

# NCERT Solutions for Class-XII Maths

## Chapter-13.2

### NCERT Maths Class 12

1. If  $P(A) = \frac{3}{5}$  and  $P(B) = \frac{1}{5}$ , find  $P(A \cap B)$  if A and B are independent events.

1. It is given that  $P(A) = \frac{3}{5}$  and  $P(B) = \frac{1}{5}$

A and B are independent events. Therefore,

$$P(A \cap B) = P(A) \cdot P(B) = \frac{3}{5} \cdot \frac{1}{5} = \frac{3}{25}$$

2. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

2. A pack of 52 cards.

As we know there are 26 cards in total which are black. Let A and B denotes respectively the events that the first and second drawn cards are black.

$$\text{Now, } P(A) = P(\text{black card in first draw}) = \frac{26}{52} = \frac{1}{2}$$

Because the second card is drawn without replacement so, now the total number of black card will be 25 and total cards will be 51. i.e. the conditional probability of B given that A has already occurred.

$$\text{Now, } P\left(\frac{B}{A}\right) = P(\text{black card in second draw}) = \frac{25}{51}$$

Thus the probability that both the cards are black:

$$\Rightarrow P(A \cap B) = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

Hence, the probability that both the cards are black =  $\frac{25}{102}$ .

3. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

3. Let A, B and C be the respective events that the first, second, and third drawn orange is good.

Therefore, probability that first drawn orange is good,  $P(A) = \frac{12}{15}$

The oranges are not replaced.

Therefore probability of getting second orange good,  $P(B) = \frac{11}{14}$

Similarly, probability of getting third orange good,  $P(C) = \frac{10}{13}$

The box is approved for sale, if all the three oranges are good.

Thus, probability of getting all the oranges good  $= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}$

Therefore, the probability that the box is approved for sale is  $\frac{44}{91}$ .

4. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

4. A fair coin and an unbiased die are tossed.

We know that the sample space S:

$S = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$

Let A be the event 'head appears on the coin':

$\Rightarrow A = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6)\}$

$$\Rightarrow P(A) = \frac{6}{12} = \frac{1}{2}$$

Now, Let B be the event 3 on the die:

$\Rightarrow B = \{(H,3), (T,3)\}$

$$\Rightarrow P(B) = \frac{2}{12} = \frac{1}{6}$$

As,  $A \cap B = \{(H,3)\}$

$$\Rightarrow P(A \cap B) = \frac{1}{12} \quad \dots(1)$$

$$\text{And } P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} \quad \dots(2)$$

From (1) and (2)  $P(A \cap B) = P(A) \cdot P(B)$

Therefore, A and B are independent events.

5. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent?

5. When a die is thrown, the sample space (S) is

$S = \{1, 2, 3, 4, 5, 6\}$

Let A: the number is even =  $\{2, 4, 6\}$

$$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$$

B: the number is red =  $\{1,2,3\}$

$$\Rightarrow P(B) = \frac{3}{6} = \frac{1}{2}$$

$$\therefore A \cap B = \{2\}$$

$$P(AB) = P(A \cap B) = \frac{1}{6}$$

$$P(A) \cdot P(B) \neq P(AB)$$

Therefore, A and B are not independent.

6. Let E and F be events with  $P(E) = \frac{3}{5}$ ,  $P(F) = \frac{3}{10}$  and  $P(E \cap F) = \frac{1}{5}$ . Are E and F independent?

6.  $P(E) = \frac{3}{5}$ ,  $P(F) = \frac{3}{10}$  and  $P(E \cap F) = \frac{1}{5}$

$$P(E) \cdot P(F) = \frac{3}{5} \times \frac{3}{10} = \frac{9}{50} \neq \frac{1}{5}$$

$$\Rightarrow P(E \cap F) \neq P(E) \cdot P(F)$$

Therefore, E and F are not independent events.

7. Given that the events A and B are such that  $P(A) = \frac{1}{2}$ ,  $P(A \cup B) = \frac{1}{5}$  and  $P(B) = p$ . Find p if they are (i) mutually exclusive (ii) independent.

