4. Inverse Trigonometric Functions

Exercise 4A

1. Question

Find the principal value of :

(i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(ii) $\sin^{-1}\left(\frac{1}{2}\right)$

(iii) $\cos^{-1}\left(\frac{1}{2}\right)$

(iv) $\tan^{-1}(1)$

(v) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(vi) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(vii) $\text{cosec}^{-1}\left(\sqrt{2}\right)$

Answer

NOTE:

Trigonometric Table

	0° (0)	**30°** $\left(\frac{\pi}{6}\right)$	**45°** $\left(\frac{\pi}{4}\right)$	**60°** $\left(\frac{\pi}{3}\right)$	**90°** $\left(\frac{\pi}{2}\right)$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
cosec	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined
cot	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

(i) Let $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\Rightarrow \frac{\sqrt{3}}{2} = \sin x$ [We know which value of x when placed in sin gives us this answer]

$\therefore x = \frac{\pi}{3}$

(ii) Let $\sin^{-1}\left(\frac{1}{2}\right) = x$

$\Rightarrow \frac{1}{2} = \sin x$ [We know which value of x when put in this expression will give us this result]

$\Rightarrow x = \frac{\pi}{6}$

(iii) Let $\cos^{-1}\left(\frac{1}{2}\right) = x$

$\Rightarrow \frac{1}{2} = \cos x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{3}$

(iv) Let $\tan^{-1}(1) = x$

$\Rightarrow 1 = \tan x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{4}$

(v) Let $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = x$

$\Rightarrow \frac{1}{\sqrt{3}} = \tan x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{6}$

(vi) Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = x$

$\Rightarrow \frac{2}{\sqrt{3}} = \sec x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{6}$

(vii) Let $\operatorname{cosec}^{-1}(\sqrt{2}) = x$

$\Rightarrow \sqrt{2} = \operatorname{cosec} x$

[We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{4}$

2. Question

Find the principal value of :

(i) $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

(ii) $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

(iii) $\tan^{-1}\left(-\sqrt{3}\right)$

(iv) $\sec^{-1}(-2)$

(v) $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

(vi) $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

Answer

(i) Let $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) = x$

$\Rightarrow -\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = x$ [Formula: $\sin^{-1}(-x) = -\sin^{-1} x$]

$\Rightarrow \frac{1}{\sqrt{2}} = -\sin x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = -\frac{\pi}{4}$

(ii) $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ [Formula: $\cos^{-1}(-x) = \pi - \cos^{-1} x$]

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\Rightarrow\left(\frac{\sqrt{3}}{2}\right) = \cos x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{6}$

Putting this value back in the equation

$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

(iii) Let $\tan^{-1}(-\sqrt{3}) = x$

$\Rightarrow -\tan^{-1}(\sqrt{3}) = x$ [Formula: $\tan^{-1}(-x) = -\tan^{-1}(x)$]

$\Rightarrow \sqrt{3} = -\tan x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{-\pi}{3}$

(iv) $\sec^{-1}(-2) = \pi - \sec^{-1}(2)$ ...(i) [Formula: $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$]

Let $\sec^{-1}(2) = x$

$\Rightarrow 2 = \sec x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{3}$

Putting the value in (i)

$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

(v) Let $\text{cosec}^{-1}(-\sqrt{2}) = x$

$\Rightarrow -\text{cosec}^{-1}(\sqrt{2}) = x$ [Formula: $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}(x)$]

$\Rightarrow \sqrt{2} = -\text{cosec}\, x$

$\therefore x = -\frac{\pi}{4}$

(vi) $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \pi - \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$ ... (i)

Let $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = x$

$\Rightarrow \frac{1}{\sqrt{3}} = \cot^{-1} x$ [We know which value of x when put in this expression will give us this result]

$\Rightarrow x = \frac{\pi}{3}$

Putting in (i)

$\pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

3. Question

Evaluate $\cos\left\{\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right\}$.

Answer

$\cos\{\pi - \frac{\pi}{6} + \frac{\pi}{6}\}$ [Refer to question 2(ii)]

$= \cos\{\pi\}$

$= \cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$

$= -1$

4. Question

Evaluate $\sin\left\{\frac{\pi}{2}-\left(\frac{-\pi}{3}\right)\right\}$

Answer

$\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$

$= \sin\left(\frac{5\pi}{6}\right)$

$= \sin\left(\pi - \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{6}$

$= \frac{1}{2}$

Exercise 4B

1. Question

Find the principal value of each of the following :

$\sin^{-1}\left(\frac{-1}{2}\right)$

Answer

$\sin^{-1}\left(\frac{-1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$ [Formula: $\sin^{-1}(-x) = \sin^{-1}(x)$]

$= -\frac{\pi}{6}$

2. Question

Find the principal value of each of the following :

$\cos^{-1}\left(\frac{-1}{2}\right)$

Answer

$\cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$ [Formula: $\cos^{-1}(-x) = -\cos^{-1}(x)$]

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

3. Question

Find the principal value of each of the following :

$\tan^{-1}(-1)$

Answer

$\tan(-1) = -\tan(1)$ [Formula: $\tan^{-1}(-x) = -\tan^{-1}(x)$]

[We know that $\tan\frac{\pi}{4} = 1$, thus $\tan^{-1}\frac{\pi}{4} = 1$]

$= -\frac{\pi}{4}$

4. Question

Find the principal value of each of the following :

$\sec^{-1}(-2)$

Answer

$\sec^{-1}(-2) = \pi - \sec^{-1}(2)$ [Formula: $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$]

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

5. Question

Find the principal value of each of the following :

$\text{cosec}^{-1}\left(-\sqrt{2}\right)$

Answer

$\text{cosec}^{-1}\left(-\sqrt{2}\right) = -\text{cosec}^{-1}\left(\sqrt{2}\right)$ [Formula: $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}(x)$]

$= -\frac{\pi}{4}$

This can also be solved as

$\text{cosec}^{-1}\left(-\sqrt{2}\right)$

Since cosec is negative in the third quadrant, the angle we are looking for will be in the third quadrant.

$= \pi + \frac{\pi}{4}$

$= \frac{5\pi}{4}$

6. Question

Find the principal value of each of the following :

$\cot^{1}(-1)$

Answer

$\cot^{-1}(-1) = \pi - \cot^{-1}(1)$ [Formula: $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$]

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

7. Question

Find the principal value of each of the following :

$\tan^{-1}\left(-\sqrt{3}\right)$

Answer

$\tan^{-1}(-\sqrt{3}) = -\tan^{-1}(\sqrt{3})$ [Formula: $\tan^{-1}(-x) = -\tan^{-1}(x)$]

$= -\frac{\pi}{3}$

8. Question

Find the principal value of each of the following :

$\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

Answer

$\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \pi - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ [Formula: $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$]

$= \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

9. Question

Find the principal value of each of the following :

$\text{cosec}^{-1}(2)$

Answer

$\text{cosec}^{-1}(2)$

Putting the value directly

$= \frac{\pi}{6}$

10. Question

Find the principal value of each of the following :

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

Answer

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$

[Formula: $\sin(\pi - x) = \sin x$]

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

[Formula: $\sin^{-1}(\sin x) = x$]

$= \frac{\pi}{3}$

11. Question

Find the principal value of each of the following :

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$$

Answer

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{4}\right)\right)$$

[Formula: tan(π – x) = -tan (x) , as tan is negative in the second quadrant.]

$$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$$

[Formula: $\tan^{-1}(\tan x) = x$]

$$= -\frac{\pi}{4}$$

12. Question

Find the principal value of each of the following :

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$$

Answer

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{6}\right)\right)$$

[Formula: cos(2π – x) = cos (x), as cos has a positive vaule in the fourth quadrant.]

$= \cos^{-1}\left(\cos\frac{5\pi}{6}\right)$ [Formula: $\cos^{-1}(\cos x) = x$

$$= \frac{5\pi}{6}$$

13. Question

Find the principal value of each of the following :

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$$

Answer

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi + \frac{\pi}{6}\right)\right)$$

[Formula: cos (2π + x) = cos x , cos is positive in the first quadrant.]

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right)$ [Formula: $\cos^{-1}(\cos x) = x$]

$$= \frac{\pi}{6}$$

14. Question

Find the principal value of each of the following :

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$$

Answer

$\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{6}\right)\right)$

[Formula: tan(π + x) = tan x, as tan is positive in the third quadrant.]

$=\tan^{-1}\left(\tan\frac{\pi}{6}\right)$ [Formula: $\tan^{-1}(\tan x) = x$]

$= \frac{\pi}{6}$

15. Question

Find the principal value of each of the following :

$\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$ 3

Answer

$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

Putting the value of $\tan^{-1}\sqrt{3}$ and using the formula

$\cot^{-1}(-x)= \pi-\cot^{-1}x$

$= \frac{\pi}{3} - \left(\pi - \cot^{-1}(\sqrt{3})\right)$

Putting the value of $\cot^{-1}(\sqrt{3})$

$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right)$

$= \frac{\pi}{3} - \frac{5\pi}{6}$

$= -\frac{3\pi}{6}$

$=-\frac{\pi}{2}$

16. Question

Find the principal value of each of the following :

$\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\}$

Answer

$\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\}$ [Formula: $\sin^{-1}(-x) = -\sin^{-1}x$]

$=\sin\left\{\frac{\pi}{3} - \left(-\sin^{-1}\frac{1}{2}\right)\right\}$

$= \sin\left\{\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right\}$

Putting value of $\sin^{-1}\left(\frac{1}{2}\right)$

$= \sin\left\{\frac{\pi}{3} + \frac{\pi}{6}\right\}$

$= \sin\frac{3\pi}{6}$

$=\sin\frac{\pi}{2}$

$= 1$

17. Question

Find the principal value of each of the following :

$\cot\left(\tan^{-1}x+\cot^{-1}x\right)$

Answer

$\cot(\tan^{-1}x + \cot^{-1}x) = \cot\left(\frac{\pi}{2}\right)$ [Formula: $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$]

Putting value of $\cot\left(\frac{\pi}{2}\right)$

= 0

18. Question

Find the principal value of each of the following :

$\operatorname{cosec}\left(\sin^{-1}x+\cos^{-1}x\right)$

Answer

$\operatorname{cosec}(\sin^{-1}x + \cos^{-1}x) = \operatorname{cosec}\frac{\pi}{2}$ [Formula: $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$]

Putting the value of $\operatorname{cosec}\frac{\pi}{2}$

= 1

19. Question

Find the principal value of each of the following :

$\sin\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right)$

Answer

$\sin(\sec^{-1}x + \operatorname{cosec}^{-1}x) = \sin\left(\frac{\pi}{2}\right)$ [Formula: $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$]

Putting the value of $\sin\left(\frac{\pi}{2}\right)$

=1

20. Question

Find the principal value of each of the following :

$\cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}$

Answer

Putting the values of the inverse trigonometric terms

$\frac{\pi}{3} + 2\times\frac{\pi}{6}$

$= \frac{\pi}{3} + \frac{\pi}{3}$

$= \frac{2\pi}{3}$

21. Question

Find the principal value of each of the following :

$$\tan^{-1}1+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$$

Answer

[Formula: $\cos^{-1}(-x)=\pi - \cos(x)$ and $\sin^{-1}(-x)= -\sin(x)$]

$$\tan^{-1}1+\left(\pi-\cos^{-1}\left(\frac{1}{2}\right)\right)+\left(-\sin^{-1}\left(\frac{1}{2}\right)\right)$$

Putting the values for each of the inverse trigonometric terms

$$=\frac{\pi}{4}+\left(\pi-\frac{\pi}{3}\right)-\frac{\pi}{6}$$

$$=\frac{\pi}{12}+\frac{2\pi}{3}$$

$$=\frac{9\pi}{12}$$

$$=\frac{3\pi}{4}$$

22. Question

Find the principal value of each of the following :

$$\sin^{-1}\left\{\sin\frac{3\pi}{5}\right\}$$

Answer

$$\sin^{-1}\left\{\sin\left(\frac{3\pi}{5}\right)\right\}$$

$$=\sin^{-1}\left\{\sin\left(\pi-\frac{2\pi}{5}\right)\right\}$$

[Formula: $\sin(\pi - x) = \sin x$, as sin is positive in the second quadrant.]

$=\sin^{-1}\left\{\sin\frac{2\pi}{5}\right\}$ [Formula: $\sin^{-1}(\sin x)=x$]

$$=\frac{2\pi}{5}$$

Exercise 4C

1 A. Question

Prove that:

$$\tan^{-1}\left(\frac{1+x}{1-x}\right)=\frac{\pi}{4}+\tan^{-1}x, x<1$$

Answer

To Prove: $\tan^{-1}\left(\frac{1+x}{1-x}\right)=\frac{\pi}{4}+\tan^{-1}x$

Formula Used: $\tan\left(\frac{\pi}{4}+A\right)=\frac{1+\tan A}{1-\tan A}$

Proof:

LHS $=\tan^{-1}\left(\frac{1+x}{1-x}\right)$ ... (1)

Let x = tan A ... (2)

Substituting (2) in (1),

$LHS = \tan^{-1}\left(\frac{1+\tan A}{1-\tan A}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + A\right)\right)$

$= \frac{\pi}{4} + A$

From (2), A = $\tan^{-1}$ x,

$\frac{\pi}{4} + A = \frac{\pi}{4} + \tan^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

1 B. Question

Prove that:

$$\tan^{-1} x + \cot^{-1}(x+1) = \tan^{-1}(x^2 + x + 1)$$

Answer

To Prove: $\tan^{-1} x + \cot^{-1}(x + 1) = \tan^{-1}(x^2 + x + 1)$

Formula Used:

1) $\cot^{-1} x = \tan^{-1}\frac{1}{x}$

2) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS = $\tan^{-1} x + \cot^{-1}(x + 1)$ ... (1)

$= \tan^{-1} x + \tan^{-1}\frac{1}{(x+1)}$

$= \tan^{-1}\left(\frac{x + \frac{1}{(x+1)}}{1 - \left(x \times \frac{1}{(x+1)}\right)}\right)$

$= \tan^{-1}\frac{x(x+1)+1}{x+1-x}$

$= \tan^{-1}(x^2 + x + 1)$

= RHS

Therefore, LHS = RHS

Hence proved.

