the positions first-

Here, $A_{11} = 2$, $A_{12} = 3$, A_{21}=-5, A_{22}=-1.

$\therefore$ Adj A= $\begin{bmatrix}A_{11} & A_{21}\\A_{12} & A_{22}\end{bmatrix}$

$= \begin{bmatrix}2 & -5\\3 & -1\end{bmatrix}$

And |A|=-1(2)-(-3)(5)=13

So $A^{-1} = \frac{1}{|A|}\text{AdjA} = \frac{1}{13}\begin{bmatrix}2 & -5\\3 & -1\end{bmatrix} = \begin{bmatrix}\frac{2}{13} & \frac{-5}{13}\\\frac{3}{13} & \frac{-1}{13}\end{bmatrix}$

7. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

7. Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Therefore, $|A| = 1(10 - 0) - 2(0 - 0) + 3\,(0 - 0) = 10 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = 10$ $\quad A_{12} = 0$ $\quad A_{13} = 0$

$A_{21} = -10$ $\quad A_{22} = 5$ $\quad A_{23} = 0$

$A_{31} = 2$ $\quad A_{32} = -4$ $\quad A_{33} = 2$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

8. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

8. Adjoint of the matrix $A = [a_{ij}]_{n\times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n\times n}$ where A_{ij} is the co-factor of the element a_{ij}.

Let's find the cofactors for all the positions first-

Here, $A_{11} = -3$, $A_{12} = 3$, $A_{13} = -9$, $A_{21} = 0$, $A_{22} = -1$, $A_{23} = -2$, $A_{31} = 0$, $A_{32} = 0$, $A_{33} = 3$.

$$\therefore \text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

And $|A| = -3$.

$$A^{-1} = \frac{1}{|A|}\text{Adj}A = \frac{1}{(-3)}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{-3}{-3} & 0 & 0 \\ \frac{3}{-3} & \frac{-1}{-3} & 0 \\ \frac{-9}{-3} & \frac{-2}{-3} & \frac{3}{-3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & \frac{1}{3} & 0 \\ 3 & \frac{2}{3} & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & \frac{1}{3} & 0 \\ 3 & \frac{2}{3} & -1 \end{bmatrix}$$

9. $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

9. Here, $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

Therefore, $|A| = 2(-1-0) - 1(4-0) + 3(8-7) = -3 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = -1$ $\quad A_{12} = -4$ $\quad A_{13} = 1$

$A_{21} = 5$ $\quad A_{22} = 23$ $\quad A_{23} = -11$

$A_{31} = 3$ $\quad A_{32} = 12$ $\quad A_{33} = -6$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

10. $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

10. Adjoint of the matrix $A = [a_{ij}]_{n\times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n\times n}$ where A_{ij} is the co-factor of the element a_{ij}.

Let's find the cofactors for all the positions first-

Here, $A_{11} = 2$, $A_{12} = -9$, $A_{13} = -6$, $A_{21} = 0$, $A_{22} = -2$, $A_{23} = -1$, $A_{31} = -1$, $A_{32} = 3$, $A_{33} = 2$.

$$\therefore \text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

And |A| = -1.

$$A^{-1} = \frac{1}{|A|}\text{Adj}A = \frac{1}{-1}\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

11. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$

11. Here, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$, therefore

$|A| = 1(-\cos_2 a - \sin^2 a) + 0(0 - 0) + 0\,(0 - 0) = -1 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = 1$ $\quad A_{12} = 0$ $\quad A_{13} = 0$

$A_{21} = 0$ $\quad A_{22} = \cos a$ $\quad A_{23} = -\sin a$

$A_{31} = 0$ $\quad A_{32} = -\sin a$ $\quad A_{33} = \cos a$

$$A^{-1} = \frac{1}{|A|}adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos a & -\sin a \\ 0 & -\sin a & \cos a \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$$

12. Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

12. We have AB= $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$ = (61)(67)-(47)(87)= -2

Here determinant of matrix=|AB|≠ 0 hence $(AB)^{-1}$ exists.

$$(AB)^{-1} = \frac{1}{|AB|}\text{Adj}(AB) = -\frac{1}{2}\begin{bmatrix} 61 & -47 \\ -87 & 67 \end{bmatrix} = \begin{bmatrix} \frac{61}{-2} & \frac{-47}{-2} \\ \frac{-87}{-2} & \frac{67}{-2} \end{bmatrix}$$

$$\{ \because \text{Adj}(AB) = \begin{bmatrix} 61 & -47 \\ -87 & 67 \end{bmatrix} \}$$

So $(AB)^{-1} = \begin{bmatrix} -\frac{61}{2} & \frac{47}{2} \\ \frac{87}{2} & -\frac{67}{2} \end{bmatrix}$

Also $|A|=1 \neq 0$ and $|B|= -2 \neq 0$.