7. It is given that  $P(A) = \frac{1}{2}$ ,  $P(A \cap B) = \frac{3}{5}$ , and  $P(B) = p$

(i) When A and B are mutually exclusive,  $A \cap B = \Phi$   
 $\therefore P(A \cap B) = 0$

It is known that,  $P(A \cup B) = \frac{3}{5}$ , and  $P(B) = p$

(i) When A and B are mutually exclusive,  $A \cap B = \Phi$   
 $\therefore P(A \cap B) = 0$

It is known that,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - 0$$

$$\Rightarrow p = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$$

(ii) When A and B are independent,  $P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2}p$

It is known that,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p$$

$$\Rightarrow \frac{3}{5} = \frac{1}{2} + \frac{p}{2}$$

$$\Rightarrow \frac{p}{2} = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$$

$$\Rightarrow p = \frac{2}{10} = \frac{1}{5}$$

8. Let A and B be independent events with  $P(A) = 0.3$  and  $P(B) = 0.4$ . Find

(i)  $P(A \cap B)$

(ii)  $P(A \cup B)$

(iii)  $P(A|B)$  (iv)  $P(B|A)$

8.  $P(A) = 0.3$  and  $P(B) = 0.4$

(i)  $P(A \cap B)$

When A and B are independent.

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

$$\Rightarrow P(A \cap B) = 0.3 \times 0.4$$

$$\Rightarrow P(A \cap B) = 0.12$$

(ii)  $P(A \cup B)$

As we know,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cup B) = 0.3 + 0.4 - 0.12$$

$$\Rightarrow P(A \cup B) = 0.58$$

(iii)  $P(A|B)$

$$\text{As we know } P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{0.12}{0.4}$$

$$\Rightarrow P(A|B) = 0.3$$

(iv)  $P(B|A)$

$$\text{As we know } P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{0.12}{0.3}$$

$$\Rightarrow P(B|A) = 0.4$$

9. If A and B are two events such that  $P(A) = \frac{1}{4}$ ,  $P(B) = \frac{1}{2}$  and  $P(A \cap B) = \frac{1}{8}$ , find  $P(\text{not A and not B})$ .

9. It is given that,  $P(A) = \frac{1}{2}$  and  $P(A \cap B) = \frac{1}{8}$

$$P(\text{not on A and not B}) = P(A' \cap B')$$

$$P(\text{not on A and not on B}) = P((A \cup B)') [A' \cap B' = (A \cup B)']$$

$$= 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[ \frac{1}{4} + \frac{1}{2} - \frac{1}{8} \right]$$

$$= 1 - \frac{5}{8}$$

$$= \frac{3}{8}$$

10. Events A and B are such that  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{7}{12}$  and  $P(\text{not A or not B}) = \frac{1}{4}$  State whether A and B are independent ?

10.  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{7}{12}$  and  $P(\text{not } A \text{ or not } B) = \frac{1}{4}$

$$\Rightarrow P(A' \cup B') = \frac{1}{4}$$

$$\Rightarrow P(A \cap B)' = \frac{1}{4}$$

$$\Rightarrow 1 - P(A \cap B) = \frac{1}{4}$$

$$\Rightarrow P(A \cap B) = 1 - \frac{1}{4}$$

$$\Rightarrow P(A \cap B) = \frac{3}{4} \dots(1)$$

And  $P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24} \dots(2)$

From (1) and (2)  $P(A \cap B) \neq P(A) \cdot P(B)$

Therefore, A and B are not independent events.

11. Given two independent events A and B such that  $P(A) = 0.3$ ,  $P(B) = 0.6$ .

Find

(i)  $P(A \text{ and } B)$

(ii)  $P(A \text{ and not } B)$

(iii)  $P(A \text{ or } B)$

(iv)  $P(\text{neither } A \text{ nor } B)$

11. it is given that  $P(A) = 0.3$  and  $P(B) = 0.6$

Also, A and B are independent events.

(i)  $\therefore P(A \text{ and } B) = P(A) \cdot P(B)$

$$\Rightarrow P(A \cap B) = 0.3 \times 0.6 = 0.18$$

(ii)  $p(A \text{ and not } B) = P(A \cap B')$

$$= P(A) - P(A \cap B)$$

$$= 0.3 - 0.18$$

$$= 0.12$$

(iii)  $P(A \text{ or } B) = P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= 0.3 + 0.6 - 0.18$$

$$= 0.72$$

(iv)  $P(\text{neither } A \text{ nor } B) = p(A' \cap B')$

$$= P((A \cup B)')$$

$$= 1 - P(A \cup B)$$

$$= 1 - 0.72$$

$$= 0.28$$

12. A die is tossed thrice. Find the probability of getting an odd number at least once.

12. A die is tossed thrice.

The sample space  $S = \{1, 2, 3, 4, 5, 6\}$

Let  $P(A)$  = probability of getting an odd number in first throw.

$$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}.$$

Let  $P(B)$  = probability of getting an even number.

$$\Rightarrow P(B) = \frac{3}{6} = \frac{1}{2}.$$

Now, probability of getting an even number in three times  $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

So, probability of getting an odd number at least once

$= 1 -$  probability of getting an odd number in no throw

$= 1 -$  probability of getting an even number in three times

$$= 1 - \frac{1}{8}$$

$\therefore$  probability of getting an odd number at least once  $= \frac{7}{8}$

**13.** Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

(i) both balls are red.