2. Question

Prove that:

$$\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1} x, |x| \le \frac{1}{\sqrt{2}}.$$

Answer

To Prove: $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$

Formula Used: sin 2A = 2 × sin A × cos A

Proof:

LHS $= \sin^{-1}(2x\sqrt{1-x^2})$ … (1)

Let x = sin A … (2)

Substituting (2) in (1),

LHS $= \sin^{-1}(2\sin A\sqrt{1-\sin^2 A})$

= $\sin^{-1}$ (2 × sin A × cos A)

= $\sin^{-1}$ (sin 2A)

= 2A

From (2), A = $\sin^{-1}$ x,

2A = 2 $\sin^{-1}$ x

= RHS

Therefore, LHS = RHS

Hence proved.

3 A. Question

Prove that:

$$\sin^{-1}\left(3x-4x^3\right)=3\sin^{-1}x, |x|\le\frac{1}{2}$$

Answer

To Prove: $\sin^{-1}(3x - 4x^3) = 3\sin^{-1}x$

Formula Used: $\sin 3A = 3\sin A - 4\sin^3 A$

Proof:

LHS = $\sin^{-1}(3x - 4x^3)$ … (1)

Let x = sin A … (2)

Substituting (2) in (1),

LHS = $\sin^{-1}(3\sin A - 4\sin^3 A)$

= $\sin^{-1}$ (sin 3A)

= 3A

From (2), A = $\sin^{-1}$ x,

3A = 3 $\sin^{-1}$ x

= RHS

Therefore, LHS = RHS

Hence proved.

3 B. Question

Prove that:

$\cos^{-1}\left(4x^3 - 3x\right) = 3\cos^{-1} x, \frac{1}{2} \le x \le 1$

Answer

To Prove: $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1} x$

Formula Used: $\cos 3A = 4\cos^3 A - 3\cos A$

Proof:

LHS = $\cos^{-1}(4x^3 - 3x)$ … (1)

Let x = cos A … (2)

Substituting (2) in (1),

LHS = $\cos^{-1}(4\cos^3 A - 3\cos A)$

= $\cos^{-1}(\cos 3A)$

= 3A

From (2), A = $\cos^{-1} x$,

3A = $3\cos^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

3 C. Question

Prove that:

$\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) = 3\tan^{-1} x, |x| < \frac{1}{\sqrt{3}}$

Answer

To Prove: $tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = 3\,tan^{-1} x$

Formula Used: $tan\,3A = \frac{3\,tan\,A - tan^3 A}{1 - 3\,tan^2 A}$

Proof:

LHS $= tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ … (1)

Let x = tan A … (2)

Substituting (2) in (1),

LHS $= tan^{-1}\left(\frac{3\,tan\,A - tan^3 A}{1 - 3\,tan^2 A}\right)$

= $\tan^{-1}(\tan 3A)$

= 3A

From (2), A = $\tan^{-1} x$,

3A = $3\tan^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

3 D. Question

Prove that:

$$\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

Answer

To Prove: $tan^{-1}x+tan^{-1}\left(\frac{2x}{1-x^2}\right)=tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

Formula Used: $tan^{-1}x+tan^{-1}y=tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS $= tan^{-1}x+tan^{-1}\left(\frac{2x}{1-x^2}\right)$ ... (1)

$$= tan^{-1}\left(\frac{x+\left(\frac{2x}{1-x^2}\right)}{1-\left(x\times\left(\frac{2x}{1-x^2}\right)\right)}\right)$$

$$= tan^{-1}\left(\frac{x(1-x^2)+2x}{1-x^2-2x^2}\right)$$

$$= tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

= RHS

Therefore, LHS = RHS

Hence proved.

4 A. Question

Prove that:

$$\cos^{-1}\left(1-2x^2\right)=2\sin^{-1}x$$

Answer

To Prove: $\cos^{-1}(1 - 2x^2) = 2\sin^{-1}x$

Formula Used: $\cos 2A = 1 - 2\sin^2 A$

Proof:

LHS = $\cos^{-1}(1 - 2x^2)$ ... (1)

Let x = sin A ... (2)

Substituting (2) in (1),

LHS = $\cos^{-1}(1 - 2\sin^2 A)$

= $\cos^{-1}(\cos 2A)$

= 2A

From (2), A = $\sin^{-1}x$,

$2A = 2 \sin^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

4 B. Question

Prove that:

$$\cos^{-1}\left(2x^2 - 1\right) = 2\cos^{-1} x$$

Answer

To Prove: $\cos^{-1} (2x^2 - 1) = 2 \cos^{-1} x$

Formula Used: $\cos 2A = 2 \cos^2 A - 1$

Proof:

LHS = $\cos^{-1} (2x^2 - 1)$ … (1)

Let $x = \cos A$ … (2)

Substituting (2) in (1),

LHS = $\cos^{-1} (2 \cos^2 A - 1)$

= $\cos^{-1} (\cos 2A)$

= 2A

From (2), $A = \cos^{-1} x$,

$2A = 2 \cos^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

4 C. Question

Prove that:

$$\sec^{-1}\left(\frac{1}{2x^2 - 1}\right) = 2\cos^{-1} x$$

Answer

To Prove: $\sec^{-1}\left(\frac{1}{2x^2-1}\right) = 2\cos^{-1} x$

Formula Used:

1) $\cos 2A = 2 \cos^2 A - 1$

2) $\cos^{-1} A = \sec^{-1}\left(\frac{1}{A}\right)$

Proof:

LHS = $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

= $\cos^{-1} (2x^2 - 1)$… (1)

Let $x = \cos A$ … (2)

Substituting (2) in (1),

LHS = $\cos^{-1}(2\cos^2 A - 1)$

$= \cos^{-1}(\cos 2A)$

$= 2A$

From (2), $A = \cos^{-1} x$,

$2A = 2\cos^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

4 D. Question

Prove that:

$$\cot^{-1}\left(\sqrt{1+x^2}-x\right)=\frac{\pi}{2}-\frac{1}{2}\cot^{-1}x$$

Answer

To Prove: $\cot^{-1}\left(\sqrt{1+x^2}-x\right)=\frac{\pi}{2}-\frac{1}{2}\cot^{-1}x$

Formula Used:

1) $\tan\left(\frac{\pi}{4}+A\right)=\frac{1+\tan A}{1-\tan A}$

2) $\operatorname{cosec}^2 A = 1 + \cot^2 A$

3) $1-\cos A = 2\sin^2\left(\frac{A}{2}\right)$

4) $\sin A = 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$

Proof:

LHS $= \cot^{-1}\left(\sqrt{1+x^2}-x\right)$

Let $x = \cot A$

LHS $= \cot^{-1}\left(\sqrt{1+\cot^2 A}-\cot A\right)$

$= \cot^{-1}(\operatorname{cosec} A - \cot A)$

$= \cot^{-1}\left(\frac{1-\cos A}{\sin A}\right)$

$= \cot^{-1}\left(\frac{2\sin^2\left(\frac{A}{2}\right)}{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}\right)$

$= \cot^{-1}\left(\tan\left(\frac{A}{2}\right)\right)$

$= \frac{\pi}{2}-\tan^{-1}\left(\tan\left(\frac{A}{2}\right)\right)$

$= \frac{\pi}{2}-\frac{A}{2}$

From (2), $A = \cot^{-1} x$,

$\frac{\pi}{2} - \frac{A}{2} = \frac{\pi}{2} - \frac{1}{2}\cot^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

5 A. Question

Prove that:

$$\tan^{-1}\left(\frac{\sqrt{x} + \sqrt{y}}{1 - \sqrt{xy}}\right) = \tan^{-1}\sqrt{x} + \tan^{-1}\sqrt{y}$$

Answer

To Prove: $\tan^{-1}(\frac{\sqrt{x}+\sqrt{y}}{1-\sqrt{xy}}) = \tan^{-1}\sqrt{x} + \tan^{-1}\sqrt{y}$

We know that, $\tan A + \tan B = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Also, $\tan^{-1}(\frac{A+B}{1-AB}) = \tan^{-1} A + \tan^{-1} B$

Taking A = √x and B = √y

We get,

$$\tan^{-1}(\frac{\sqrt{x}+\sqrt{y}}{1-\sqrt{xy}}) = \tan^{-1}\sqrt{x} + \tan^{-1}\sqrt{y}$$

Hence, Proved.

5 B. Question

Prove that:

$$\tan^{-1}\left(\frac{x + \sqrt{x}}{1 - x^{3/2}}\right) = \tan^{-1} x + \tan^{-1}\sqrt{x}$$

Answer

We know that,

$$\tan^{-1}(\frac{A+B}{1-AB}) = \tan^{-1} A + \tan^{-1} B$$

Now, taking A = x and B = √x

We get,

$$\tan^{-1} x + \tan^{-1}\sqrt{x} = \tan^{-1}(\frac{x+\sqrt{x}}{1-x^{3/2}})$$

As, $x.x^{1/2} = x^{3/2}$

Hence, Proved.

5 C. Question

Prove that:

$$\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \frac{x}{2}$$

Answer

To Prove: $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \frac{x}{2}$

Formula Used:

1) $\sin A = 2 \times \sin\frac{A}{2} \times \cos\frac{A}{2}$

2) $1 + \cos A = 2\cos^2\frac{A}{2}$

Proof:

LHS $= \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

$= \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$= \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{x}{2}\right)$

$= \frac{x}{2}$

= RHS

Therefore LHS = RHS

Hence proved.

6 A. Question

Prove that:

$$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\frac{3}{4}$$

Answer

To Prove: $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\frac{3}{4}$

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS $= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11}$

$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\left(\frac{1}{2}\times\frac{2}{11}\right)}\right)$

$= \tan^{-1}\left(\frac{11+4}{22-2}\right)$

$= \tan^{-1}\frac{15}{20}$

$= \tan^{-1}\frac{3}{4}$

= RHS

Therefore LHS = RHS

Hence proved.

6 B. Question

Prove that:

$$\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$$

Answer

To Prove: $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$

Formula Used: $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS = $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}$

$$=\tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\left(\frac{2}{11}\times\frac{7}{24}\right)}\right)$$

$$=\tan^{-1}\left(\frac{48+77}{264-14}\right)$$

$$=\tan^{-1}\frac{125}{250}$$

$$=\tan^{-1}\frac{1}{2}$$

= RHS

Therefore LHS = RHS

Hence proved.

6 C. Question

Prove that:

$$\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$$

Answer

To Prove: $\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$

Formula Used: $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS = $\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}$

$$=\tan^{-1}1+\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\times\frac{1}{3}\right)}\right)$$

$$=\tan^{-1}1\ +\ \tan^{-1}\left(\frac{5}{6-1}\right)$$

$$=\tan^{-1}1\ +\ \tan^{-1}1$$

$= \frac{\pi}{4} + \frac{\pi}{4}$

$= \frac{\pi}{2}$

= RHS

Therefore LHS = RHS

Hence proved.

6 D. Question

Prove that:

$$2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$$

Answer

To Prove: $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS = $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{3}}{1 - \left(\frac{1}{3} \times \frac{1}{3}\right)}\right) + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{6}{9-1}\right) + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{\frac{3}{4} + \frac{1}{7}}{1 - \left(\frac{3}{4} \times \frac{1}{7}\right)}\right)$

$= \tan^{-1}\left(\frac{21+4}{28-3}\right)$

$= \tan^{-1}\frac{25}{25}$

$= \tan^{-1} 1$

$= \frac{\pi}{4}$

= RHS

Therefore LHS = RHS

Hence proved.

6 E. Question

Prove that:

$\tan^{-1} 2 - \tan^{-1} 1 = \tan^{-1}\frac{1}{3}$

Answer

To Prove: $\tan^{-1} 2 - \tan^{-1} 1 = \tan^{-1}\frac{1}{3}$

Formula Used: $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ where $xy > -1$

Proof:

LHS = $\tan^{-1} 2 - \tan^{-1} 1$

$= \tan^{-1}\left(\frac{2-1}{1+2}\right)$

$= \tan^{-1}\left(\frac{1}{3}\right)$

= RHS

Therefore LHS = RHS

Hence proved.

6 F. Question

Prove that:

$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

Answer

To Prove: $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

Formula Used: $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy > 1$

Proof:

LHS $= \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$

$= \frac{\pi}{4} + \pi + \tan^{-1}\left(\frac{2+3}{1-(2\times3)}\right)$ {since 2 × 3 = 6 > 1}

$= \frac{5\pi}{4} + \tan^{-1}\left(\frac{5}{-5}\right)$

$= \frac{5\pi}{4} + \tan^{-1}(-1)$

$= \frac{5\pi}{4} - \frac{\pi}{4}$

$= \pi$

= RHS

Therefore LHS = RHS

Hence proved.

6 G. Question

Prove that:

$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

Answer

To Prove: $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

Formula Used: $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy<1$

Proof:

LHS $=\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}$

$$=\tan^{-1}\frac{1}{2}+\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\left(\frac{1}{5}\times\frac{1}{8}\right)}\right)$$

$$=\tan^{-1}\frac{1}{2}+\tan^{-1}\left(\frac{8+5}{40-1}\right)$$

$$=\tan^{-1}\frac{1}{2}+\tan^{-1}\left(\frac{13}{39}\right)$$

$$=\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}$$

$$=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\times\frac{1}{3}\right)}\right)$$

$$=\tan^{-1}\left(\frac{3+2}{6-1}\right)$$

$=\tan^{-1}1$

$$=\frac{\pi}{4}$$

= RHS

Therefore LHS = RHS

Hence proved.

