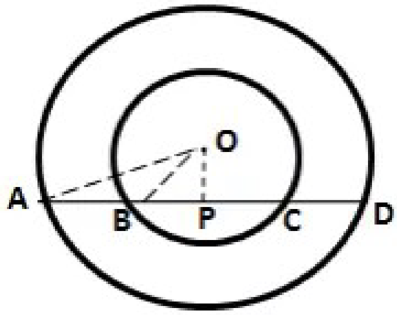
Solution 1:

Exercise 17(B)



Drop $OP \perp AD$

$\therefore$ OP bisects AD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \quad AP = PD \quad ----(i)$

Now, BC is a chord for the inner circle and $OP \perp BC$

$\therefore$ OP bisects BC

(Perpendicular drawn from the centre of a circle to a chord bisects it)

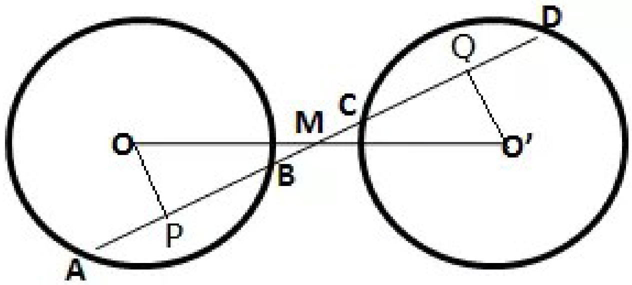
$\Rightarrow \quad BP = PC \quad ----(ii)$

Subtracting (ii) from (i),

$AP - BP = PD - PC$

$\Rightarrow \quad AB = CD$

Solution 2:



Given: A straight line Ad intersects two circles of equal radii at A, B, C and D.

The line joining the centres OO' intersect AD at M and M is the midpoint of OO'.

To prove: AB = CD.

Construction: From O, draw $OP \perp AB$ and from O', draw $O'Q \perp CD$.

Proof:

In ΔOMP and $\Delta O'MQ$,

$\angle OMP = \angle O'MQ$ (Vertically opposite angles)

$\angle OPM = \angle O'QM$ $\left(\text{each} = 90^0\right)$

$OM = O'M$ (Given)

By Angle-Angle-Side criterion of congruence,

$\therefore$ $\Delta OMP \cong \Delta O'MQ$, (by AAS)

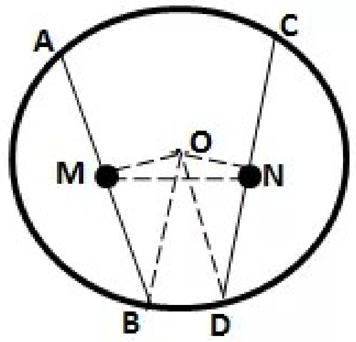
The corresponding parts of the congruent triangles are congruent

$\therefore$ $OP = O'Q$ (c.p.c.t)

We know that two chords of a circle or equal circles which are equidistant from the centre are equal.

$\therefore$ $AB = CD$

Solution 3:



Drop $OM \perp AB$ and $ON \perp CD$

$\therefore$ OM bisects AB and ON bisects CD.

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \quad BM = \frac{1}{2}AB = \frac{1}{2}CD = DN \quad ----(1)$

Applying Pythagoras theorem,

$OM^2 = OB^2 - BM^2$

$= OD^2 - DN^2 \quad$ (by (1))

$= ON^2$

$\therefore \quad OM = ON$

$\Rightarrow \quad \angle OMN = \angle ONM \quad ----(2)$

(Angles opp to equal sides are equal)

(i) $\quad \angle OMB = \angle OND \quad$ (both 90^0)

Subtracting (2) from above,

$\angle BMN = \angle DNM$

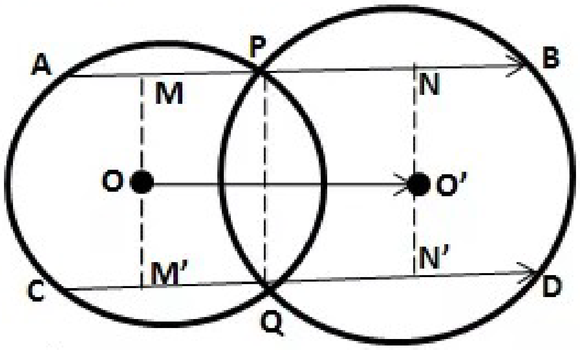
(ii) $\quad \angle OMA = \angle ONC \quad$ (both 90^0)

Adding (2) to above,

$\angle AMN = \angle CNM$

Solution 4:

Drop OM and O'N perpendicular on AB and OM' and O'N' perpendicular on CD.