$\therefore$ A^{-1} and B^{-1} will also exist and are given by-

$$(A)^{-1} = \frac{1}{|A|}\text{Adj}(A) = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$(B)^{-1} = \frac{1}{|B|}\text{Adj}(B) = -\frac{1}{2}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

And hence,

$$(B)^{-1}(A)^{-1} = -\frac{1}{2}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 61 & -47 \\ -87 & 67 \end{bmatrix} = \begin{bmatrix} \frac{61}{-2} & \frac{-47}{-2} \\ \frac{-87}{-2} & \frac{67}{-2} \end{bmatrix}$$

13. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = 0$. Hence, find A^{-1}.

13. LHS = $A^2 - 5A + 7I = AA - 5A + 7I$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = \text{RHS}$$

$\Rightarrow A^2 - 5A + 7I = 0$

$\Rightarrow A^2 - 5A = -7I$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$AAA^{-1} - 5AA^{-1} = -7IA^{-1}$

$\Rightarrow AI - 5I = -7A^{-1}$ [Because $AA^{-1} = I$]

$$\Rightarrow 7A^{-1} = 5I - A = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

14. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = 0$.

14. We have $A^2 = A.A= \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 5 & 5 \end{bmatrix}$

Since $A^2 + aA + bI = \begin{bmatrix} 10 & 5 \\ 5 & 5 \end{bmatrix} + a\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

So $A^2 + aA + bI = \begin{bmatrix} 10+3a+b & 5+a \\ 5+a & 5+2a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence 10+3a+b=0 …(i)

5+a=0 …(ii)

5+2a+b=0 …(iii)

From (ii) a=-5

Putting a in (iii) we get b=5

So a=-5 and b=5 satisfy the equation.

15. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, show that $A^3 - 6A^2 + 5a + 11\,I = 0$. Hence, find A^{-1}.

15. $A^2 = A.A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

LHS = $A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5+11 & 7-12+5+0 & 1-6+5+0 \\ -23+18+5+0 & 27-48+10+11 & -69+84-15+0 \\ 32-42+10+0 & -13+18-5+0 & 58-84+15+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{RHS}$$

$= A^3 - 6A^2 + 5A + 11I = 0 \qquad \Rightarrow A^3 - 6A^2 + 5A = -11I$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$A^2AA^{-1} - 6AAA^{-1} + 5AA^{-1} = -11IA^{-1}$

$= A^2I - 6AI + 5I = -11A^{-1}$

[Because $AA^{-1} = 1$] $= 11A^{-1} = -A^2 + 6A - 5I$

$$\Rightarrow 11A^{-1} = -\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix} + \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} -4+6-5 & -2+6-0 & -1+6+0 \\ 3+6-0 & -8+12-5 & 14-18+0 \\ -7+12+0 & 3-6+0 & -14+18-5 \end{bmatrix} = \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

16. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}.

16. Here $A^2 = A.A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$

And hence $A^3 = A. A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$

$\therefore A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} - \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} = 0$$

Thus, $A^3 - 6A^2 + 9A - 4I = 0$

Now, $A^3 - 6A^2 + 9A - 4I = 0$,

$\rightarrow$ (A.A.A)- 6 (A.A) +9A = 4I

Post-multiply with A^{-1} on both sides-

$\rightarrow (A.A.A.A^{-1}) - 6\,(A.A.A^{-1}) + 9A.A^{-1} = 4I.\ A^{-1}$

$\rightarrow (A.A.I) - 6(A.I) + 9I = 4I.\ A^{-1}$ {since $A.A^{-1} = I$}

$\rightarrow (A.A) - 6A + 9I = 4A^{-1}$ {since X.I = X}

$\rightarrow A^{-1} = \frac{1}{4}\left(A^2 - 6A + 9I\right)$

$$= \frac{1}{4}\left(\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

Hence $A^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$

17. Let A be a non-singular square matrix of order 3 × 3. Then |adj A| is equal to:

(A) $|A|$ (B) $|A|^2$

(C) $|A|^3$ (D) $3|A|$

17. We know that adj A = |A|I

$$\Rightarrow (adj\ A)A = |A|\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \Rightarrow |(adj\ A)A| = |A|\begin{vmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{vmatrix} = \begin{vmatrix}|A| & 0 & 0\\0 & |A| & 0\\0 & 0 & |A|\end{vmatrix} = |A|^3$$

$\Rightarrow$ |adj A| = $|A|^2$

Hence, the option (B) is correct.

18. If A is an ivertible matrix of order 2, then det (A^{-1}) is equal to:

(A) det(A)

(B) $\dfrac{1}{\det(A)}$

(C) 1

(D) 0

18. $(A)^{-1} = \dfrac{1}{|A|}Adj(A)$

So $|(A)^{-1}| = \left|\dfrac{1}{|A|}Adj(A)\right| = \dfrac{1}{|A|^n}|Adj(A)| = \dfrac{1}{|A|^n}|A|^{n-1} = \dfrac{1}{|A|^1}$

{since adj(A) is of order n and $|Adj(A)|=|A|^{n-1}$}

Alternative-

We know that $AA^{-1}=I$

So $|A||A^{-1}|=|I|=1$

Hence $|(A)^{-1}| = \dfrac{1}{|A|}$