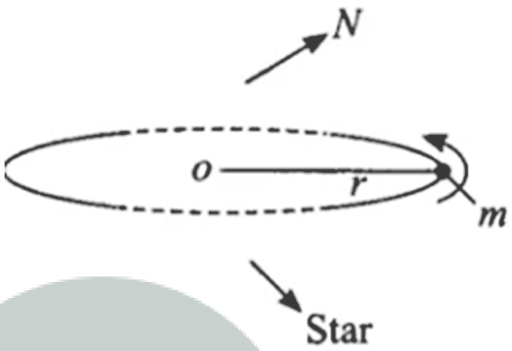
Class 11 Physics Chapter 7: Gravitation

Long Answers

34. A star like the sun has several bodies moving around it at different distances. Consider that all of them are moving in circular orbits. Let r be the distance of the body from the centre of the star and let its linear velocity be v, angular velocity ω, kinetic energy K, gravitational potential energy U, total energy E, and angular momentum l. As the radius r of the orbit increases, determine which of the above quantities increase and which ones decrease.

Answer:



For circular orbits: $GMm/r^2 = mv^2/r$, so $v = \sqrt{GM/r}$

Analysis:

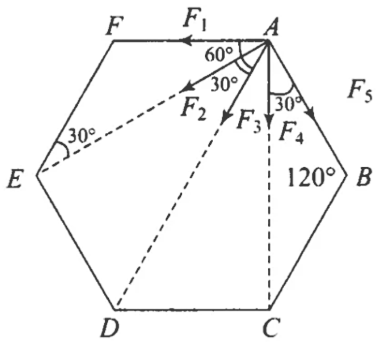
- **Linear velocity $v \propto 1/\sqrt{r}$ → DECREASES**
- **Angular velocity $\omega = v/r \propto 1/r^{3/2}$ → DECREASES**
- **Kinetic energy $K = \frac{1}{2}mv^2 \propto 1/r$ → DECREASES**
- **Potential energy $U = -GMm/r \propto 1/r$ → INCREASES** (becomes less negative)
- **Total energy $E = -GMm/2r \propto 1/r$ → INCREASES** (becomes less negative)
- **Angular momentum $l = mvr \propto \sqrt{r}$ → INCREASES**

35. Six point masses of mass m each are at the vertices of a regular hexagon of side l. Calculate the force on any of the masses.

Answer:

Consider mass at vertex A. Distances from A:

- To adjacent vertices (B,F): l
- To next vertices (C,E): $\sqrt{3}l$
- To opposite vertex (D): 2l



Force components along AD:

- From B,F: $F_1 = Gm^2/l^2 \times \cos(30°) = Gm^2/l^2 \times (\sqrt{3}/2)$
- From C,E: $F_2 = Gm^2/(3l^2) \times \cos(30°) = Gm^2/(3l^2) \times (\sqrt{3}/2)$
- From D: $F_3 = Gm^2/(4l^2)$

Total force = $2F_1 + 2F_2 + F_3 = Gm^2/l^2 \times (\sqrt{3} + 1/\sqrt{3} + 1/4)$

$F_{total} = (Gm^2/l^2) \times (11\sqrt{3} + 3)/12$

36. A satellite is to be placed in equatorial geostationary orbit around earth for communication: a) calculate height of such a satellite b) find out the minimum number of satellites that are needed to cover entire earth

Answer:

Given:

- $M_{earth} = 6 \times 10^{24}$ kg
- $R_{earth} = 6.4 \times 10^6$ m
- T = 24 × 3600 = 86,400 s
- $G = 6.67 \times 10^{-11}$ N·m²/kg²

a) Height calculation: For circular orbit: $GM/r^3 = 4\pi^2/T^2$

$r^3 = GMT^2/4\pi^2 = (6.67\times10^{-11} \times 6\times10^{24} \times (86{,}400)^2)/(4\pi^2)$

$r = 4.22 \times 10^7$ m

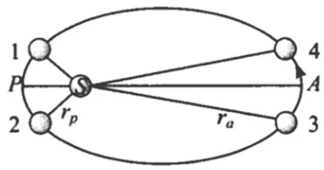
Height h = r - R_earth = 42,200 - 6,400 = 35,800 km

b) Minimum satellites = 3

Three satellites equally spaced (120° apart) can provide complete equatorial coverage.

37. Earth's orbit is an ellipse with eccentricity 0.0167. Thus, earth's distance from the sun and speed as it moves around the sun varies from day to day. This means that the length of the solar day is not constant through the year. Find out the length of the shortest and the longest day.

Answer:



Given: $e = 0.0167$

At perihelion: $r_p = a(1-e) = a(1-0.0167) = 0.9833a$ At aphelion: $r_a = a(1+e) = a(1+0.0167) = 1.0167a$

Angular velocities ratio: $\omega_p/\omega_a = (r_a/r_p)^2 = (1.0167/0.9833)^2 = 1.0691$

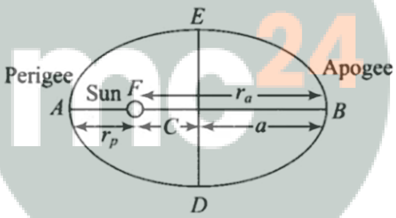
Day length variation:

- Shortest day: ≈ 24h - 7.9 minutes
- Longest day: ≈ 24h + 7.9 minutes

However, this simplified analysis doesn't account for Earth's axial tilt, which is the primary cause of actual seasonal day length variation.

38. A satellite is in an elliptic orbit around the earth with aphelion of 6R and perihelion of 2R where R = 6400 km is the radius of the earth. Find eccentricity of the orbit. Find the velocity of the satellite at apogee and perigee. What should be done if this satellite has to be transferred to a circular orbit of radius 6R?

Answer:



Given:

- $r_p = 2R = 12{,}800$ km
- $r_a = 6R = 38{,}400$ km

Eccentricity: $e = (r_a - r_p)/(r_a + r_p) = (6R - 2R)/(6R + 2R) = 4R/8R = 0.5$

Semi-major axis: $a = (r_a + r_p)/2 = 4R = 25{,}600$ km

Velocities using energy and angular momentum conservation: From conservation of angular momentum: $r_p v_p = r_a v_a$ Therefore: $v_p = 3v_a$

From energy conservation: $\frac{1}{2}v_p^2 - GM/r_p = \frac{1}{2}v_a^2 - GM/r_a$

Solving:

- **$v_a = 2.28$ km/s** (at apogee)
- **$v_p = 6.84$ km/s** (at perigee)

For circular orbit at r = 6R: $v_c = \sqrt{GM/6R} = 3.23$ km/s

Required velocity change at apogee: $\Delta v = v_c - v_a = 3.23 - 2.28 =$ **0.95 km/s**

The satellite needs a prograde burn of 0.95 km/s at apogee to circularize the orbit.