$\therefore$ OM, O'N, OM' and O'N' bisect AP, PB, CQ and QD respectively

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\therefore \quad MP = \frac{1}{2}AP,\ PN = \frac{1}{2}BP,\ M'Q = \frac{1}{2}CQ,\ QN' = \frac{1}{2}QD$

Now, $OO' = MN = MP + PN = \frac{1}{2}(AP + BP) = \frac{1}{2}AB$ ---(i)

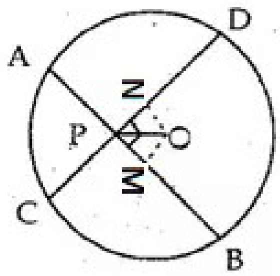
and $OO' = M'N' = M'Q + QN' = \frac{1}{2}(CQ + QD) = \frac{1}{2}CD$ ---(ii)

By (i) and (ii),

$AB = CD$

Solution 5:

Drop OM and ON perpendicular on AB and CD.
Join OP, OB and OD.



$\therefore$ OM and ON bisect AB and CD respectively

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\therefore \quad MB = \frac{1}{2}AB = \frac{1}{2}CD = ND \qquad ---(i)$

In rtΔOMB, $\quad OM^2 = OB^2 - MB^2 \qquad ---(ii)$

In rtΔOND, $\quad ON^2 = OD^2 - ND^2 \qquad ---(iii)$

From (i), (ii) and (iii),

$OM = ON$

In ΔOPM and ΔOPN,

$\angle OMP = \angle ONP \qquad$ (both 90^0)

$OP = OP \qquad$ (Common)

$OM = ON \qquad$ (Proved above)

By Right Angle-Hypotenuse-Side criterion of congruence,

$\therefore \quad \Delta OPM \cong \Delta OPN$ (by RHS)

The corresponding parts of the congruent triangles are congruent.

$\therefore \quad PM = PN \qquad$ (cp.c.t)

Adding (i) to both sides,

$MB + PM = ND + PN$

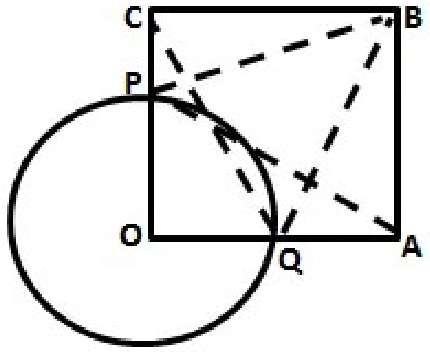
$\Rightarrow \quad BP = DP$

Now, $\quad AB = CD$

$\therefore \quad AB - BP = CD - DP \qquad (\because BP = DP)$

$\Rightarrow \quad AP = CP$

Solution 6:



(i)

In ΔOPA and ΔOQC,

$OP = OQ$	(radii of same circle)
$\angle AOP = \angle COQ$	(both 90^0)
$OA = OC$	(sides of the square)

By Side – Angle – Side criterion of congruence,

$\therefore \quad \Delta OPA \cong \Delta OQC$ (by SAS)

(ii)

Now,	$OP = OQ$	(radii)
and	$OC = OA$	(sides of the square)
$\therefore$	$OC - OP = OA - OQ$	
$\Rightarrow$	$CP = AQ$	----(1)

