$2x + (k - 2)y = k$

$\Rightarrow 2x + (k - 2)y - k = 0$ ....(i)

And, $6x + (2k - 1)y = (2k + 5)$

$\Rightarrow 6x + (2k - 1)y - (2k + 5) = 0$ ...(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0,\ a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1 = (k - 2)$, $c_1 = -k$ and $a_2 = 6$, $b_2 = (2k - 1)$, $c_2 = -(2k + 5)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{6} = \frac{(k-2)}{(2k-1)} = \frac{-k}{-(2k+5)}$

$\Rightarrow \frac{1}{3} = \frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

Now, we have the following three cases:

Case I:

$\frac{1}{3} = \frac{(k-2)}{(2k-1)}$

$\Rightarrow (2k - 1) = 3(k - 2)$

$\Rightarrow 2k - 1 = 3k - 6 \Rightarrow k = 5$

Case II:

$\frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

$\Rightarrow (k - 2)(2k + 5) = k(2k - 1)$

$\Rightarrow 2k^2 + 5k - 4k - 10 = 2k^2 - k$

$\Rightarrow k + k = 10 \Rightarrow 2k = 10 \Rightarrow k = 5$

Case III:

$\frac{1}{3} = \frac{k}{(2k+5)}$

$\Rightarrow 2k + 5 = 3k \Rightarrow k = 5$

Hence, the given system of equations has an infinite number of solutions when k is equal to 5.

17. Find the value of k for which the system of linear equations has an infinite number of solutions:

$kx + 3y = (2k + 1)$,

$2(k + 1)x + 9y = (7k + 1)$.

Sol:

The given system of equations:

$kx + 3y = (2k + 1)$

$\Rightarrow kx + 3y - (2k + 1) = 0$ ....(i)

And, $2(k + 1)x + 9y = (7k + 1)$

$\Rightarrow 2(k + 1)x + 9y - (7k + 1) = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0, a_2x+b_2y+c_2 = 0$

where, $a_1 = k, b_1= 3, c_1 = -(2k + 1)$ and $a_2 = 2(k + 1), b_2 = 9, c_2 = -(7k + 1)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., $\frac{k}{2(k+1)} = \frac{3}{9} = \frac{-(2k+1)}{-(7k+1)}$

$\Rightarrow \frac{k}{2(k+1)} = \frac{1}{3} = \frac{(2k+1)}{(7k+1)}$

Now, we have the following three cases:

Case I:

$\frac{k}{2(k+1)} = \frac{1}{3}$

$\Rightarrow 2(k + 1) = 3k$

$\Rightarrow 2k + 2 = 3k$

$\Rightarrow k = 2$

Case II:

$\frac{1}{3} = \frac{(2k+1)}{(7k+1)}$

$\Rightarrow (7k + 1) = 6k + 3$

$\Rightarrow k = 2$

Case III:

$\frac{k}{2(k+1)} = \frac{(2k+1)}{(7k+1)}$

$\Rightarrow k(7k + 1) = (2k + 1) \times 2(k + 1)$

$\Rightarrow 7k^2 + k = (2k + 1) (2k + 2)$

$\Rightarrow 7k^2 + k = 4k^2 + 4k + 2k + 2$

$\Rightarrow 3k^2 – 5k – 2 = 0$

$\Rightarrow 3k^2 – 6k + k – 2 = 0$

$\Rightarrow 3k(k – 2) + 1(k – 2) = 0$

$\Rightarrow (3k + 1) (k – 2) = 0$

$\Rightarrow k = 2$ or $k = \frac{-1}{3}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 2.

18. Find the value of k for which the system of linear equations has an infinite number of solutions:

$5x + 2y = 2k$,

$2(k + 1)x + ky = (3k + 4)$.