6 H. Question

Prove that:

$$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\tan^{1}\frac{4}{3}$$

Answer

To Prove: $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\tan^{-1}\frac{4}{3}\Rightarrow 2\left(\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}\right)=\tan^{-1}\frac{4}{3}$

Formula Used: $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy<1$

Proof:

LHS $=2(\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9})$

$$=2\left(\tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\left(\frac{1}{4}\times\frac{2}{9}\right)}\right)\right)$$

$$=2\tan^{-1}\left(\frac{9+8}{36-2}\right)$$

$= 2\tan^{-1}\frac{17}{34}$

$= 2\tan^{-1}\frac{1}{2}$

$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{2}}{1-\left(\frac{1}{2}\times\frac{1}{2}\right)}\right)$

$= \tan^{-1}\left(\frac{1}{\frac{4-1}{4}}\right)$

$= \tan^{-1}\frac{4}{3}$

= RHS

Therefore LHS = RHS

Hence proved.

7 A. Question

Prove that:

$\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

Answer

To Prove: $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

Formula Used: $\cos^{-1}x + \cos^{-1}y = \cos^{-1}(xy - \sqrt{1-x^2} \times \sqrt{1-y^2})$

Proof:

LHS $= \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$

$= \cos^{-1}\left(\frac{4}{5}\times\frac{12}{13} - \sqrt{1-\left(\frac{4}{5}\right)^2} \times \sqrt{1-\left(\frac{12}{13}\right)^2}\right)$

$= \cos^{-1}\left(\frac{48}{65} - \sqrt{1-\frac{16}{25}} \times \sqrt{1-\frac{144}{169}}\right)$

$= \cos^{-1}\left(\frac{48}{65} - \left(\sqrt{\frac{25-16}{25}} \times \sqrt{\frac{169-144}{169}}\right)\right)$

$= \cos^{-1}\left(\frac{48}{65} - \left(\sqrt{\frac{9}{25}} \times \sqrt{\frac{25}{169}}\right)\right)$

$= \cos^{-1}\left(\frac{48}{65} - \frac{3}{13}\right)$

$= \cos^{-1}\left(\frac{48-15}{65}\right)$

$= \cos^{-1}\frac{33}{65}$

= RHS

Therefore, LHS = RHS

Hence proved.

7 B. Question

Prove that:

$$\sin^{-1}\frac{1}{\sqrt{5}}+\sin^{-1}\frac{2}{\sqrt{5}}=\frac{\pi}{2}$$

Answer

To Prove: $\sin^{-1}\frac{1}{\sqrt{5}}+\sin^{-1}\frac{2}{\sqrt{5}}=\frac{\pi}{2}$

Formula Used: $\sin^{-1}x+\sin^{-1}y=\sin^{-1}(x\times\sqrt{1-y^2}+y\times\sqrt{1-x^2})$

Proof:

LHS $= \sin^{-1}\frac{1}{\sqrt{5}}+\sin^{-1}\frac{2}{\sqrt{5}}$

$= \sin^{-1}\left(\frac{1}{\sqrt{5}}\times\sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2}+\frac{2}{\sqrt{5}}\times\sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2}\right)$

$= \sin^{-1}\left(\frac{1}{\sqrt{5}}\times\sqrt{1-\frac{4}{5}}+\frac{2}{\sqrt{5}}\times\sqrt{1-\frac{1}{5}}\right)$

$= \sin^{-1}\left(\frac{1}{\sqrt{5}}\times\frac{1}{\sqrt{5}}+\frac{2}{\sqrt{5}}\times\frac{2}{\sqrt{5}}\right)$

$= \sin^{-1}\left(\frac{1}{5}+\frac{4}{5}\right)$

$= \sin^{-1}\frac{5}{5}$

$= \sin^{-1} 1$

$= \frac{\pi}{2}$

= RHS

Therefore, LHS = RHS

Hence proved.

7 C. Question

Prove that:

$$\cos^{-1}\frac{3}{5}+\sin^{-1}\frac{12}{13}=\sin^{-1}\frac{56}{65}$$

Answer

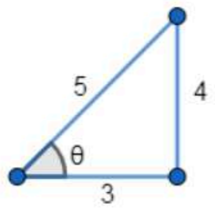
To Prove: $\cos^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13} = \sin^{-1}\frac{56}{65}$

Formula Used: $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x \times \sqrt{1-y^2} + y \times \sqrt{1-x^2})$

Proof:

LHS $= \cos^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13}$ ... (1)

Let $\cos\theta = \frac{3}{5}$



Therefore $\theta = \cos^{-1}\frac{3}{5}$ ... (2)

From the figure, $\sin\theta = \frac{4}{5}$

$\Rightarrow \theta = \sin^{-1}\frac{4}{5}$ ... (3)

From (2) and (3),

$\cos^{-1}\frac{3}{5} = \sin^{-1}\frac{4}{5}$

Substituting in (1), we get

LHS $= \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{12}{13}$

$= \sin^{-1}\left(\frac{4}{5} \times \sqrt{1-\left(\frac{12}{13}\right)^2} + \frac{12}{13} \times \sqrt{1-\left(\frac{4}{5}\right)^2}\right)$

$= \sin^{-1}\left(\frac{4}{5} \times \sqrt{1-\frac{144}{169}} + \frac{12}{13} \times \sqrt{1-\frac{16}{25}}\right)$

$= \sin^{-1}\left(\frac{4}{5} \times \sqrt{\frac{25}{169}} + \frac{12}{13} \times \sqrt{\frac{9}{25}}\right)$

$= \sin^{-1}\left(\frac{4}{5} \times \frac{5}{13} + \frac{12}{13} \times \frac{3}{5}\right)$

$= \sin^{-1}\left(\frac{20}{65} + \frac{36}{65}\right)$

$= \sin^{-1}\frac{56}{65}$

= RHS

Therefore, LHS = RHS

Hence proved.

7 D. Question

Prove that:

$$\cos^{-1}\frac{4}{5}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{27}{11}$$

Answer

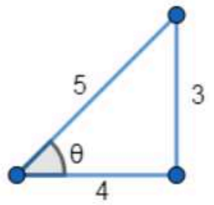
To Prove: $\cos^{-1}\frac{4}{5}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{27}{11}$

Formula Used: $\sin^{-1}x+\sin^{-1}y=\sin^{-1}(x\times\sqrt{1-y^2}\ +y\times\sqrt{1-x^2})$

Proof:

LHS $=\cos^{-1}\frac{4}{5}+\sin^{-1}\frac{3}{5}$ ... (1)

Let $\cos\theta=\frac{4}{5}$



Therefore $\theta=\cos^{-1}\frac{4}{5}$ ... (2)

From the figure, $\sin\theta=\frac{3}{5}$

$\Rightarrow\theta=\sin^{-1}\frac{3}{5}$ ... (3)

From (2) and (3),

$$\cos^{-1}\frac{4}{5}=\sin^{-1}\frac{3}{5}$$

Substituting in (1), we get

LHS $=\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{3}{5}$

$$=\sin^{-1}\left(2\times\frac{3}{5}\times\sqrt{1-\left(\frac{3}{5}\right)^2}\right)$$

$$=\sin^{-1}\left(2\times\frac{3}{5}\times\sqrt{1-\frac{9}{25}}\right)$$

$$=\sin^{-1}\left(2\times\frac{3}{5}\times\sqrt{\frac{16}{25}}\right)$$

$$=\sin^{-1}\left(2\times\frac{3}{5}\times\frac{4}{5}\right)$$

$$=\sin^{-1}\frac{24}{25}$$

7 E. Question

Prove that:

$$\tan^{-1}\frac{1}{3}+\sec^{-1}\frac{\sqrt{5}}{2}=\frac{\pi}{4}$$

Answer

To Prove: $\tan^{-1}\frac{1}{3}+\sec^{-1}\frac{\sqrt{5}}{2}=\frac{\pi}{4}$

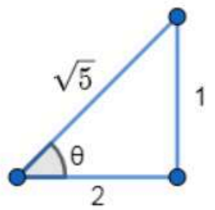
Formula Used: $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy<1$

Proof:

LHS $=\tan^{-1}\frac{1}{3}+\sec^{-1}\frac{\sqrt{5}}{2}$ ... (1)

Let $\sec\theta=\frac{\sqrt{5}}{2}$

Therefore $\theta=\sec^{-1}\frac{\sqrt{5}}{2}$ ... (2)



From the figure, $\tan\theta=\frac{1}{2}$

$\Rightarrow\theta=\tan^{-1}\frac{1}{2}$ ... (3)

From (2) and (3),

$\sec^{-1}\frac{\sqrt{5}}{2}=\tan^{-1}\frac{1}{2}$

Substituting in (1), we get

LHS $=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$

$=\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\left(\frac{1}{3}\times\frac{1}{2}\right)}\right)$

$=\tan^{-1}\left(\frac{2+3}{6-1}\right)$

$=\tan^{-1}\frac{5}{5}$

$=\tan^{-1}1$

$=\frac{\pi}{4}$

= RHS

Therefore, LHS = RHS

Hence proved.

7 F. Question

Prove that:

$\sin^{-1}\frac{1}{\sqrt{17}}+\cos^{-1}\frac{9}{\sqrt{85}}=\tan^{-1}\frac{1}{2}$

Answer

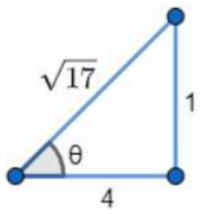
To Prove: $\sin^{-1}\frac{1}{\sqrt{17}} + \cos^{-1}\frac{9}{\sqrt{85}} = \tan^{-1}\frac{1}{2}$

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy < 1$

Proof:

LHS $= \sin^{-1}\frac{1}{\sqrt{17}} + \cos^{-1}\frac{9}{\sqrt{85}}$ ... (1)

Let $\sin\theta = \frac{1}{\sqrt{17}}$



Therefore $\theta = \sin^{-1}\frac{1}{\sqrt{17}}$ ... (2)

From the figure, $\tan\theta = \frac{1}{4}$

$\Rightarrow \theta = \tan^{-1}\frac{1}{4}$ ... (3)

From (2) and (3),

$\sin^{-1}\frac{1}{\sqrt{17}} = \tan^{-1}\frac{1}{4}$ ... (3)

Now, let $\cos\theta = \frac{9}{\sqrt{85}}$

Therefore $\theta = \cos^{-1}\frac{9}{\sqrt{85}}$ ... (4)

From the figure, $\tan\theta = \frac{2}{9}$

$\Rightarrow \theta = \tan^{-1}\frac{2}{9}$ ... (5)

From (4) and (5),

$\cos^{-1}\frac{9}{\sqrt{85}} = \tan^{-1}\frac{2}{9}$ ... (6)

Substituting (3) and (6) in (1), we get

LHS $= \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}$

$= \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\left(\frac{1}{4}\times\frac{2}{9}\right)}\right)$

$= \tan^{-1}\left(\frac{9+8}{36-2}\right)$

$= \tan^{-1}\frac{17}{34}$

$= \tan^{-1}\frac{1}{2}$

= RHS

Therefore, LHS = RHS

Hence proved.

7 G. Question

Prove that:

$$2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$$

Answer

To Prove: $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$

Formula Used:

1) $2\sin^{-1}x=\sin^{-1}(2x\times\sqrt{1-x^2})$

2) $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy<1$

Proof:

LHS $=2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}$ … (1)

$$2\sin^{-1}\frac{3}{5}=\sin^{-1}\left(2\times\frac{3}{5}\times\sqrt{1-\left(\frac{3}{5}\right)^2}\right)$$

$$=\sin^{-1}\left(\frac{6}{5}\times\frac{4}{5}\right)$$

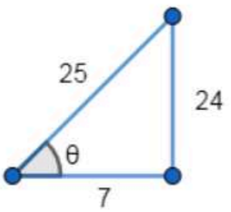
$=\sin^{-1}\frac{24}{25}$ … (2)

Substituting (2) in (1), we get

LHS $=\sin^{-1}\frac{24}{25}-\tan^{-1}\frac{17}{31}$ … (3)

Let $\sin\theta=\frac{24}{25}$

Therefore $\theta=\sin^{-1}\frac{24}{25}$ … (4)



From the figure, $\tan\theta=\frac{24}{7}$

$\Rightarrow\theta=\tan^{-1}\frac{24}{7}$ … (5)

From (4) and (5),

$\sin^{-1}\frac{24}{25}=\tan^{-1}\frac{24}{7}$ … (6)

Substituting (6) in (3), we get

LHS $=\tan^{-1}\frac{24}{7}-\tan^{-1}\frac{17}{31}$

$$=\tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\left(\frac{24}{7}\times\frac{17}{31}\right)}\right)$$

$= \tan^{-1}\left(\frac{744 - 119}{217 + 408}\right)$

$= \tan^{-1}\frac{625}{625}$

$= \tan^{-1} 1$

$= \frac{\pi}{4}$

= RHS

Therefore, LHS = RHS

Hence proved.

8 A. Question

Solve for x:

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$$

Answer

To find: value of x

Formula Used: $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy < 1$

Given: $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$

LHS $= \tan^{-1}\left(\frac{x+1+x-1}{1-\{(x+1)\times(x-1)\}}\right)$

$= \tan^{-1}\frac{2x}{1-(x^2 - x + x - 1)}$

$= \tan^{-1}\frac{2x}{2 - x^2}$

Therefore, $\tan^{-1}\frac{2x}{2-x^2} = \tan^{-1}\frac{8}{31}$

Taking tangent on both sides, we get

$\frac{2x}{2 - x^2} = \frac{8}{31}$

$\Rightarrow 62x = 16 - 8x^2$

$\Rightarrow 8x^2 + 62x - 16 = 0$

$\Rightarrow 4x^2 + 31x - 8 = 0$

$\Rightarrow 4x^2 + 32x - x - 8 = 0$

$\Rightarrow 4x \times (x + 8) - 1 \times (x + 8) = 0$

$\Rightarrow (4x - 1) \times (x + 8) = 0$

$\Rightarrow x = \frac{1}{4}$ or $x = -8$

Therefore, $x = \frac{1}{4}$ or $x = -8$ are the required values of x.