(ii) first ball is black and second is red.

(iii) one of them is black and other is red.

**13.** Total number of balls = 18

Number of red balls = 8

Number of black balls = 10

(i) Probability of getting a red ball in the first draw  $= \frac{8}{18} = \frac{4}{9}$

The ball is replaced after the first draw.

$\therefore$  Probability of getting a red ball in the second draw  $= \frac{8}{18} = \frac{4}{9}$

Therefore, probability of getting both the balls red  $= \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$

(ii) Probability of getting first ball black  $= \frac{10}{18} = \frac{5}{9}$

The ball is replaced after the first draw.

Probability of getting second ball as red  $= \frac{8}{18} = \frac{4}{9}$

Therefore, probability of getting first ball as black and second ball as red  $= \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$

(iii) Probability of getting first ball as red  $= \frac{8}{18} = \frac{4}{9}$

The ball is replaced after the first draw.

Probability of getting second ball as black  $= \frac{10}{18} = \frac{5}{9}$

Therefore, probability of getting first ball as black and second ball as red  $= \frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$

Therefore, probability that one of them is black and other is red

= Probability of getting first ball black and second as red+ Probability of getting first ball black

$$= \frac{20}{81} + \frac{20}{81}$$

$$= \frac{40}{81}$$

14. Probability of solving specific problem independently by A and B are  $\frac{1}{2}$  and  $\frac{1}{3}$  respectively. If both try to solve the problem independently, find the probability that
- the problem is solved
  - exactly one of them solves the problem.

14.  $P(A)$  = Probability of solving the problem by A =  $\frac{1}{2}$

$P(B)$  = Probability of solving the problem by B =  $\frac{1}{3}$

Because A and B both are independent.

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

$$\Rightarrow P(A \cap B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$$P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(B') = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$$

- (i) the problem is solved

The problem is solved, i.e. it is either solved by A or it is solved by B.

$$= P(A \cup B)$$

$$\text{As we know, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6}$$

$$\Rightarrow P(A \cup B) = \frac{2}{3}$$

- (ii) exactly one of them solves the problem i.e. either problem is solved by A but not by B or vice versa i.e.  $P(A) \cdot P(B') + P(A') \cdot P(B)$

$$= \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3}$$

$$= \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$$

$$\Rightarrow P(A) \cdot P(B') + P(A') \cdot P(B) = \frac{1}{2}$$

15. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?

- (i) E : 'the card drawn is a spade'

F : 'the card drawn is an ace'

- (ii) E : 'the card drawn is black'

F : 'the card drawn is a king'

(iii) E : 'the card drawn is a king or queen'

F : 'the card drawn is a queen or jack'.

15. A deck of 52 cards.

(i) In a deck of 52 cards, 13 cards are spade and 4 cards are ace and only one card is there which is spade and ace both.

$$\text{Hence, } P(E) = \text{The card drawn is a spade} = \frac{13}{52} = \frac{1}{4}$$

$$P(F) = \text{The card drawn is an ace} = \frac{4}{52} = \frac{1}{13}$$

$$P(E \cap F) = \text{The card drawn is a spade and ace both} = \frac{1}{52} \quad \dots(1)$$

And  $P(E) \cdot P(F)$

$$= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \quad \dots(2)$$

From (1) and (2)

$$\Rightarrow P(E \cap F) = P(E) \cdot P(F)$$

Hence, E and F are independent events.

(ii) In a deck of 52 cards, 26 cards are black and 4 cards are king and only two card are there which are black and king both.

$$\text{Hence, } P(E) = \text{The card drawn is of black} = \frac{26}{52} = \frac{1}{2}$$

$$P(F) = \text{The card drawn is a king} = \frac{4}{52} = \frac{1}{13}$$

$$P(E \cap F) = \text{The card drawn is a black and king both} = \frac{2}{52} = \frac{1}{26}$$

...(1)

And  $P(E) \cdot P(F)$

$$= \frac{1}{2} \times \frac{1}{13} = \frac{1}{26}$$

...(2)

From (1) and (2)

$$\Rightarrow P(E \cap F) = P(E) \cdot P(F)$$

Hence, E and F are independent events.