Sol:

The given system of equations:

$5x + 2y = 2k$

$\Rightarrow 5x + 2y - 2k = 0 \quad ....(i)$

And, $2(k + 1)x + ky = (3k + 4)$

$\Rightarrow 2(k + 1)x + ky - (3k + 4) = 0 \quad ...(ii)$

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 5$, $b_1= 2$, $c_1 = -2k$ and $a_2 = 2(k + 1)$, $b_2 = k$, $c_2 = -(3k + 4)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{5}{2(k+1)} = \frac{2}{k} = \frac{-2k}{-(3k+4)}$

$\Rightarrow \frac{5}{2(k+1)} = \frac{2}{k} = \frac{2k}{(3k+4)}$

Now, we have the following three cases:

Case I:

$\frac{5}{2(k+1)} = \frac{2}{k}$

$\Rightarrow 2 \times 2(k + 1) = 5k$

$\Rightarrow 4(k + 1) = 5k$

$\Rightarrow 4k + 4 = 5k$

$\Rightarrow k = 4$

Case II:

$\frac{2}{k} = \frac{2k}{(3k+4)}$

$\Rightarrow 2k^2 = 2 \times (3k + 4)$

$\Rightarrow 2k^2 = 6k + 8 \Rightarrow 2k^2 – 6k – 8 = 0$

$\Rightarrow 2(k^2 – 3k – 4) = 0$

$\Rightarrow k^2 – 4k + k – 4 = 0$

$\Rightarrow k(k – 4) + 1(k – 4) = 0$

$\Rightarrow (k + 1) (k – 4) = 0$

$\Rightarrow (k + 1) = 0$ or $(k – 4) = 0$

$\Rightarrow$ k = -1 or k = 4

Case III:

$\frac{5}{2(k+1)} = \frac{2k}{(3k+4)}$

$\Rightarrow 15k + 20 = 4k^2 + 4k$

$\Rightarrow 4k^2 - 11k - 20 = 0$

$\Rightarrow 4k^2 - 16k + 5k - 20 = 0$

$\Rightarrow 4k(k - 4) + 5(k - 4) = 0$

$\Rightarrow (k - 4)(4k + 5) = 0$

$\Rightarrow k = 4$ or $k = \frac{-5}{4}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 4.

19. Find the value of k for which the system of linear equations has an infinite number of solutions:

$(k - 1)x - y = 5,$

$(k + 1)x + (1 - k)y = (3k + 1).$

Sol:

The given system of equations:

$(k - 1)x - y = 5$

$\Rightarrow (k - 1)x - y - 5 = 0$ ….(i)

And, $(k + 1)x + (1 - k)y = (3k + 1)$

$\Rightarrow (k + 1)x + (1 - k)y - (3k + 1) = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = (k - 1)$, $b_1 = -1$, $c_1 = -5$ and $a_2 = (k + 1)$, $b_2 = (1 - k)$, $c_2 = -(3k + 1)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., $\frac{(k-1)}{(k+1)} = \frac{-1}{-(k-1)} = \frac{-5}{-(3k+1)}$

$\Rightarrow \frac{(k-1)}{(k+1)} = \frac{1}{(k-1)} = \frac{5}{(3k+1)}$

Now, we have the following three cases:

Case I:

$\frac{(k-1)}{(k+1)} = \frac{1}{(k-1)}$

$\Rightarrow (k - 1)^2 = (k + 1)$

$\Rightarrow k^2 + 1 - 2k = k + 1$

$\Rightarrow k^2 - 3k = 0 \Rightarrow k(k-3) = 0$

$\Rightarrow k = 0$ or $k = 3$

Case II:

$\frac{1}{(k-1)} = \frac{5}{(3k+1)}$

$\Rightarrow 3k + 1 = 5k - 5$

$\Rightarrow 2k = 6 \Rightarrow k = 3$

Case III:

$\frac{(k-1)}{(k+1)} = \frac{5}{(3k+1)}$

$\Rightarrow (3k+1)(k-1) = 5(k+1)$

$\Rightarrow 3k^2 + k - 3k - 1 = 5k + 5$

$\Rightarrow 3k^2 - 2k - 5k - 1 - 5 = 0$

$\Rightarrow 3k^2 - 7k - 6 = 0$

$\Rightarrow 3k^2 - 9k + 2k - 6 = 0$

$\Rightarrow 3k(k-3) + 2(k-3) = 0$

$\Rightarrow (k-3)(3k+2) = 0$

$\Rightarrow (k-3) = 0$ or $(3k+2) = 0$

$\Rightarrow k = 3$ or $k = \frac{-2}{3}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 3.

20. Find the value of k for which the system of linear equations has a unique solution:
$(k-3)x + 3y - k$, $kx + ky - 12 = 0$.