8 B. Question

Solve for x:

$$\tan^{-1}(2+x)+\tan^{-1}(2-x)=\tan^{-1}\frac{2}{3}$$

Answer

To find: value of x

Formula Used: $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy<1$

Given: $\tan^{-1}(2+x)+\tan^{-1}(2-x)=\tan^{-1}\frac{2}{3}$

LHS $=\tan^{-1}\left(\frac{2+x+2-x}{1-\{(2+x)\times(2-x)\}}\right)$

$$=\tan^{-1}\frac{4}{1-(4-2x+2x-x^2)}$$

$$=\tan^{-1}\frac{4}{x^2-3}$$

Therefore, $\tan^{-1}\frac{4}{x^2-3}=\tan^{-1}\frac{2}{3}$

Taking tangent on both sides, we get

$$\frac{4}{x^2-3}=\frac{2}{3}$$

$\Rightarrow 12 = 2x^2 - 6$

$\Rightarrow 2x^2 = 18$

$\Rightarrow x^2 = 9$

$\Rightarrow x = 3$ or $x = -3$

Therefore, $x = \pm 3$ are the required values of x.

8 C. Question

Solve for x:

$$\cos\left(\sin^{-1}x\right)=\frac{1}{9}$$

Answer

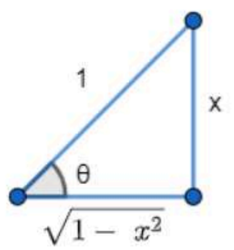
To find: value of x

Given: $\cos(\sin^{-1}x)=\frac{1}{9}$

LHS = $\cos(\sin^{-1} x)$ … (1)

Let $\sin\theta = x$

Therefore $\theta = \sin^{-1} x$ … (2)



From the figure, $\cos\theta=\sqrt{1-x^2}$

$\Rightarrow \theta = \cos^{-1}\sqrt{1-x^2}$ … (3)

From (2) and (3),

$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2}$ … (4)

Substituting (4) in (1), we get

LHS $= \cos(\cos^{-1}\sqrt{1-x^2})$

$= \sqrt{1-x^2}$

Therefore, $\sqrt{1-x^2} = \frac{1}{9}$

Squaring and simplifying,

$\Rightarrow 81 - 81x^2 = 1$

$\Rightarrow 81x^2 = 80$

$\Rightarrow x^2 = \frac{80}{81}$

$\Rightarrow x = \pm\frac{4\sqrt{5}}{9}$

Therefore, $x = \pm\frac{4\sqrt{5}}{9}$ are the required values of x.

8 D. Question

Solve for x:

$\cos\left(2\sin^{-1} x\right) = \frac{1}{9}$

Answer

To find: value of x

Formula Used: $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$

Given: $\cos(2\sin^{-1} x) = \frac{1}{9}$

LHS = $\cos(2\sin^{-1} x)$

Let $\theta = \sin^{-1} x$

So, $x = \sin\theta$ … (1)

LHS = $\cos(2\theta)$

$= 1 - 2\sin^2\theta$

Substituting in the given equation,

$1 - 2\sin^2\theta = \frac{1}{9}$

$2\sin^2\theta = \frac{8}{9}$

$\sin^2\theta = \frac{4}{9}$

Substituting in (1),

$x^2 = \frac{4}{9}$

$x = \pm\frac{2}{3}$

Therefore, $x = \pm\frac{2}{3}$ are the required values of x.

8 E. Question

Solve for x:

$$\sin^{-1}\frac{8}{x} + \sin^{-1}\frac{15}{x} = \frac{\pi}{2}$$

Answer

To find: value of x

Given: $\sin^{-1}\frac{8}{x} + \sin^{-1}\frac{15}{x} = \frac{\pi}{2}$

We know $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

Let $\sin^{-1}\frac{8}{x} = P$

$\Rightarrow \sin P = \frac{8}{x}$

Therefore, $\cos P = \frac{\sqrt{x^2-64}}{x}$

$P = \cos^{-1}\frac{\sqrt{x^2-64}}{x}$

$\cos^{-1}\frac{\sqrt{x^2-64}}{x} + \sin^{-1}\frac{15}{x} = \frac{\pi}{2}$

Therefore, $\frac{\sqrt{x^2-64}}{x} = \frac{15}{x}$

$\Rightarrow \sqrt{x^2-64} = 15$

Squaring both sides,

$\Rightarrow x^2 - 64 = 225$

$\Rightarrow x^2 = 289$

$\Rightarrow x = \pm 17$

Therefore, $x = \pm 17$ are the required values of x.

9 A. Question

Solve for x :

$$\cos\left(\sin^{-1}x\right) = \frac{1}{2}$$

Answer

To find: value of x

Given: $\cos(\sin^{-1}x) = \frac{1}{2}$

LHS $= \cos(\sin^{-1}x)$

$= \cos(\cos^{-1}(\sqrt{1-x^2}))$

$= \sqrt{1-x^2}$

Therefore, $\sqrt{1-x^2} = \frac{1}{2}$

Squaring both sides,

$1 - x^2 = \frac{1}{4}$

$x^2 = 1 - \frac{1}{4}$

$x^2 = \frac{3}{4}$

$x = \pm\frac{\sqrt{3}}{2}$

Therefore, $x = \pm\frac{\sqrt{3}}{2}$ are the required values of x.

9 B. Question

Solve for x :

$$\tan^{-1} x = \sin^{-1}\frac{1}{\sqrt{2}}$$

Answer

To find: value of x

Given: $\tan^{-1} x = \sin^{-1}\frac{1}{\sqrt{2}}$

We know that $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Therefore, $\frac{\pi}{4} = \sin^{-1}\frac{1}{\sqrt{2}}$

Substituting in the given equation,

$\tan^{-1} x = \frac{\pi}{4}$

$x = \tan\frac{\pi}{4}$

$\Rightarrow x = 1$

Therefore, x = 1 is the required value of x.

9 C. Question

Solve for x :

$$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$$

Answer

Given: $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$

We know that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

So, $\sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x$

Substituting in the given equation,

$$\frac{\pi}{2} - \cos^{-1}x - \cos^{-1}x = \frac{\pi}{6}$$

Rearranging,

$$2\cos^{-1}x = \frac{\pi}{2} - \frac{\pi}{6}$$

$$2\cos^{-1}x = \frac{\pi}{3}$$

$$\cos^{-1}x = \frac{\pi}{6}$$

$$x = \frac{\sqrt{3}}{2}$$

Therefore, $x = \frac{\sqrt{3}}{2}$ is the required value of x.

Exercise 4D

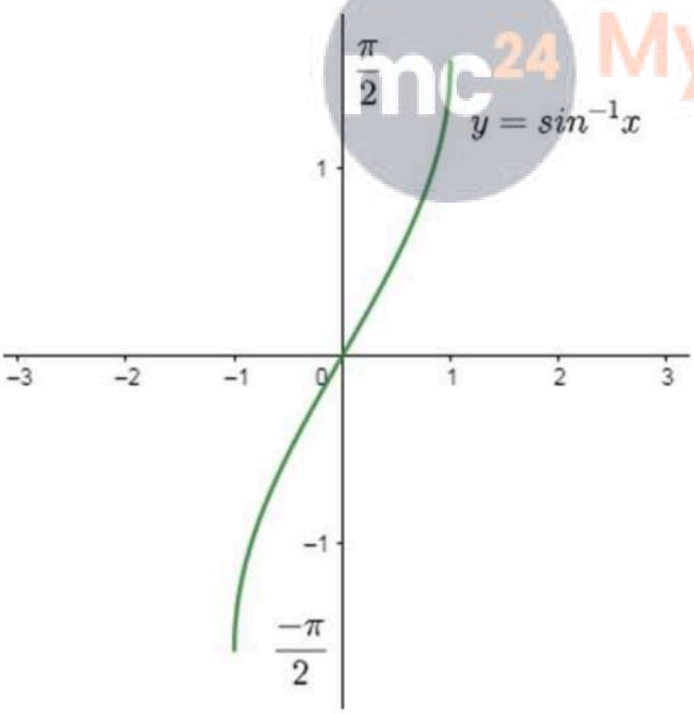
1. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\sin^{-1} x$

Answer

Principal value branch of $\sin^{-1} x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$



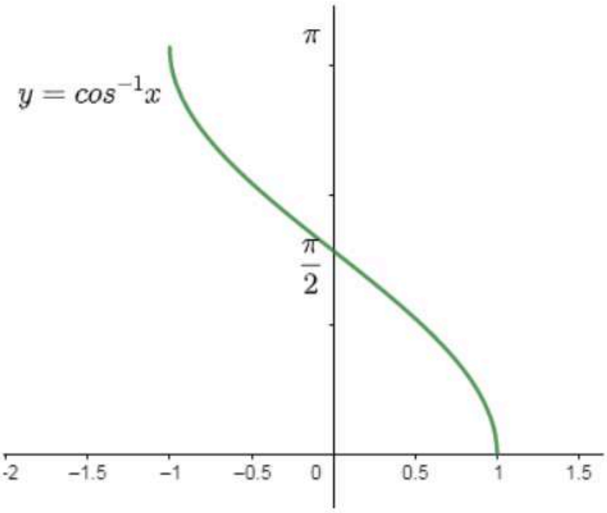
2. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\cos^{-1} x$

Answer

Principal value branch of $\cos^{-1} x$ is $[0, \pi]$



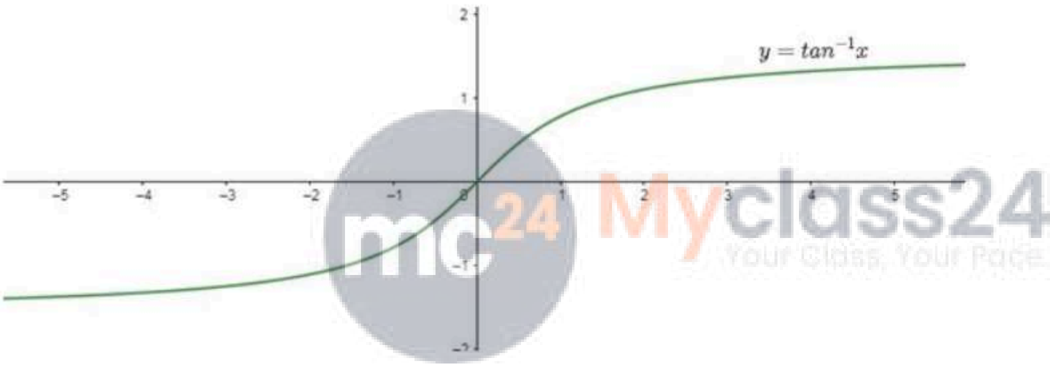
3. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\tan^{-1} x$

Answer

Principal value branch of $\tan^{-1} x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$



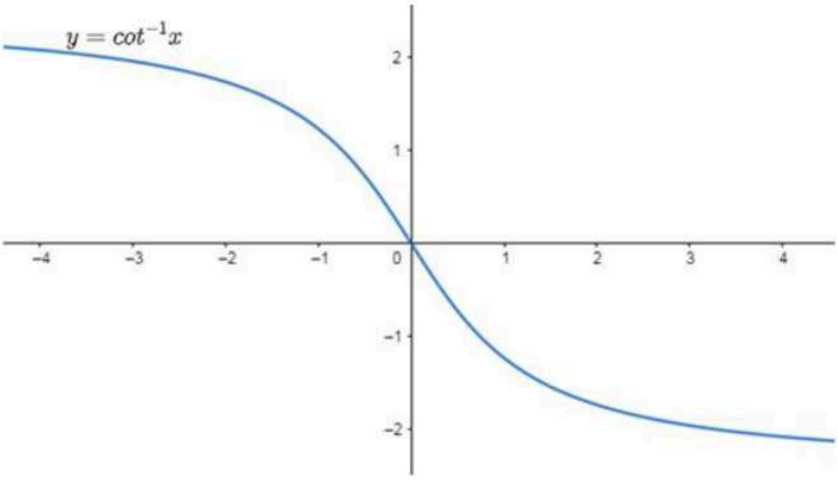
4. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\cot^{-1} x$

Answer

Principal value branch of $\cot^{-1} x$ is $(0, \pi)$



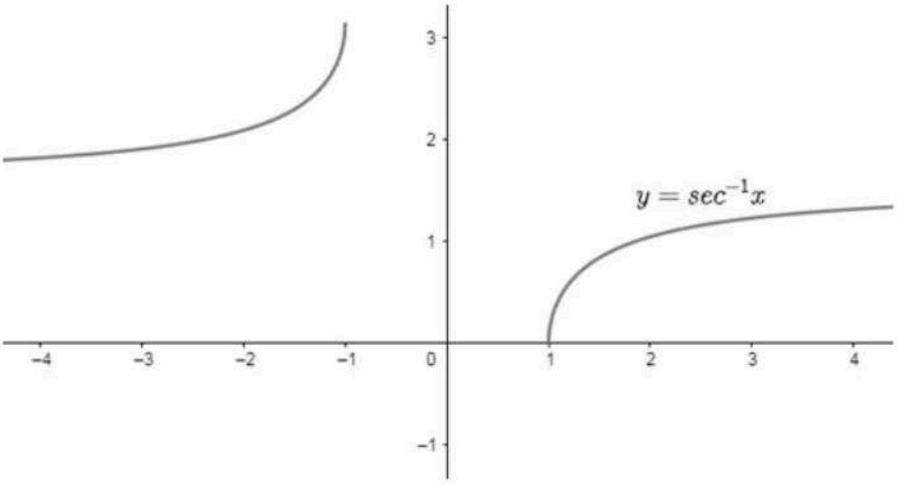
5. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\sec^{-1} x$

Answer

Principal value branch of $\sec^{-1} x$ is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