(iii) In a deck of 52 cards, 4 cards are queen, 4 cards are king and 4 cards are jack.

$$\text{Hence, } P(E) = \text{The card drawn is either king or queen} = \frac{8}{52} = \frac{2}{13}$$

$$P(F) = \text{The card drawn is either queen or jack} = \frac{8}{52} = \frac{2}{13}$$

There are 4 cards which are either king or queen and either queen or jack.

$$P(E \cap F) = \text{The card drawn is either king or queen and either queen or jack} = \frac{4}{52} = \frac{1}{13}$$

...(1)

And  $P(E) \cdot P(F)$

$$= \frac{2}{13} \times \frac{2}{13} = \frac{4}{169}$$

...(2)

From (1) and (2)

$$\Rightarrow P(E \cap F) \neq P(E) \cdot P(F)$$

Hence, E and F are not independent events.

16. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.
- (a) Find the probability that she reads neither Hindi nor English newspapers.  
(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.  
(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.
16. Let H denote the students who read Hindi newspaper and E denote the students who read English newspaper.

It is given that,

$$P(H) = 60\% = \frac{60}{100} = \frac{3}{5}$$

$$P(E) = 40\% = \frac{40}{100} = \frac{2}{5}$$

$$P(H \cap E) = 20\% = \frac{20}{100} = \frac{1}{5}$$

- i. Probability that a student reads Hindi or English newspaper is,

$$\begin{aligned} (H \cup E)' &= 1 - P(H \cup E) \\ &= 1 - \{P(H) + P(E) - P(H \cap E)\} \\ &= 1 - \left(\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right) \\ &= 1 - \frac{4}{5} \\ &= \frac{1}{5} \end{aligned}$$

- ii. Probability that a randomly chosen student reads English newspaper, if she reads Hindi news paper, is given by  $P(E|H)$

$$\begin{aligned} P(E|H) &= \frac{P(E \cap H)}{P(H)} \\ &= \frac{\frac{1}{5}}{\frac{3}{5}} \\ &= \frac{1}{3} \end{aligned}$$

- iii. Probability that a randomly chosen student reads Hindi newspaper, if she reads English newspaper, is given by  $P(H|E)$

$$P(H|E) = \frac{P(H \cap E)}{P(E)}$$

$$= \frac{1}{\frac{5}{\frac{2}{5}}}$$

$$= \frac{1}{2}$$

17. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

- A. 0                      B.  $\frac{1}{3}$                       C.  $\frac{1}{12}$                       D.  $\frac{1}{36}$

17. A pair of dice is rolled.

Hence the number of outcomes = 36

Let P(E) be the probability to get an even prime number on each die.

As we know the only even prime number is 2.

So, E = {2,2}

$$\Rightarrow P(E) = \frac{1}{36}$$

The correct answer is D.

18. Two events A and B will be independent, if

(A) A and B are mutually exclusive

(B)  $P(A'B') = [1 - P(A)] [1 - P(B)]$

(C)  $P(A) = P(B)$

(D)  $P(A) + P(B) = 1$

18. Two events A and B will be independent.

As A and B are independent events.

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

We solve it using options.

Let  $P(A) = a$ ,  $P(B) = b$

As, A and B are mutually exclusive

$$P(A \cap B) = \phi$$

Now,  $P(A) \cdot P(B) = a \cdot b \neq P(A \cap B)$

$$\Rightarrow P(A \cap B) \neq P(A) \cdot P(B)$$

Hence, it shows A and B are not Independent events.

(B)  $P(A'B') = [1 - P(A)] [1 - P(B)]$

$$\Rightarrow P(A' \cap B') = 1 - P(A) - P(B) + P(A)P(B)$$

$$\Rightarrow 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A)P(B)$$

$$= - [P(A) + P(B) - P(A \cap B)] = - P(A) - P(B) + P(A)P(B)$$

$$= - P(A) - P(B) + P(A \cap B) = - P(A) - P(B) + P(A)P(B)$$

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

Hence, it shows A and B are Independent events.

(C)  $P(A) = P(B)$

As,  $P(A) = P(B)$

Let we take the example of a coin

$P(A)$  = probability of getting head =  $\frac{1}{2}$

$P(B)$  = probability of getting tail =  $\frac{1}{2}$

$A \cap B = \phi$

$P(A \cap B)$  = probability of getting head and tail both = 0

Now,  $P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$

$\Rightarrow P(A \cap B) \neq P(A) \cdot P(B)$

Hence, it shows A and B are not Independent events.

(D)  $P(A) + P(B) = 1$

Let we take the example of a coin

$P(A)$  = probability of getting head =  $\frac{1}{2}$

$P(B)$  = probability of getting tail =  $\frac{1}{2}$

Now,  $P(A) + P(B) = \frac{1}{2} + \frac{1}{2} = 1$

But it doesnot inferred that A and B are independent.

Hence the correct option is B.