Sol:

The given system of equations can be written as

$(k-3)x + 3y - k = 0$

$kx + ky - 12 = 0$

This system is of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1 = 3$, $c_1 = -k$ and $a_2 = k$, $b_2 = k$, $c_2 = -12$

For the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{k-3}{k} = \frac{3}{k} = \frac{-k}{-12}$

$\Rightarrow k - 3 = 3$ and $k^2 = 36$

$\Rightarrow$ k = 6 and k = ±6
$\Rightarrow$ k = 6
Hence, k = 6.

21. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
(a – 1) x + 3y = 2, 6x + (1 – 2b)y = 6
Sol:
The given system of equations can be written as
(a – 1) x + 3y = 2
$\Rightarrow$(a – 1) x + 3y – 2 = 0 ….(i)
and 6x + (1 – 2b)y = 6
$\Rightarrow$6x + (1 – 2b)y – 6 = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = (a - 1)$, $b_1 = 3$, $c_1 = -2$ and $a_2 = 6$, $b_2 = (1 - 2b)$, $c_2 = -6$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{a-1}{6} = \frac{3}{(1-2b)} = \frac{-2}{-6}$
$\Rightarrow \frac{a-1}{6} = \frac{3}{(1-2b)} = \frac{1}{3}$
$\Rightarrow \frac{a-1}{6} = \frac{1}{3}$ and $\frac{3}{(1-2b)} = \frac{1}{3}$
$\Rightarrow$ 3a – 3 = 6 and 9 = 1 – 2b
$\Rightarrow$ 3a = 9 and 2b = -8
$\Rightarrow$ a = 3 and b = -4
$\therefore$ a = 3 and b = -4

22. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
(2a – 1) x + 3y = 5, 3x + (b – 1)y = 2.
Sol:
The given system of equations can be written as
(2a – 1) x + 3y = 5
$\Rightarrow$(2a – 1) x + 3y – 5 = 0 ….(i)
and 3x + (b – 1)y = 2
$\Rightarrow$3x + (b – 1)y – 2 = 0 ….(ii)
These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = (2a – 1)$, $b_1 = 3$, $c_1 = -5$ and $a_2 = 3$, $b_2 = (b – 1)$, $c_2 = -2$
For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{(2a-1)}{3} = \frac{3}{(b-1)} = \frac{-5}{-2}$

$\Rightarrow \frac{(2a-1)}{6} = \frac{3}{(b-1)} = \frac{5}{2}$

$\Rightarrow \frac{(2a-1)}{6} = \frac{5}{2}$ and $\frac{3}{(b-1)} = \frac{5}{2}$

$\Rightarrow 2(2a – 1) = 15$ and $6 = 5(b – 1)$

$\Rightarrow 4a – 2 = 15$ and $6 = 5b – 5$

$\Rightarrow 4a = 17$ and $5b = 11$

$\therefore a = \frac{17}{4}$ and $b = \frac{11}{5}$

23. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
$2x - 3y = 7$, $(a + b)x - (a + b – 3)y = 4a + b$.

Sol:

The given system of equations can be written as
$2x - 3y = 7$

$\Rightarrow 2x - 3y - 7 = 0$ ….(i)

and $(a + b)x - (a + b – 3)y = 4a + b$

$\Rightarrow (a + b)x - (a + b – 3)y - 4a + b = 0$ ….(ii)

These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
Here, $a_1 = 2$, $b_1 = -3$, $c_1 = -7$ and $a_2 = (a + b)$, $b_2 = -(a + b - 3)$, $c_2 = -(4a + b)$
For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{a+b} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)}$

$\Rightarrow \frac{2}{a+b} = \frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$

$\Rightarrow \frac{2}{a+b} = \frac{7}{(4a+b)}$ and $\frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$

$\Rightarrow 2(4a + b) = 7(a + b)$ and $3(4a + b) = 7(a + b - 3)$

$\Rightarrow 8a + 2b = 7a + 7b$ and $12a + 3b = 7a + 7b - 21$

$\Rightarrow 4a = 17$ and $5b = 11$

$\therefore a = 5b$ …….(iii)

and $5a = 4b – 21$ ……(iv)

On substituting $a = 5b$ in (iv), we get:

25b = 4b – 21
⇒21b = -21
⇒b = -1
On substituting b = -1 in (iii), we get:
a = 5(-1) = -5
∴a = -5 and b = -1.

24. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
2x + 3y = 7, (a + b + 1)x - (a + 2b + 2)y = 4(a + b) + 1.
Sol:
The given system of equations can be written as
2x + 3y = 7
⇒2x + 3y - 7 = 0 ….(i)
and (a + b + 1)x - (a + 2b + 2)y = 4(a + b) + 1
(a + b + 1)x - (a + 2b + 2)y – [4(a + b) + 1] = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = (a + b + 1)$, $b_2 = (a + 2b + 2)$, $c_2 = – [4(a + b) + 1]$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)} = \frac{-7}{-[4(a+b)+1]}$
$\Rightarrow \frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)} = \frac{7}{[4(a+b)+1]}$
$\Rightarrow \frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)}$ and $\frac{3}{(a+2b+2)} = \frac{7}{[4(a+b)+1]}$
⇒ 2(a + 2b + 2) = 3(a + b + 1) and 3[4(a + b) + 1] = 7(a + 2b + 2)
⇒ 2a + 4b + 4 = 3a + 3b + 3 and 3(4a + 4b + 1) = 7a + 14b + 14
⇒ a – b - 1=0 and 12a + 12b + 3 = 7a + 14b + 14
⇒ a - b = 1 and 5a – 2b = 11
a = (b + 1) …….(iii)
5a - 2b = 11 ……(iv)
On substituting a = (b + 1) in (iv), we get:
5(b + 1) – 2b = 11
⇒5b + 5 – 2b = 11
⇒ 3b = 6
⇒ b = 2
On substituting b = 2 in (iii), we get:
a = 3

$\therefore a = 3$ and $b = 2$.

25. Find the values of a and b for which the system of linear equations has an infinite number of solutions:

$2x + 3y = 7, (a + b)x + (2a - b)y = 21.$

Sol:

The given system of equations can be written as

$2x + 3y - 7 = 0$ ….(i)

$(a + b)x + (2a - b)y – 21 = 0$ ….(ii)

This system is of the form:

$a_1x+b_1y+c_1 = 0, a_2x+b_2y+c_2 = 0$

where, $a_1 = 2, b_1= 3, c_1 = -7$ and $a_2 = a + b, b_2 = 2a - b, c_2 = – 21$

For the given system of linear equations to have an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{a+b} = \frac{3}{2a-b} = \frac{-7}{-21}$

$\Rightarrow \frac{2}{a+b} = \frac{-7}{-21} = \frac{1}{3}$ and $\frac{3}{2a-b} = \frac{-7}{-21} = \frac{1}{3}$

$\Rightarrow a + b = 6$ and $2a – b = 9$

Adding $a + b = 6$ and $2a – b = 9$,we get

$3a = 15 \Rightarrow a = \frac{15}{3} = 3$

Now substituting $a = 5$ in $a + b = 6$, we have

$5 + b = 6 \Rightarrow b = 6 – 5 = 1$

Hence, $a = 5$ and $b = 1$.

26. Find the values of a and b for which the system of linear equations has an infinite number of solutions:

$2x + 3y = 7, 2ax + (a + b)y = 28.$

Sol:

The given system of equations can be written as

$2x + 3y - 7 = 0$ ….(i)

$2ax + (a + b)y – 28 = 0$ ….(ii)

This system is of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = 2, b_1= 3, c_1 = -7$ and $a_2 = 2a, b_2 = a + b, c_2 = – 28$

For the given system of linear equations to have an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$
$\Rightarrow \frac{2}{2a} = \frac{-7}{-28} = \frac{1}{4}$ and $\frac{3}{a+b} = \frac{-7}{-28} = \frac{1}{4}$
$\Rightarrow$ a = 4 and a + b = 12
Substituting a = 4 in a + b = 12, we get
4 + b = 12 $\Rightarrow$ b = 12 – 4 = 8
Hence, a = 4 and b = 8.

27. Find the value of k for which the system of equations
8x + 5y = 9, kx + 10y = 15
has a non-zero solution.
Sol:
The given system of equations:
8x + 5y = 9
8x + 5y - 9 = 0 ….(i)
kx + 10y = 15
kx + 10y - 15 = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 8$, $b_1 = 5$, $c_1 = -9$ and $a_2 = k$, $b_2 = 10$, $c_2 = -15$
In order that the given system has no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
i.e., $\frac{8}{k} = \frac{5}{10} \neq \frac{-9}{-15}$
i.e., $\frac{8}{k} = \frac{1}{2} \neq \frac{3}{5}$
$\frac{8}{k} = \frac{1}{2}$ and $\frac{8}{k} \neq \frac{3}{5}$
$\Rightarrow$ k = 16 and k $\neq \frac{40}{3}$
Hence, the given system of equations has no solutions when k is equal to 16.