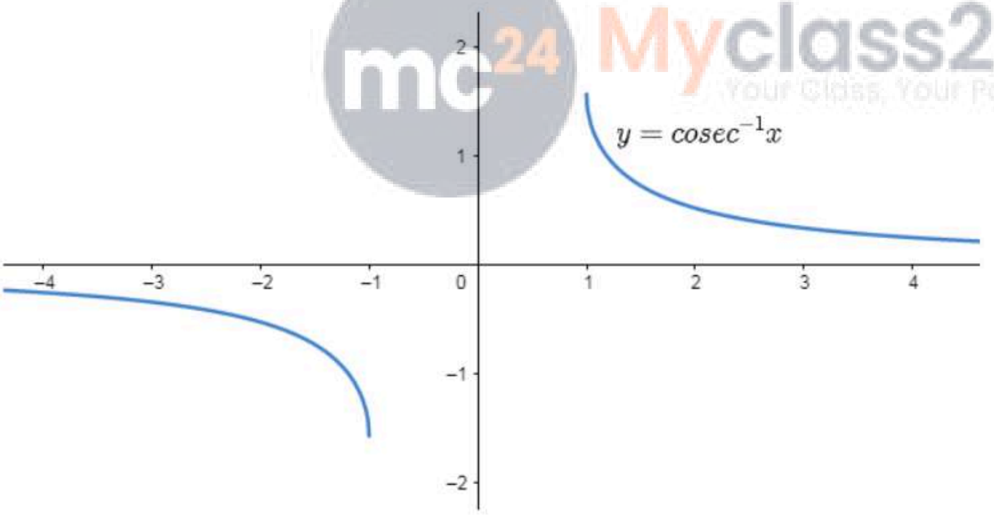
6. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\text{cosec}^{-1} x$

Answer

Principal value branch of $\text{cosec}^{-1} x$ is $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



Objective Questions

1. Question

Mark the tick against the correct answer in the following:

The principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is

A. $\frac{\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{7\pi}{6}$

D. none of these

Answer

To Find:The Principle value of $\cos^{-1}(\frac{\sqrt{3}}{2})$

Let the principle value be given by x

Now, let x = $\cos^{-1}(\frac{\sqrt{3}}{2})$

⇒ cos x=$\frac{\sqrt{3}}{2}$

⇒ cos x=cos($\frac{\pi}{6}$) ($\because \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$)

⇒ x = $\frac{\pi}{6}$

2. Question

Mark the tick against the correct answer in the following:

The principal value of $\text{cosec}^{-1}(2)$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer

To Find: The Principle value of $\text{cosec}^{-1}(2)$

Let the principle value be given by x

Now, let x = $\text{cosec}^{-1}(2)$

⇒ cosec x =2

⇒ cosec x=cosec($\frac{\pi}{6}$) ($\because \cos\left(\frac{\pi}{6}\right) = 2$)

⇒ x = $\frac{\pi}{6}$

3. Question

Mark the tick against the correct answer in the following:

The principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is

A. $\frac{-\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{4}$

Answer

To Find: The Principle value of $\cos^{-1}(\frac{-1}{\sqrt{2}})$

Let the principle value be given by x

Now, let x = $\cos^{-1}(\frac{-1}{\sqrt{2}})$

⇒ cos x =$\frac{-1}{\sqrt{2}}$

⇒ cos x= - cos($\frac{\pi}{4}$) ($\because \cos(\frac{\pi}{4})=\frac{1}{\sqrt{2}}$)

⇒ cos x=cos($\pi - \frac{\pi}{4}$) ($\because -\cos(\theta) = \cos(\pi - \theta\ )$)

⇒ x = $\frac{3\pi}{4}$

4. Question

Mark the tick against the correct answer in the following:

The principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is

A. $\frac{-\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{7\pi}{6}$

D. none of these

Answer

To Find: The Principle value of $\sin^{-1}(\frac{-1}{2})$

Let the principle value be given by x

Now, let x = $\sin^{-1}(\frac{-1}{2})$

⇒ sin x =$\frac{-1}{2}$

⇒ sin x= - sin($\frac{\pi}{6}$) ($\because \sin(\frac{\pi}{6})=\frac{1}{2}$)

⇒ sin x=sin($-\frac{\pi}{6}$) ($\because -\sin(\theta) = \sin(-\theta\ )$)

⇒ x = $-\frac{\pi}{4}$

5. Question

Mark the tick against the correct answer in the following:

The principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is

A. $\frac{-\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{4\pi}{3}$

D. $\frac{\pi}{3}$

Answer

To Find: The Principle value of $\cos^{-1}(\frac{-1}{2})$

Let the principle value be given by x

Now, let x = $\cos^{-1}(\frac{-1}{2})$

$\Rightarrow \cos x = \frac{-1}{2}$

$\Rightarrow \cos x = -\cos(\frac{\pi}{3})$ ($\because \cos(\frac{\pi}{3}) = \frac{1}{2}$)

$\Rightarrow \cos x = \cos(\pi - \frac{\pi}{3})$ ($\because -\cos(\theta) = \cos(\pi - \theta)$)

$\Rightarrow x = \frac{2\pi}{3}$

6. Question

Mark the tick against the correct answer in the following:

The principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ is

A. $\frac{2\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{-\pi}{3}$

D. none of these

Answer

To Find: The Principle value of $\tan^{-1}(-\sqrt{3})$

Let the principle value be given by x

Now, let x = $\tan^{-1}(-\sqrt{3})$

$\Rightarrow$ tan x $=-\sqrt{3}$

$\Rightarrow$ tan x= - tan($\frac{\pi}{3}$) ($\because \tan(\frac{\pi}{3}) = -\sqrt{3}$)

$\Rightarrow$ $\tan x = \tan(-\frac{\pi}{3})$ ($\because -\tan(\theta) = \tan(-\theta)$)

$\Rightarrow$ x $= -\frac{\pi}{3}$

7. Question

Mark the tick against the correct answer in the following:

The principal value of $\cot^{-1}$ (-1) is

A. $\frac{-\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{5\pi}{4}$

D. $\frac{3\pi}{4}$

Answer

To Find: The Principle value of $\cot^{-1}(-1)$

Let the principle value be given by x

Now, let x = $\cot^{-1}(-1)$

$\Rightarrow$ cot x =-1

$\Rightarrow$ cot x= - cot($\frac{\pi}{4}$) ($\because \cot(\frac{\pi}{4}) = 1$)

$\Rightarrow$ cot x=cot($\pi - \frac{\pi}{4}$) ($\because -\cot(\theta) = \cot(\pi - \theta)$)

$\Rightarrow$ x $= \frac{3\pi}{4}$

8. Question

Mark the tick against the correct answer in the following:

The principal value of $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is

A. $\frac{\pi}{6}$

B. $\frac{-\pi}{6}$

C. $\frac{5\pi}{6}$

D. $\frac{7\pi}{6}$

Answer

To Find: The Principle value of $\sec^{-1}(\frac{-2}{\sqrt{3}})$

Let the principle value be given by x

Now, let x = $\sec^{-1}(\frac{-2}{\sqrt{3}})$

⇒ sec x =$\frac{-2}{\sqrt{3}}$

⇒ sec x= - sec($\frac{\pi}{6}$) ($\because$ $\sec(\frac{\pi}{6})= \frac{2}{\sqrt{3}}$)

⇒ sec x=sec($\pi - \frac{\pi}{6}$) ($\because -\sec(\theta) = \sec(\pi - \theta\,)$)

⇒ x = $\frac{5\pi}{6}$

9. Question

Mark the tick against the correct answer in the following:

The principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is

A. $\frac{-\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{5\pi}{4}$

D. none of these

Answer

To Find: The Principle value of $\operatorname{cosec}^{-1}(-\sqrt{2})$

Let the principle value be given by x

Now, let x = $\operatorname{cosec}^{-1}(-\sqrt{2})$

⇒ cosec x =$-\sqrt{2}$

⇒ cosec x= - cosec($\frac{\pi}{4}$) ($\because$ $cosec(\frac{\pi}{4})= \sqrt{2}$)

⇒ cosec x=cosec($-\frac{\pi}{4}$) ($\because -cosec(\theta) = \operatorname{cosec}(-\theta\,)$)

⇒ x = $-\frac{\pi}{4}$

10. Question

Mark the tick against the correct answer in the following:

The principal value of $\cot^{-1}\left(-\sqrt{3}\right)$ is

A. $\frac{2\pi}{6}$

B. $\frac{\pi}{6}$

C. $\frac{7\pi}{6}$

D. $\frac{5\pi}{6}$

Answer

To Find: The Principle value of $\cot^{-1}(-\sqrt{3})$

Let the principle value be given by x

Now, let x = $\cot^{-1}(-\sqrt{3})$

⇒ cot x =$-\sqrt{3}$

⇒ cot x= - cot($\frac{\pi}{6}$) (∵ $cot(\frac{\pi}{6})= \sqrt{3}$)

⇒ cot x=cot($\pi - \frac{\pi}{6}$) (∵ $-cot(\theta) = \cot(\pi - \theta)$)

⇒ x = $\frac{5\pi}{6}$

11. Question

Mark the tick against the correct answer in the following:

The value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

A. $\frac{2\pi}{3}$

B. $\frac{5\pi}{3}$

C. $\frac{\pi}{3}$

D. none of these

Answer

To Find: The value of $\sin^{-1}(\sin(\frac{2\pi}{3}))$

Now, let x =$\sin^{-1}(\sin(\frac{2\pi}{3}))$

⇒ sin x =sin ($\frac{2\pi}{3}$)

Here range of principle value of sine is [$-\frac{\pi}{2},\frac{\pi}{2}$]

⇒ x = $\frac{2\pi}{3} \notin [-\frac{\pi}{2},\frac{\pi}{2}]$

Hence for all values of x in range $[-\frac{\pi}{2},\frac{\pi}{2}]$,the value of

$\sin^{-1}(\sin(\frac{2\pi}{3}))$ is

⇒ sin x =sin $(\pi-\frac{\pi}{3})$ (∵sin $(\frac{2\pi}{3})$= sin $(\pi-\frac{\pi}{3})$)

⇒ sin x =sin $(\frac{\pi}{3})$ (∵sin $(\pi-\theta)$= sin θas here θ *lies in II quadrant and sine is positive*)

⇒ x = $\frac{\pi}{3}$

12. Question

Mark the tick against the correct answer in the following:

The value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is

A. $\frac{13\pi}{6}$

B.

C. $\frac{5\pi}{6}$

D. $\frac{\pi}{6}$ $\frac{7\pi}{6}$

Answer

To Find: The value of $\cos^{-1}(\cos(\frac{13\pi}{6}))$

Now, let x =$\cos^{-1}(\cos(\frac{13\pi}{6}))$

⇒ cos x =cos $(\frac{13\pi}{6})$

Here ,range of principle value of cos is $[0,\pi]$

⇒ x = $\frac{13\pi}{6}\notin[0,\pi]$

Hence for all values of x in range $[0,\pi]$,the value of

$\cos^{-1}(\cos(\frac{13\pi}{6}))$ is

⇒ cos x =cos $(2\pi-\frac{\pi}{6})$ (∵cos $(\frac{13\pi}{6})$= cos $(2\pi-\frac{\pi}{6})$)

⇒ cos x =cos $(\frac{\pi}{6})$ (∵cos $(2\pi-\theta)$= cos θ)

⇒ x = $\frac{\pi}{6}$

13. Question

Mark the tick against the correct answer in the following:

The value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ is

A. $\frac{7\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{\pi}{6}$

D. none of these

Answer

To Find: The value of $\tan^{-1}(\tan(\frac{7\pi}{6}))$

Now, let x =$\tan^{-1}(\tan(\frac{7\pi}{6}))$

⇒ tan x =tan $(\frac{7\pi}{6})$

Here range of principle value of tan is $[-\frac{\pi}{2},\frac{\pi}{2}]$

⇒ x = $\frac{7\pi}{6} \notin [-\frac{\pi}{2},\frac{\pi}{2}]$

Hence for all values of x in range $[-\frac{\pi}{2},\frac{\pi}{2}]$,the value of

$\tan^{-1}(\tan(\frac{13\pi}{6}))$ is

⇒ tan x =tan $(\pi + \frac{\pi}{6})$ (∵tan $(\frac{7\pi}{6})$= tan $(\pi + \frac{\pi}{6})$)

⇒ tan x =tan $(\frac{\pi}{6})$ (∵tan $(\pi + \theta)$= tan θ)

⇒ x = $\frac{\pi}{6}$

14. Question

Mark the tick against the correct answer in the following:

The value of $\cot^{-1}\left(\cot\frac{5\pi}{4}\right)$ is

A. $\frac{\pi}{4}$

B. $\frac{-\pi}{4}$

C. $\frac{3\pi}{4}$

D. none of these

Answer

To Find: The value of $\cot^{-1}(\cot(\frac{5\pi}{4}))$

Now, let x =$\cot^{-1}(\cot(\frac{5\pi}{4}))$

$\Rightarrow$ cot x =cot $(\frac{5\pi}{4})$

Here range of principle value of cot is $[-\frac{\pi}{2},\frac{\pi}{2}]$

$\Rightarrow$ x = $\frac{5\pi}{4} \notin [-\frac{\pi}{2},\frac{\pi}{2}]$

Hence for all values of x in range $[-\frac{\pi}{2},\frac{\pi}{2}]$,the value of

$\cot^{-1}(\cot(\frac{5\pi}{4}))$ is

$\Rightarrow$ cot x =cot $(\pi + \frac{\pi}{4})$ ($\because$cot $(\frac{5\pi}{4})$= cot $(\pi + \frac{\pi}{4})$)

$\Rightarrow$ cot x =cot $(\frac{\pi}{4})$ ($\because$cot $(\pi + \theta)$= cot θ)

$\Rightarrow$ x = $\frac{\pi}{4}$

15. Question

Mark the tick against the correct answer in the following:

The value of $\sec^{-1}\left(\sec\frac{8\pi}{5}\right)$ is

A. $\frac{2\pi}{5}$

B. $\frac{3\pi}{5}$

C. $\frac{8\pi}{5}$

D. none of these

Answer

To Find: The value of $\sec^{-1}(\sec(\frac{8\pi}{5}))$

Now, let x =$\sec^{-1}(\sec(\frac{8\pi}{5}))$

$\Rightarrow$ sec x =sec $(\frac{8\pi}{5})$

Here range of principle value of sec is $[0,\pi]$

$\Rightarrow$ x = $\frac{8\pi}{5} \notin [0,\pi]$

Hence for all values of x in range $[0,\pi]$,the value of

$\sec^{-1}(\sec(\frac{8\pi}{5}))$ is

$\Rightarrow$ sec x =sec $(2\pi - \frac{2\pi}{5})$ ($\because$sec $(\frac{8\pi}{5})$= sec $(2\pi - \frac{2\pi}{5})$)