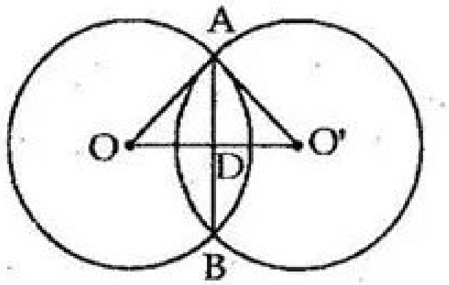
In ΔBPC and ΔBQA,

$BC = BA$	(sides of the square)
$\angle PCB = \angle QAB$	(both 90^0)
$PC = QA$	(by (1))

By Side – Angle – Side criterion of congruence,

$\therefore \quad \Delta BPC \cong \Delta BQA$ (by SAS)

Solution 7:



$OA = 25 cm \quad$ and $AB = 30$ cm

$\therefore \quad AD = \frac{1}{2} \times AB = \left(\frac{1}{2} \times 30\right) cm = 15$ cm

Now in right angled Δ ADO,

$OA^2 = AD^2 + OD^2$

$\Rightarrow \quad OD^2 = OA^2 - OD^2 = 25^2 - 15^2$

$= 625 - 225 = 400$

$\therefore \quad OD = \sqrt{400} = 20$ cm

Again, we have O'A = 17 cm.

In right angle Δ ADO'

$O'A^2 = AD^2 + O'D^2$

$\Rightarrow \quad O'D^2 = O'A^2 - AD^2 = 17^2 - 15^2$

$= 289 - 225 = 64$

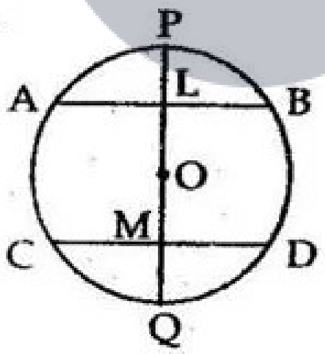
$\therefore \quad O'D = 8$ cm

$\therefore \quad OO' = (OD + O'D)$

$= (20 + 8) = 28$ cm

$\therefore$ the distance between their centres is 28 cm.

Solution 8:



Given: AB and CD are the two chords of a circle with centre O.
L and M are the midpoints of AB and CD and O lies in the line joining ML

To Prove : AB || CD.

Proof: AB and CD are two chords of a circle with centre O.
Line LOM bisects them at L and M.

Then, $\quad OL \perp AB$

and, $\quad OM \perp CD$

$\therefore \quad \angle ALM = \angle LMD = 90^0$

But they are alternate angles

$\therefore \quad AB \parallel CD$.

Solution 9:

In the circle with centre Q, $QO \perp AD$

$\therefore \quad OA = OD \quad ----(1)$

(Perpendicular drawn from the centre of a circle to a chord bisects it)

In the circle with centre P, $PO \perp BC$

$\therefore \quad OB = OC \quad ----(2)$

(Perpendicular drawn from the centre of a circle to a chord bisects it)

(i)

(1) − (2) gives,

$AB = CD \quad ----(3)$

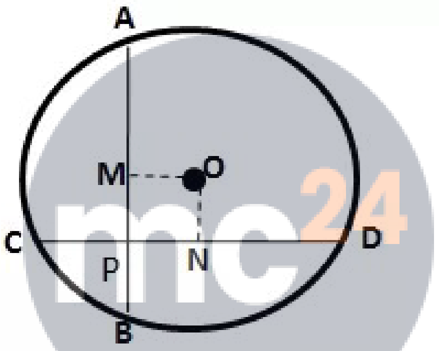
(ii)

Adding BC to both sides of equation (3)

$AB + BC = CD + BC$

$\Rightarrow \quad AC = BD$

Solution 10:



Clearly, all the angles of OMPN are 90^{0}.

$OM \perp AB$ and $ON \perp CD$

$\therefore \quad BM = \frac{1}{2}AB = \frac{1}{2}CD = CN \quad ----(i)$

(Perpendicular drawn from the centre of a circle to a chord bisects it)

As the two equal chords AB and CD intersect at point P inside the circle,

$\therefore \quad AP = DP$ and $CP = BP \quad ----(ii)$

Now, $CN - CP = BM - BP$ (by (i) and (ii))

$\Rightarrow \quad PN = MP$

$\therefore$ Quadrilateral OMPN is a square.