28. Find the value of k for which the system of equations
kx + 3y = 3, 12x + ky = 6 has no solution.
Sol:
The given system of equations:
kx + 3y = 3
kx + 3y - 3 = 0 ….(i)
12x + ky = 6
12x + ky - 6 = 0 ….(ii)
These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = k$, $b_1= 3$, $c_1 = -3$ and $a_2 = 12$, $b_2 = k$, $c_2 = –6$
In order that the given system has no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
i.e., $\frac{k}{12} = \frac{3}{k} \neq \frac{-3}{-6}$
$\frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{1}{2}$
$\Rightarrow k^2 = 36$ and $k \neq 6$
$\Rightarrow k = \pm 6$ and $k \neq 6$
Hence, the given system of equations has no solution when k is equal to -6.

29. Find the value of k for which the system of equations
$3x - y = 5$, $6x - 2y = k$
has no solution.
Sol:
The given system of equations:
$3x - y - 5 = 0$ ….(i)
And, $6x - 2y + k = 0$ ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 3$, $b_1= -1$, $c_1 = -5$ and $a_2 = 6$, $b_2 = -2$, $c_2 = k$
In order that the given system has no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
i.e., $\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{k}$
$\Rightarrow \frac{-1}{-2} \neq \frac{-5}{k} \Rightarrow k \neq -10$
Hence, equations (i) and (ii) will have no solution if $k \neq -10$.

30. Find the value of k for which the system of equations
$kx + 3y + 3 - k = 0$, $12x + ky - k = 0$
has no solution.
Sol:
The given system of equations can be written as
$kx + 3y + 3 - k = 0$ ….(i)
$12x + ky - k = 0$ ….(ii)
This system of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = k$, $b_1= 3$, $c_1 = 3 - k$ and $a_2 = 12$, $b_2 = k$, $c_2 = –k$

For the given system of linear equations to have no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{12} = \frac{3}{k} \neq \frac{3-k}{-k}$

$\Rightarrow \frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{3-k}{-k}$

$\Rightarrow k^2 = 36$ and $-3 \neq 3 - k$

$\Rightarrow k = \pm 6$ and $k \neq 6$

$\Rightarrow k = -6$

Hence, k = -6.

31. Find the value of k for which the system of equations

$5x - 3y = 0$, $2x + ky = 0$

has a non-zero solution.

Sol:

The given system of equations:

$5x - 3y = 0$ ….(i)

$2x + ky = 0$ ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 5$, $b_1 = -3$, $c_1 = 0$ and $a_2 = 2$, $b_2 = k$, $c_2 = 0$

For a non-zero solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \frac{5}{2} = \frac{-3}{k}$

$\Rightarrow 5k = -6 \Rightarrow k = \frac{-6}{5}$

Hence, the required value of k is $\frac{-6}{5}$.

Linear equations in two variables – 3E

32. 5 chairs and 4 tables together cost ₹5600, while 4 chairs and 3 tables together cost ₹ 4340. Find the cost of each chair and that of each table.

Sol:

Let the cost of a chair be ₹ x and that of a table be ₹ y, then

$5x + 4y = 5600$ ……..(i)

$4x + 3y = 4340$ ……..(ii)

Multiplying (i) by 3 and (ii) by 4, we get

$15x – 16x = 16800 – 17360$

$\Rightarrow -x = -560$

$\Rightarrow x = 560$

Substituting x = 560 in (i), we have

$5 \times 560 + 4y = 5600$

$\Rightarrow 4y = 5600 – 2800$

$\Rightarrow y = \frac{2800}{4} = 700$

Hence, the cost of a chair and that a table are respectively ₹ 560 and ₹ 700.

33. 23 spoons and 17 forks cost Rs.1770, while 17 spoons and 23 forks cost Rs.1830. Find the cost of each spoon and that of a fork.