$\Rightarrow$ sec x =sec $(\frac{2\pi}{5})$ ($\because$sec $(2\pi - \theta)$= sec θ)

$\Rightarrow$ x = $\frac{2\pi}{5}$

16. Question

Mark the tick against the correct answer in the following:

The value of $\text{cosec}^{-1}\left(\text{cosec}\frac{4\pi}{3}\right)$ is

A. $\frac{\pi}{3}$

B. $\frac{-\pi}{3}$

C. $\frac{2\pi}{3}$

D. none of these

Answer

To Find: The value of $\text{cosec}^{-1}(\text{cosec}(\frac{4\pi}{3}))$

Now, let x =$\text{cosec}^{-1}(\text{cosec}(\frac{4\pi}{3}))$

⇒ cosec x =cosec $(\frac{4\pi}{3})$

Here range of principle value of cosec is $[-\frac{\pi}{2},\frac{\pi}{2}]$

⇒ $x = \frac{4\pi}{3} \notin [-\frac{\pi}{2},\frac{\pi}{2}]$

Hence for all values of x in range $[-\frac{\pi}{2},\frac{\pi}{2}]$,the value of

$\text{cosec}^{-1}(\text{cosec}(\frac{4\pi}{3}))$ is

⇒ cosec x =cosec $(\pi + \frac{\pi}{3})$ (∵cosec $(\frac{4\pi}{3})$= cosec $(\pi + \frac{\pi}{3})$)

⇒ cosec x =cosec $(-\frac{\pi}{3})$ (∵cosec $(\pi + \theta)$= cosec$(-\theta)$)

⇒ $x = -\frac{\pi}{3}$

17. Question

Mark the tick against the correct answer in the following:

The value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{-\pi}{4}$

D. none of these

Answer

To Find: The value of $\tan^{-1}(\tan(\frac{3\pi}{4}))$

Now, let x $=\tan^{-1}(\tan(\frac{3\pi}{4}))$

$\Rightarrow$ tan x =tan $(\frac{3\pi}{4})$

Here range of principle value of tan is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\Rightarrow$ x $= \frac{3\pi}{4} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$

Hence for all values of x in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$,the value of

$\tan^{-1}(\tan(\frac{3\pi}{4}))$ is

$\Rightarrow$ tan x =tan $(\pi - \frac{\pi}{4})$ ($\because$tan $(\frac{3\pi}{4})$= tan $(\pi - \frac{\pi}{4})$)

$\Rightarrow$ tan x =tan $(-\frac{\pi}{4})$ ($\because$tan $(\pi - \theta)$= tan$(-\theta)$)

$\Rightarrow$ x $= -\frac{\pi}{4}$

18. Question

Mark the tick against the correct answer in the following:

$$\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right) = ?$$

A. 0

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{2}$

D. π

Answer

To Find: The value of $\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})$

Now, let x $=\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})$

$\Rightarrow$ x $=\frac{\pi}{3} - (-\sin^{-1}(\frac{1}{2}))$ ($\because \sin(-\theta) = -\sin(\theta)$)

$\Rightarrow$ x $= \frac{\pi}{3} - (-\frac{\pi}{6})$ ($\because sin\ \frac{\pi}{6} = \frac{1}{2}$)

$\Rightarrow$ x $= \frac{\pi}{3} + \frac{\pi}{6}$

$\Rightarrow$ x $= \frac{3\pi}{6} = \frac{\pi}{2}$

19. Question

Mark the tick against the correct answer in the following:

The value of $\sin\left(\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}\right)=?$

A. 0

B. 1

C. -1

D. none of these

Answer

To Find: The value of $\sin(\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2})$

Now, let x = $\sin(\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2})$

⇒ x = sin $(\frac{\pi}{2})$ ($\because \sin^{-1}\theta+\cos^{-1}\theta=\frac{\pi}{2}$)

⇒ x = 1 ($\because \sin\left(\frac{\pi}{2}\right)=1$)

20. Question

Mark the tick against the correct answer in the following:

If x ≠ 0 then cos ($\tan^{-1}$ x + $\cot^{-1}$ x) = ?

A. -1

B. 1

C. 0

D. none of these

Answer

Given: x ≠ 0

To Find: The value of $\cos(\tan^{-1}x+\cot^{-1}x)$

Now, let x = $\cos(\tan^{-1}x+\cot^{-1}x)$

⇒ x = cos $(\frac{\pi}{2})$ ($\because \tan^{-1}\theta+\cot^{-1}\theta=\frac{\pi}{2}$)

⇒ x = 0 ($\because \cos\left(\frac{\pi}{2}\right)=0$)

21. Question

Mark the tick against the correct answer in the following:

The value of $\sin\left(\cos^{-1}\frac{3}{5}\right)$ is

A. $\frac{2}{5}$

B. $\frac{4}{5}$

C. $\frac{-2}{5}$

D. none of these

Answer

To Find: The value of sin($\cos^{-1}\frac{3}{5}$)

Now, let x = $\cos^{-1}\frac{3}{5}$

⇒ cos x = $\frac{3}{5}$

Now ,sin x = $\sqrt{1-\cos^2 x}$

= $\sqrt{1-(\frac{3}{5})^2}$

= $\frac{4}{5}$

⇒ x = $\sin^{-1}\frac{4}{5}$ = $\cos^{-1}\frac{3}{5}$

Therefore,

sin($\cos^{-1}\frac{3}{5}$) = sin($\sin^{-1}\frac{4}{5}$)

Let , Y= sin($\sin^{-1}\frac{4}{5}$)

⇒ $\sin^{-1}$Y = $\sin^{-1}\frac{4}{5}$

⇒ Y = $\frac{4}{5}$

22. Question

Mark the tick against the correct answer in the following:

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=?$$

A. $\frac{4\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{5\pi}{3}$

D. π

Answer

To Find: The value of $\cos^{-1}(\cos(\frac{2\pi}{3}))+\sin^{-1}(\sin(\frac{2\pi}{3}))$

Here,consider $\cos^{-1}(\cos(\frac{2\pi}{3}))$ ($\because$ *the principle value of* cos *lies in the range* $[0,\pi]$ *and since* $\frac{2\pi}{3}\in[0,\pi]$)

⇒ $\cos^{-1}(\cos(\frac{2\pi}{3}))=\frac{2\pi}{3}$

Now,consider $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

Since here the principle value of sine lies in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and since $\frac{2\pi}{3} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\Rightarrow \sin^{-1}(\sin(\frac{2\pi}{3})) = \sin^{-1}(\sin(\pi - \frac{\pi}{3}))$

$= \sin^{-1}(\sin(\frac{\pi}{3}))$

$= \frac{\pi}{3}$

Therefore,

$\cos^{-1}(\cos(\frac{2\pi}{3})) + \sin^{-1}(\sin(\frac{2\pi}{3})) = \frac{2\pi}{3} + \frac{\pi}{3}$

$= \frac{3\pi}{3}$

$= \pi$

23. Question

Mark the tick against the correct answer in the following:

$$\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2) = ?$$

A. $\frac{\pi}{3}$

B. $\frac{-\pi}{3}$

C. $\frac{5\pi}{3}$

D. none of these

Answer

To Find: The value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

Let , x = $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

$\Rightarrow$x = $\frac{\pi}{3}$ - [π- $\sec^{-1}(2)$] ($\because \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ *and* $\sec^{-1}(-\theta) = \pi - \sec^{-1}(\theta)$)

$\Rightarrow$x = $\frac{\pi}{3}$ - [π - $\frac{\pi}{3}$]

$\Rightarrow$x = $\frac{\pi}{3}$ - [$\frac{2\pi}{3}$]

$\Rightarrow$x = - $\frac{\pi}{3}$

24. Question

Mark the tick against the correct answer in the following:

$$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2} = ?$$

A. $\frac{2\pi}{3}$

B. $\frac{3\pi}{2}$

C. 2π

D. none of these

Answer

To Find: The value of $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$

Now, let x = $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$

$\Rightarrow$x= $\frac{\pi}{3}$ +2($\frac{\pi}{6}$) ($\because$cos ($\frac{\pi}{3}$)= $\frac{1}{2}$ and sin ($\frac{\pi}{6}$)= $\frac{1}{2}$)

$\Rightarrow$x= $\frac{\pi}{3}$ +$\frac{\pi}{3}$

$\Rightarrow$x= $\frac{2\pi}{3}$

25. Question

Mark the tick against the correct answer in the following:

$$\tan^{-1}1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right) = ?$$

A. π

B. $\frac{2\pi}{3}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{2}$

Answer

To Find: The value of $\tan^{-1}1 + \cos^{-1}(\frac{-1}{2}) + \sin^{-1}(\frac{-1}{2})$

Now, let x = $\tan^{-1}1 + \cos^{-1}(\frac{-1}{2}) + \sin^{-1}(\frac{-1}{2})$

$\Rightarrow$ x = $\frac{\pi}{4}$ + [π- $\cos^{-1}(\frac{1}{2})$] + [-$\sin^{-1}\frac{1}{2}$] ($\because \tan\left(\frac{\pi}{4}\right) = 1$ *and* $\cos^{-1}(-\theta) = [\pi - \cos^{-1}\theta]$*and* $\sin^{-1}(-\theta) = -\sin^{-1}\theta$)

$\Rightarrow$ x = $\frac{\pi}{4}$ + [π- $\frac{\pi}{3}$] + [- $\frac{\pi}{6}$]

$\Rightarrow$ x = $\frac{\pi}{4}$ + $\frac{2\pi}{3}$ - $\frac{\pi}{6}$

$\Rightarrow$ x = $\frac{3\pi}{4}$

26. Question

Mark the tick against the correct answer in the following:

$$\tan\left[2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right] = ?$$

A. $\frac{7}{17}$

B. $\frac{-7}{17}$

C. $\frac{7}{12}$

D. $\frac{-7}{12}$

Answer

To Find: The value of tan(2 $\tan^{-1}\frac{1}{5}$ - $\frac{\pi}{4}$)

Consider , tan(2 $\tan^{-1}\frac{1}{5}$ - $\frac{\pi}{4}$) =tan($\tan^{-1}(\frac{2(\frac{1}{5})}{1-(\frac{1}{5})^2})$ - $\frac{\pi}{4}$)

($\because 2\tan^{-1}x = \tan^{-1}(\frac{2x}{1-x^2})$)

= tan($\tan^{-1}(\frac{\frac{2}{5}}{1-\frac{1}{25}})$ - $\frac{\pi}{4}$)

= tan($\tan^{-1}(\frac{5}{12})$ - $\frac{\pi}{4}$)

= tan($\tan^{-1}(\frac{5}{12})$ - $\tan^{-1}(1)$) ($\because$tan($\frac{\pi}{4}$)=1)

= tan($\tan^{-1}(\frac{\frac{5}{12}-1}{1+\frac{5}{12}})$)

($\tan^{-1}x$ - $\tan^{-1}y$ =$\tan^{-1}(\frac{x-y}{1+xy})$

= tan($\tan^{-1}(\frac{-7}{17})$)

tan(2 $\tan^{-1}\frac{1}{5}$ - $\frac{\pi}{4}$) = $\frac{-7}{17}$

27. Question

Mark the tick against the correct answer in the following:

$$\tan\frac{1}{2}\left(\cos^{-1}\frac{\sqrt{5}}{3}\right) = ?$$

A. $\frac{\left(3-\sqrt{5}\right)}{2}$

B. $\frac{\left(3+\sqrt{5}\right)}{2}$

C. $\frac{\left(5-\sqrt{3}\right)}{2}$

D. $\frac{\left(5+\sqrt{3}\right)}{2}$

Answer

To Find: The value of tan $\frac{1}{2}(\cos^{-1}\frac{\sqrt{5}}{3})$

Let , x = $\cos^{-1}\frac{\sqrt{5}}{3}$

$\Rightarrow$cos x = $\frac{\sqrt{5}}{3}$

Now, tan $\frac{1}{2}(\cos^{-1}\frac{\sqrt{5}}{3})$ becomes

tan $\frac{1}{2}(\cos^{-1}\frac{\sqrt{5}}{3})$= tan $\frac{1}{2}$(x) =tan $\frac{x}{2}$

$=\sqrt{\frac{1-cosx}{1+cosx}}$

$=\sqrt{\frac{1-(\frac{\sqrt{5}}{3})}{1+\frac{\sqrt{5}}{3}}}$

$=\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$

$=\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}\times\sqrt{\frac{3-\sqrt{5}}{3-\sqrt{5}}}$

tan $\frac{1}{2}(\cos^{-1}\frac{\sqrt{5}}{3})$ = $\frac{3-\sqrt{5}}{2}$

28. Question

Mark the tick against the correct answer in the following:

$\sin\left(\cos^{-1}\frac{3}{5}\right)=?$

A. $\frac{3}{4}$

B. $\frac{4}{5}$

C. $\frac{3}{5}$

D. none of these

Answer

To Find: The value of sin($\cos^{-1}\frac{3}{5}$)

Let, x = $\cos^{-1}\frac{3}{5}$

$\Rightarrow$cos x = $\frac{3}{5}$

Now , sin($\cos^{-1}\frac{3}{5}$) becomes sin (x)

Since we know that sin x = $\sqrt{1-\cos^2 x}$

= $\sqrt{1-(\frac{3}{5})^2}$

sin($\cos^{-1}\frac{3}{5}$) = Sin x = $\frac{4}{5}$

29. Question

Mark the tick against the correct answer in the following:

$$\cos\left(\tan^{-1}\frac{3}{4}\right)=?$$

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{4}{9}$

D. none of these

Answer

To Find: The value of cos($\tan^{-1}\frac{3}{4}$)

Let x = $\tan^{-1}\frac{3}{4}$

⇒tan x = $\frac{3}{4}$

⇒tan x = $\frac{3}{4}$ = $\frac{opposite\ side}{adjacent\ side}$

We know that by pythagorus theorem ,

(Hypotenuse $)^2$ = (opposite side $)^2$ + (adjacent side $)^2$

Therefore, Hypotenuse = 5

⇒cos x = $\frac{adjacent\ side}{hypotenuse}$ = $\frac{4}{5}$

Since here x = $\tan^{-1}\frac{3}{4}$ hence cos($\tan^{-1}\frac{3}{4}$) becomes cos x