Sol:

Let the cost of a spoon be Rs.x and that of a fork be Rs.y. Then

$23x + 17y = 1770$ ……….(i)

$17x + 23y = 1830$ ………(ii)

Adding (i) and (ii), we get

$40x + 40y = 3600$

$\Rightarrow x + y = 90$ ………..(iii)

Now, subtracting (ii) from (i), we get

$6x – 6y = -60$

$\Rightarrow x – y = -10$ ………..(iv)

Adding (iii) and (iv), we get

$2x = 80 \Rightarrow x = 40$

Substituting x = 40 in (iii), we get

$40 + y = 90 \Rightarrow y = 50$

Hence, the cost of a spoon that of a fork is Rs.40 and Rs.50 respectively.

34. A lady has only 50-paisa coins and 25-paisa coins in her purse. If she has 50 coins in all totaling Rs.19.50, how many coins of each kind does she have?

Sol:

Let x and y be the number of 50-paisa and 25-paisa coins respectively. Then

$x + y = 50$ ……….(i)

$0.5x + 0.25y = 19.50$ ………(ii)

Multiplying (ii) by 2 and subtracting it from (i), we get

$0.5y = 50 - 39$

$\Rightarrow y = \frac{11}{0.5} = 22$

Subtracting y = 22 in (i), we get

$x + 22 = 50$

$\Rightarrow x = 50 – 22 = 28$

Hence, the number of 25-paisa and 50-paisa coins is 22 and 28 respectively.

35. The sum of two numbers is 137 and their differences are 43. Find the numbers.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x + y = 137$ ……….(i)

$x - y = 43$ ………(ii)

On adding (i) and (ii), we get

$2x = 180 \Rightarrow x = 90$

On substituting x = 90 in (i), we get

$90 + y = 137$

$\Rightarrow y = (137 – 90) = 47$

Hence, the required numbers are 90 and 47.

36. Find two numbers such that the sum of twice the first and thrice the second is 92, and four times the first exceeds seven times the second by 2.

Sol:

Let the first number be x and the second number be y.

Then, we have:

$2x + 3y = 92$ ……….(i)

$4x - 7y = 2$ ……….(ii)

On multiplying (i) by 7 and (ii) by 3, we get

$14x + 21y = 644$ ………..(iii)

$12x - 21y = 6$ ………..(iv)

On adding (iii) and (iv), we get

$26x = 650$

$\Rightarrow x = 25$

On substituting x = 25 in (i), we get

$2 \times 25 + 3y = 92$

$\Rightarrow 50 + 3y = 92$

$\Rightarrow 3y = (92 – 50) = 42$

$\Rightarrow y = 14$

Hence, the first number is 25 and the second number is 14.

37. Find the numbers such that the sum of thrice the first and the second is 142, and four times the first exceeds the second by 138.

Sol:

Let the first number be x and the second number be y.

Then, we have:

$3x + y = 142$ ……….(i)
$4x - y = 138$ ………(ii)
On adding (i) and (ii), we get
$7x = 280$

$\Rightarrow x = 40$
On substituting x = 40 in (i), we get:

$3 \times 40 + y = 142$

$\Rightarrow y = (142 – 120) = 22$

$\Rightarrow y = 22$
Hence, the first number is 40 and the second number is 22.

38. If 45 is subtracted from twice the greater of two numbers, it results in the other number. If 21 is subtracted from twice the smaller number, it results in the greater number. Find the numbers.
Sol:
Let the greater number be x and the smaller number be y.
Then, we have:
$25x – 45 = y$ or $2x – y = 45$ ……….(i)
$2y - 21 = x$ or $–x + 2y = 21$ ……….(ii)
On multiplying (i) by 2, we get:
$4x - 2y = 90$ ………..(iii)
On adding (ii) and (iii), we get
$3x = (90 + 21) = 111$

$\Rightarrow x = 37$
On substituting x = 37 in (i), we get

$2 \times 37 - y = 45$

$\Rightarrow 74 - y = 45$

$\Rightarrow y = (74 - 45) = 29$
Hence, the greater number is 37 and the smaller number is 29.

39. If three times the larger of two numbers is divided by the smaller, we get 4 as the quotient and 8 as the remainder. If five times the smaller is divided by the larger, we get 3 as the quotient and 5 as the remainder. Find the numbers.
Sol:
We know:

Dividend = Divisor × Quotient + Remainder
Let the larger number be x and the smaller be y.