Hence , cos($\tan^{-1}\frac{3}{4}$) = cos x = $\frac{4}{5}$

30. Question

Mark the tick against the correct answer in the following:

$$\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right\}=?$$

A. 1

B. 0

C. $\frac{-1}{2}$

D. none of these

Answer

To Find: The value of of sin $\{\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})\}$

Let, x = sin $\{\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})\}$

⇒ x = sin $\{\frac{\pi}{3} - (-\sin^{-1}\frac{1}{2})\}$ $(\because \sin^{-1}(-\theta) = -\sin\theta$)

⇒ x = sin $(\frac{\pi}{3} + \frac{\pi}{6})$

⇒ x = sin $(\frac{3\pi}{6})$ = sin $(\frac{\pi}{2})$ = 1

31. Question

Mark the tick against the correct answer in the following:

$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = ?$

A. $\frac{1}{\sqrt{5}}$

B. $\frac{2}{\sqrt{5}}$

C. $\frac{1}{\sqrt{10}}$

D. $\frac{2}{\sqrt{10}}$

Answer

To Find: The value of sin$(\frac{1}{2}\cos^{-1}\frac{4}{5})$

Let x = $\cos^{-1}\frac{4}{5}$

⇒cos x = $\frac{4}{5}$

Therefore sin$(\frac{1}{2}\cos^{-1}\frac{4}{5})$ becomes sin$(\frac{1}{2}x)$,i.e sin $(\frac{x}{2})$

We know that sin $(\frac{x}{2})$ = $\sqrt{\frac{1-\cos x}{2}}$

= $\sqrt{\frac{1-\frac{4}{5}}{2}}$

= $\sqrt{\frac{\frac{1}{5}}{2}}$

sin $(\frac{x}{2})$ = $\frac{1}{\sqrt{10}}$

32. Question

Mark the tick against the correct answer in the following:

$$\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\} = ?$$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. $\frac{2\pi}{3}$

Answer

To Find: The value of $\tan^{-1}\{2\cos(2\sin^{-1}\frac{1}{2})\}$

Let , x = $\tan^{-1}\{2\cos(2\sin^{-1}\frac{1}{2})\}$

⇒x = $\tan^{-1}\{2\cos(2(\frac{\pi}{6}))\}$ ($\because$sin $(\frac{\pi}{6})= \frac{1}{2}$)

⇒x = $\tan^{-1}(2\cos\frac{\pi}{3})$

⇒x = $\tan^{-1}(2(\frac{1}{2})) = \tan^{-1} 1 = \frac{\pi}{4}$ ($\because$cos $(\frac{\pi}{3})= \frac{1}{2}$ $and \tan\left(\frac{\pi}{4}\right) = 1$)

33. Question

Mark the tick against the correct answer in the following:

If $\cot^{-1}\left(\frac{-1}{5}\right) = x$ then sin x = ?

A. $\frac{1}{\sqrt{26}}$

B. $\frac{5}{\sqrt{26}}$

C. $\frac{1}{\sqrt{24}}$

D. none of these

Answer

Given: $\cot^{-1}\frac{-1}{5}$ = x

To Find: The value of sin x

Since , x = $\cot^{-1}\frac{-1}{5}$

⇒cot x = $\frac{-1}{5} = \frac{adjacent\ side}{opposite\ side}$

By pythagorus theroem ,

(Hypotenuse $)^2$ = (opposite side $)^2$ + (adjacent side $)^2$

Therefore, Hypotenuse = $\sqrt{26}$

$\Rightarrow$sin x = $\frac{opposite\ side}{hypotenuse} = \frac{5}{\sqrt{26}}$

34. Question

Mark the tick against the correct answer in the following:

$$\sin^{-1}\left(\frac{-1}{2}\right)+2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)=?$$

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. none of these

Answer

To Find: The value of $\sin^{-1}(\frac{-1}{2}) + 2\cos^{-1}(\frac{-\sqrt{3}}{2})$

Let , x = $\sin^{-1}(\frac{-1}{2}) + 2\cos^{-1}(\frac{-\sqrt{3}}{2})$

$\Rightarrow$ x = - $\sin^{-1}(\frac{1}{2})$ + 2 $[\pi - \cos^{-1}(\frac{\sqrt{3}}{2})]$ ($\because \sin^{-1}(-\theta) = -\sin^{-1}(\theta)$ *and* $\cos^{-1}(-\theta) = \pi - \cos^{-1}(\theta)$)

$\Rightarrow$ x = - $(\frac{\pi}{6})$ + 2 $[\pi - \frac{\pi}{6}]$

$\Rightarrow$ x = - $(\frac{\pi}{6})$ + 2 $[\frac{5\pi}{6}]$

$\Rightarrow$ x = - $\frac{\pi}{6} + \frac{5\pi}{3}$

$\Rightarrow$ x = $\frac{3\pi}{2}$

Tag:

35. Question

Mark the tick against the correct answer in the following:

$$\tan^{-1}(-1)+\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)=?$$

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. $\frac{2\pi}{3}$

Answer

To Find: The value of $\tan^{-1}(-1) + \cos^{-1}(\frac{-1}{\sqrt{2}})$

Let , x = $\tan^{-1}(-1) + \cos^{-1}(\frac{-1}{\sqrt{2}})$

⇒x = - $\tan^{-1}(1) + (\pi - \cos^{-1}(\frac{1}{\sqrt{2}}))$

($\because \tan^{-1}(-\theta) = -\tan^{-1}(\theta)$ *and* $\cos^{-1}(-\theta) = \pi - \cos^{-1}(\theta)$)

⇒x = $-\frac{\pi}{4} + (\pi - \frac{\pi}{4})$

⇒x = $-\frac{\pi}{4} + \frac{3\pi}{4}$

⇒x = $\frac{\pi}{2}$

36. Question

Mark the tick against the correct answer in the following:

$\cot\left(\tan^{-1}x + \cot^{-1}x\right) = ?$

A. 1

B. $\frac{1}{2}$

C. 0

D. none of these

Answer

To Find: The value of cot $(\tan^{-1}x + \cot^{-1}x)$

Let , x = cot $(\tan^{-1}x + \cot^{-1}x)$

⇒x = cot $(\frac{\pi}{2})$ ($\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$)

⇒x = 0

37. Question

Mark the tick against the correct answer in the following:

$\tan^{-1}1 + \tan^{-1}\frac{1}{3} = ?$

A. $\tan^{-1}\frac{4}{3}$

B. $\tan^{-1}\frac{2}{3}$

C. $\tan^{-1}2$

D. $\tan^{-1}3$

Answer

To Find: The value of $\tan^{-1} 1 + \tan^{-1}\frac{1}{3}$

Let , x = $\tan^{-1} 1 + \tan^{-1}\frac{1}{3}$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1} 1 + \tan^{-1}\frac{1}{3} = \tan^{-1}(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}) = \tan^{-1} 2$

38. Question

Mark the tick against the correct answer in the following:

$$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = ?$$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer

To Find: The value of $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$

Let , x = $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1} 1 + \tan^{-1}\frac{1}{3} = \tan^{-1}(\frac{\frac{1}{2}+\frac{1}{3}}{1-(\frac{1}{3}\times\frac{1}{2})}) = \tan^{-1} 1 = \frac{\pi}{4}$

39. Question

Mark the tick against the correct answer in the following:

$$2\tan^{-1}\frac{1}{3} = ?$$

A. $\tan^{-1}\frac{3}{2}$

B. $\tan^{-1}\frac{3}{4}$

C. $\tan^{-1}\frac{4}{3}$

D. none of these

Answer

To Find: The value of 2 $\tan^{-1}\frac{1}{3}$ i.e, $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{3}$

Let , x = $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{3}$

Since we know that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow\ \tan^{-1}1+\tan^{-1}\frac{1}{3}=\tan^{-1}(\frac{\frac{1}{3}+\frac{1}{3}}{1-(\frac{1}{3}\times\frac{1}{3})})=\tan^{-1}\frac{3}{4}$

40. Question

Mark the tick against the correct answer in the following:

$$\cos\left(2\tan^{-1}\frac{1}{2}\right)=?$$

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{7}{8}$

D. none of these

Answer

To Find: The value of cos $(2\tan^{-1}\frac{1}{2})$

Let , x = cos $(2\tan^{-1}\frac{1}{2})$

$\Rightarrow$x = cos $(\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{2})$

Since we know that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow\ \tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{2}=\tan^{-1}(\frac{\frac{1}{2}+\frac{1}{2}}{1-(\frac{1}{2}\times\frac{1}{2})})=\tan^{-1}\frac{4}{3}$

$\Rightarrow$x = cos $(\tan^{-1}\frac{4}{3})$

Now , let y = $\tan^{-1}\frac{4}{3}$

$\Rightarrow$ tan y = $\frac{4}{3}=\frac{opposite\ side}{adjacent\ side}$

By pythagorus theroem ,

$(\text{Hypotenuse})^2=(\text{opposite side})^2+(\text{adjacent side})^2$

Therefore, Hypotenuse = 5

$\Rightarrow$ cos $(\tan^{-1}\frac{4}{3})$=cos y = $\frac{3}{5}$

41. Question

Mark the tick against the correct answer in the following:

$\sin\left[2\tan^{-1}\frac{5}{8}\right]$

A. $\frac{25}{64}$

B. $\frac{80}{89}$

C. $\frac{75}{128}$

D. none of these

Answer

To Find: The value of sin $(2\tan^{-1}\frac{5}{8})$

Let , x = $\sin(2\tan^{-1}\frac{5}{8})$

We know that $2\tan^{-1}x = \sin^{-1}(\frac{2x}{1+x^2})$

$\Rightarrow$ x = $\sin(\sin^{-1}(\frac{2(\frac{5}{8})}{1+(\frac{5}{8})^2})) = \sin(\sin^{-1}(\frac{80}{89})) = \frac{80}{89}$

42. Question

Mark the tick against the correct answer in the following:

$\sin\left[2\sin^{-1}\frac{4}{5}\right]$

A. $\frac{12}{25}$

B. $\frac{16}{25}$

C. $\frac{24}{25}$

D. None of these

Answer

To Find: The value of sin $(2\sin^{-1}\frac{4}{5})$

Let , x = $\sin^{-1}\frac{4}{5}$

$\Rightarrow$sin x = $\frac{4}{5}$

We know that ,cos x = $\sqrt{1-\sin^2 x}$

$= \sqrt{1-(\frac{4}{5})^2}$

$=\frac{3}{5}$

Now since, x = $\sin^{-1}\frac{4}{5}$,hence sin (2 $\sin^{-1}\frac{4}{5}$) becomes sin(2x)

Here, sin(2x)= 2 sin x cos x

=2× $\frac{4}{5}$ × $\frac{3}{5}$

=$\frac{24}{25}$

43. Question

Mark the tick against the correct answer in the following:

If $\tan^{-1}x = \frac{\pi}{4} - \tan^{-1}\frac{1}{3}$ then x = ?

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{6}$

D. None of these

Answer

To Find: The value of $\tan^{-1}x = \frac{\pi}{4}$ - $\tan^{-1}\frac{1}{3}$

Now , $\tan^{-1}x = \tan^{-1}1$ - $\tan^{-1}\frac{1}{3}$ ($\because \tan\frac{\pi}{4} = 1$)

Since we know that $\tan^{-1}x$- $\tan^{-1}y = \tan^{-1}(\frac{x-y}{1+xy})$

⇒ $\tan^{-1}1$+ $\tan^{-1}\frac{1}{3} = \tan^{-1}(\frac{1-\frac{1}{3}}{1+\frac{1}{3}}) = \tan^{-1}\frac{1}{2}$

⇒ $\tan^{-1}x = \tan^{-1}\frac{1}{2}$

⇒x = $\frac{1}{2}$

44. Question

Mark the tick against the correct answer in the following:

If $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$ then x = ?

A. 1

B. -1

C. 0

D. $\frac{1}{2}$

Answer

To Find: The value of $\tan^{-1}(1+x)$ + $\tan^{-1}(1-x)$= $\frac{\pi}{2}$

Since we know that $\tan^{-1}x + \tan^{-1}y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1}(1+x) + \tan^{-1}(1-x) = \tan^{-1}(\frac{(1+x)+(1-x)}{1-(1+x)(1-x)})$

$= \tan^{-1}(\frac{2}{1-(1-x^2)})$

$=\tan^{-1}(\frac{2}{x^2})$

Here since $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}(\frac{2}{x^2}) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}(\frac{2}{x^2}) = \tan^{-1}(\infty)$ ($\because \tan\frac{\pi}{2} = \infty$)

$\Rightarrow \frac{2}{x^2} = \infty$

$\Rightarrow x^2 = \frac{2}{\infty}$

$\Rightarrow$ x = 0

45. Question

Mark the tick against the correct answer in the following:

If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ then $\left(\cos^{-1}x + \cos^{-1}y\right) = ?$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. π

D. $\frac{2\pi}{3}$

Answer

Given: $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$

To Find: The value of $\cos^{-1}x + \cos^{-1}y$

Since we know that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

Similarly $\cos^{-1}y = \frac{\pi}{2} - \sin^{-1}y$

Now consider $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2} - \sin^{-1}x + \frac{\pi}{2} - \sin^{-1}y$

$= \frac{2\pi}{2} - [\sin^{-1}x + \sin^{-1}y]$

$= \pi - \frac{2\pi}{3}$

$= \frac{\pi}{3}$

46. Question

Mark the tick against the correct answer in the following:

($\tan^{-1} 2 + \tan^{-1} 3$) = ?