Then, we have:

$3x = y \times 4 + 8$ or $3x - 4y = 8$ ……….(i)

$5y = x \times 3 + 5$ or $-3x + 5y = 5$ ………(ii)

On adding (i) and (ii), we get:

$y = (8 + 5) = 13$

On substituting y = 13 in (i) we get

$3x - 4 \times 13 = 8$

$\Rightarrow 3x = (8 + 52) = 60$

$\Rightarrow x = 20$

Hence, the larger number is 20 and the smaller number is 13.

40. If 2 is added to each of two given numbers, their ratio becomes 1 : 2. However, if 4 is subtracted from each of the given numbers, the ratio becomes 5 : 11. Find the numbers.

Sol:

Let the required numbers be x and y.

Now, we have:

$\frac{x+2}{y+2} = \frac{1}{2}$

By cross multiplication, we get:

$2x + 4 = y + 2$

$\Rightarrow 2x - y = -2$ ……(i)

Again, we have:

$\frac{x-4}{y-4} = \frac{5}{11}$

By cross multiplication, we get:

$11x - 44 = 5y - 20$

$\Rightarrow 11x - 5y = 24$ ……(ii)

On multiplying (i) by 5, we get:

$10x - 5y = -10$

On subtracting (iii) from (ii), we get:

$x = (24 + 10) = 34$

On substituting x = 34 in (i), we get:

$2 \times 34 - y = -2$

$\Rightarrow 68 - y = -2$

$\Rightarrow y = (68 + 2) = 70$

Hence, the required numbers are 34 and 70.

41. The difference between two numbers is 14 and the difference between their squares is 448. Find the numbers.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x - y = 14$ or $x = 14 + y$ ……….(i)

$x^2 - y^2 = 448$ ………(ii)

On substituting $x = 14 + y$ in (ii) we get

$(14 + y)^2 - y^2 = 448$

$\Rightarrow 196 + y^2 + 28y - y^2 = 448$

$\Rightarrow 196 + 28y = 448$

$\Rightarrow 28y = (448 - 196) = 252$

$\Rightarrow y = \frac{252}{28} = 9$

On substituting $y = 9$ in (i), we get:

$x = 14 + 9 = 23$

Hence, the required numbers are 23 and 9.

42. The sum of the digits of a two-digit number is 12. The number obtained by interchanging its digits exceeds the given number by 18. Find the number.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number $= (10x + y)$

$x + y = 12$ ……….(i)

Number obtained on reversing its digits $= (10y + x)$

$\therefore (10y + x) - (10x + y) = 18$

$\Rightarrow 10y + x - 10x - y = 18$

$\Rightarrow 9y - 9x = 18$

$\Rightarrow y - x = 2$ ……..(ii)

On adding (i) and (ii), we get:

$2y = 14$

$\Rightarrow y = 7$

On substituting $y = 7$ in (i) we get

$x + 7 = 12$

$\Rightarrow x = (12 - 7) = 5$

Number $= (10x + y) = 10 \times 5 + 7 = 50 + 7 = 57$

Hence, the required number is 57.

43. A number consisting of two digits is seven times the sum of its digits. When 27 is subtracted from the number, the digits are reversed. Find the number.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$10x + y = 7(x + y)$

$10x + 7y = 7x + 7y$ or $3x - 6y = 0$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$(10x + y) - 27 = (10y + x)$

$\Rightarrow 10x - x + y - 10y = 27$

$\Rightarrow 9x - 9y = 27$

$\Rightarrow 9(x - y) = 27$

$\Rightarrow x - y = 3$ ……..(ii)

On multiplying (ii) by 6, we get:

$6x - 6y = 18$ ………(iii)

On subtracting (i) from (ii), we get:

$3x = 18$

$\Rightarrow x = 6$

On substituting $x = 6$ in (i) we get

$3 \times 6 - 6y = 0$

$\Rightarrow 18 - 6y = 0$

$\Rightarrow 6y = 18$

$\Rightarrow y = 3$

Number = $(10x + y) = 10 \times 6 + 3 = 60 + 3 = 63$

Hence, the required number is 63.