A. $\frac{-\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. π

Answer

To Find: The value of $\tan^{-1} 2 + \tan^{-1} 3$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1} 2 + \tan^{-1} 3 = \tan^{-1}(\frac{2+3}{1-(2\times 3)})$

$= \tan^{-1}(\frac{5}{-5})$

$= \tan^{-1}(-1)$

Since the principle value of tan lies in the range $[0,\pi]$

$\Rightarrow \tan^{-1}(-1) = \frac{3\pi}{4}$

47. Question

Mark the tick against the correct answer in the following:

If $\tan^{-1} x + \tan^{-1} 3 = \tan^{-1} 8$ then x = ?

A. $\frac{1}{3}$

B. $\frac{1}{5}$

C. 3

D. 5

Answer

Given: $\tan^{-1} x + \tan^{-1} 3 = \tan^{-1} 8$

To Find: The value of x

Here $\tan^{-1} x + \tan^{-1} 3 = \tan^{-1} 8$ can be written as

$\tan^{-1} x = \tan^{-1} 8 - \tan^{-1} 3$

Since we know that $\tan^{-1} x - \tan^{-1} y = \tan^{-1}(\frac{x-y}{1+xy})$

$\tan^{-1} x = \tan^{-1} 8 - \tan^{-1} 3 = \tan^{-1}(\frac{8-3}{1+(8\times3)})$

$= \tan^{-1}(\frac{5}{25})$

$= \tan^{-1}(\frac{1}{5})$

$\Rightarrow x = \frac{1}{5}$

48. Question

Mark the tick against the correct answer in the following:

If $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$ then x = ?

A. $\frac{1}{2}$ or -2

B. $\frac{1}{3}$ or -3

C. $\frac{1}{4}$ or -2

D. $\frac{1}{6}$ or -1

Answer

Given: $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$

To Find: The value of x

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1} 3x + \tan^{-1} 2x = \tan^{-1}(\frac{3x+2x}{1-(3x\times2x)})$

$= \tan^{-1}(\frac{5x}{1-6x^2})$

Now since $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$

$\tan^{-1} 3x + \tan^{-1} 2x = \tan^{-1} 1$ ($\because \tan\frac{\pi}{4} = 1$)

$\Rightarrow \tan^{-1}(\frac{5x}{1-6x^2}) = \tan^{-1} 1$

$\Rightarrow \frac{5x}{1-6x^2} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow x = \frac{1}{6}$ or x= -1

49. Question

Mark the tick against the correct answer in the following:

$\tan\left\{\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right\} = ?$

A. $\frac{13}{6}$

B. $\frac{17}{6}$

C. $\frac{19}{6}$

D. $\frac{23}{6}$

Answer

To Find: The value of tan $\{\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\}$

Let x = $\cos^{-1}\frac{4}{5}$

$\Rightarrow$cos x = $\frac{4}{5}$ =$\frac{adjacent\ side}{hypotenuse}$

By pythagorus theroem ,

(Hypotenuse)^2 = (opposite side)^2 + (adjacent side)^2

Therefore , opposite side = 3

$\Rightarrow$tan x= $\frac{opposite\ side}{adjacent\ side} = \frac{3}{4}$

$\Rightarrow$x =$\tan^{-1}\frac{3}{4}$

Now tan $\{\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\}$ = tan $\{\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\}$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

tan $\{\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\}$ = tan $(\tan^{-1}(\frac{\frac{3}{4}+\frac{2}{3}}{1-(\frac{3}{4}\times\frac{2}{3})}))$

= tan $(\tan^{-1}(\frac{17}{6}))$

= $\frac{17}{6}$

50. Question

Mark the tick against the correct answer in the following:

$$\cos^{-1}9 + \operatorname{cosec}^{-1}\frac{\sqrt{41}}{4} = ?$$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{3\pi}{4}$

Answer

To Find: The value of $\cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4}$

Now $\cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4}$ can be written in terms of tan inverse as

$\cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4} = \tan^{-1} \frac{1}{9} + \tan^{-1} \frac{4}{5}$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1} \frac{1}{9} + \tan^{-1} \frac{4}{5} = \tan^{-1}(\frac{\frac{1}{9}+\frac{4}{5}}{1-(\frac{1}{9}\times\frac{4}{5})})$

$= \tan^{-1}(\frac{41}{41})$

$=\tan^{-1}(1) = \frac{\pi}{4}$

51. Question

Mark the tick against the correct answer in the following:

Range of $\sin^{-1}$ x is

A. $\left[0, \frac{\pi}{2}\right]$

B. [0, π]

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. None of these

Answer

To Find: The range of $\sin^{-1} x$

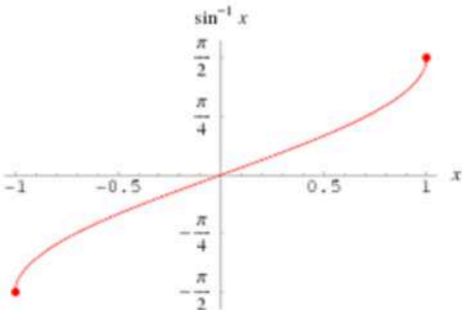
Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\sin^{-1}(x)$ can be obtained from the graph of

Y = sin x by interchanging x and y axes.i.e, if (a,b) is a point on Y = sin x then (b,a) is

The point on the function y = $\sin^{-1}(x)$

Below is the Graph of range of $\sin^{-1}(x)$



From the graph, it is clear that the range of $\sin^{-1}(x)$ is restricted to the interval

$[-\frac{\pi}{2}, \frac{\pi}{2}]$

52. Question

Mark the tick against the correct answer in the following:

Range of $\cos^{-1} x$ is

A. $[0, \pi]$

B. $\left[0, \frac{\pi}{2}\right]$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. None of these

Answer

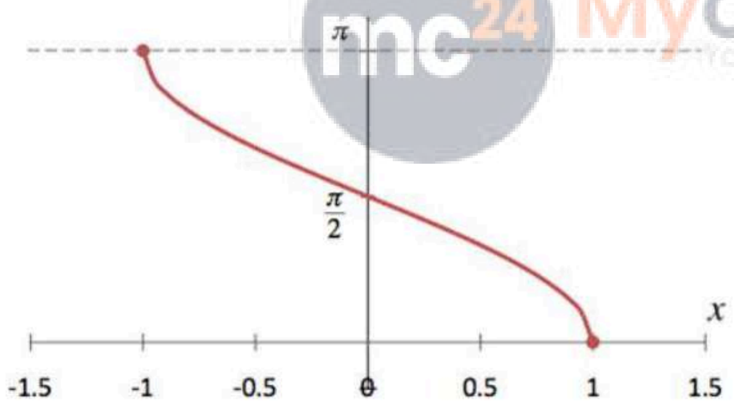
To Find: The range of $\cos^{-1} x$

Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\cos^{-1}(x)$ can be obtained from the graph of

Y = cos x by interchanging x and y axes.i.e, if (a,b) is a point on Y = cos x then (b,a) is the point on the function y = $\cos^{-1}(x)$

Below is the Graph of the range of $\cos^{-1}(x)$



From the graph, it is clear that the range of $\cos^{-1}(x)$ is restricted to the interval

$[0, \pi]$

53. Question

Mark the tick against the correct answer in the following:

Range of $\tan^{-1} x$ is

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. None of these

Answer

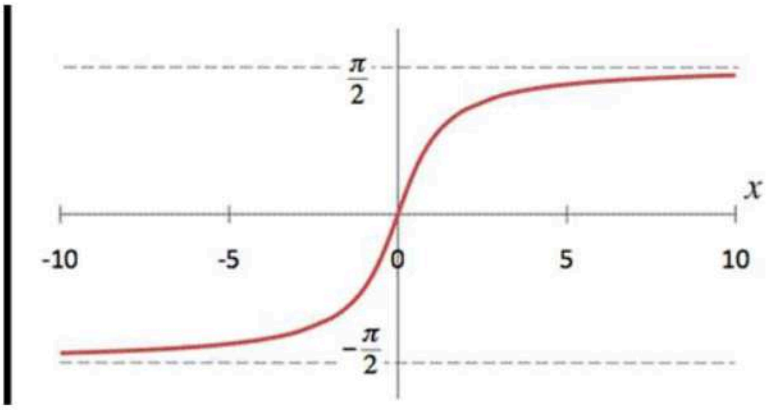
To Find: The range of $\tan^{-1}$ x

Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\tan^{-1}(x)$ can be obtained from the graph of

Y = tan x by interchanging x and y axes.i.e, if (a,b) is a point on Y = tan x then (b,a) is the point on the function y = $\tan^{-1}(x)$

Below is the Graph of the range of $\tan^{-1}(x)$



From the graph, it is clear that the range of $\tan^{-1}(x)$ is restricted to any of the intervals like $[-\frac{3\pi}{2}, -\frac{\pi}{2}]$, $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $[\frac{\pi}{2}, \frac{3\pi}{2}]$ and so on. Hence the range is given by

$(-\frac{\pi}{2}, \frac{\pi}{2})$.

54. Question

Mark the tick against the correct answer in the following:

Range of $\sec^{-1}$ x is

A. $\left[0, \frac{\pi}{2}\right]$

B. [0, π]

C. $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

D. None of these

Answer

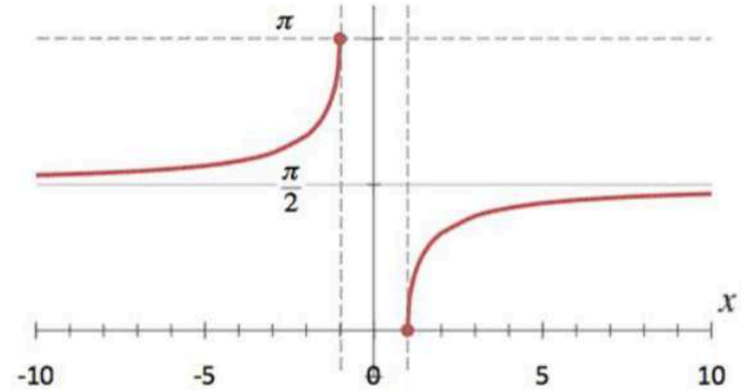
To Find:The range of $\sec^{-1}(x)$

Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\sec^{-1}(x)$ can be obtained from the graph of

Y = sec x by interchanging x and y axes.i.e, if (a,b) is a point on Y = sec x then (b,a) is the point on the function y = $\sec^{-1}(x)$

Below is the Graph of the range of $\sec^{-1}(x)$



From the graph, it is clear that the range of $\sec^{-1}(x)$ is restricted to interval

$[0,\pi] - \{\frac{\pi}{2}\}$

55. Question

Mark the tick against the correct answer in the following:

Range of coses^{-1} x is

A. $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

B. $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

C. $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$

D. None of these

Answer

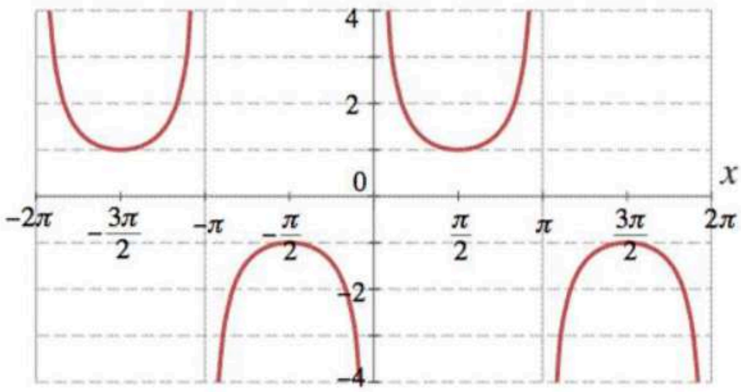
To Find: The range of $\text{cosec}^{-1}(x)$

Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\text{cosec}^{-1}(x)$ can be obtained from the graph of

Y = cosec x by interchanging x and y axes.i.e, if (a,b) is a point on Y = cosec x then (b,a) is the point on the function y = $\text{cosec}^{-1}(x)$

Below is the Graph of the range of $\text{cosec}^{-1}(x)$



From the graph it is clear that the range of $\text{cosec}^{-1}(x)$ is restricted to interval

$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

56. Question

Mark the tick against the correct answer in the following:

Domain of cos-1 x is

A. [0, 1]

B. [-1, 1]

C. [-1, 0]

D. None of these

Answer

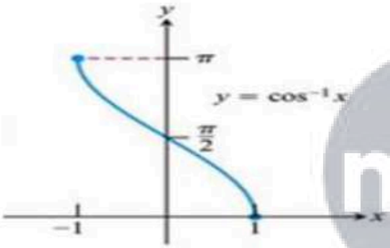
To Find: The Domain of $\cos^{-1}(x)$

Here,the inverse function of cos is given by y = $f^{-1}(x)$

The graph of the function y = $\cos^{-1}(x)$ can be obtained from the graph of

Y = cos x by interchanging x and y axes.i.e, if (a,b) is a point on Y = cos x then (b,a) is the point on the function y = $\cos^{-1}(x)$

Below is the Graph of the domain of $\cos^{-1}(x)$



From the graph, it is clear that the domain of $\cos^{-1}(x)$ is [-1,1]

57. Question

Mark the tick against the correct answer in the following:

Domain of $\sec^{-1}$ x is

A. [-1, 1]

B. R – {0}

C. R – [-1, 1]

D. R – {-1, 1}

Answer

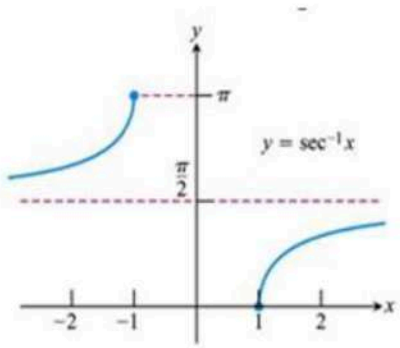
To Find: The Domain of $\sec^{-1}(x)$

Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\sec^{-1}(x)$ can be obtained from the graph of

Y = sec x by interchanging x and y axes.i.e, if (a,b) is a point on Y = sec x then (b,a) is the point on the function y = $\sec^{-1}(x)$

Below is the Graph of the domain of $\sec^{-1}(x)$



From the graph, it is clear that the domain of $\sec^{-1}(x)$ is a set of all real numbers excluding -1 and 1 i.e, R – [-1,1]