44. The sum of the digits of a two-digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$x + y = 15$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10y + x) - (10x + y) = 9$

$\Rightarrow 10y + x - 10x - y = 9$

$\Rightarrow 9y - 9x = 9$

$\Rightarrow y - x = 1$ ……..(ii)
On adding (i) and (ii), we get:
$2y = 16$

$\Rightarrow y = 8$
On substituting y = 8 in (i) we get
$x + 8 = 15$

$\Rightarrow x = (15 - 8) = 7$

Number = $(10x + y) = 10 \times 7 + 8 = 70 + 8 = 78$
Hence, the required number is 78.

45. A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.
Sol:
Let the tens and the units digits of the required number be x and y, respectively.
Required number = $(10x + y)$
$10x + y = 4(x + y) + 3$

$\Rightarrow 10x + y = 4x + 4y + 3$

$\Rightarrow 6x - 3y = 3$

$\Rightarrow 2x - y = 1$ ………(i)
Again, we have:
$10x + y + 18 = 10y + x$

$\Rightarrow 9x - 9y = -18$

$\Rightarrow x - y = -2$ ……..(ii)
On subtracting (ii) from (i), we get:
$x = 3$
On substituting x = 3 in (i) we get

$2 \times 3 - y = 1$

$\Rightarrow y = 6 - 1 = 5$

Required number = $(10x + y) = 10 \times 3 + 5 = 30 + 5 = 35$
Hence, the required number is 35.

46. A number consists of two digits. When it is divided by the sum of its digits, the quotient is 6 with no remainder. When the number is diminished by 9, the digits are reversed. Find the number.
Sol:

We know:

Dividend = Divisor × Quotient + Remainder

Let the tens and the units digits of the required number be x and y, respectively.
Required number = $(10x + y)$

$10x + y = (x + y) \times 6 + 0$

$\Rightarrow 10x – 6x + y – 6y = 0$

$\Rightarrow 4x – 5y = 0$ …….(i)

Number obtained on reversing its digits = $(10y + x)$

$\therefore 10x + y - 9 = 10y + x$

$\Rightarrow 9x – 9y = 9$

$\Rightarrow x – y = 1$ ……..(ii)

On multiplying (ii) by 5, we get:

$5x – 5y = 5$ ……..(iii)

On subtracting (i) from (iii), we get:

$x = 5$

On substituting $x = 5$ in (i) we get

$4 \times 5 – 5y = 0$

$\Rightarrow 20 - 5y = 0$

$\Rightarrow y = 4$

$\therefore$ The number = $(10x + y) = 10 \times 5 + 4 = 50 + 4 = 54$

Hence, the required number is 54.

47. A two-digit number is such that the product of its digits is 35. If 18 is added to the number, the digits interchange their places. Find the number.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Then, we have:

$xy = 35$ …….(i)

Required number = $(10x + y)$

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10x + y) + 18 = 10y + x$

$\Rightarrow 9x – 9y = -18$

$\Rightarrow 9(y – x) = 18$

$\Rightarrow y – x = 2$ ……..(ii)

We know:

$(y + x)^2 – (y – x)^2 = 4xy$

$\Rightarrow (y + x) = \pm \sqrt{(y-x)^2 + 4xy}$

$\Rightarrow (y + x) = \pm \sqrt{4 + 4 \times 35} = \pm \sqrt{144} = \pm 12$

$\Rightarrow y + x = 12$ ……..(iii) ($\because$ x and y cannot be negative)

On adding (ii) and (iii), we get:

$2y = 2 + 12 = 14$

$\Rightarrow y = 7$

On substituting y = 7in (ii) we get

$7 – x = 2$

$\Rightarrow x = (7 – 2) = 5$

$\therefore$ The number = $(10x + y) = 10 \times 5 + 7 = 50 + 7 = 57$

Hence, the required number is 57.

48. A two-digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Then, we have:

$xy = 18$ …….(i)

Required number = $(10x + y)$

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10x + y) - 63 = 10y + x$

$\Rightarrow 9x – 9y = 63$

$\Rightarrow 9(x – y) = 63$

$\Rightarrow x – y = 7$ ……..(ii)

We know:

$(x + y)^2 – (x – y)^2 = 4xy$

$\Rightarrow (x + y) = \pm \sqrt{(x-y)^2 + 4xy}$

$\Rightarrow (x + y) = \pm \sqrt{49 + 4 \times 18}$

$= \pm \sqrt{49 + 72}$

$= \pm \sqrt{121} = \pm 11$

$\Rightarrow x + y = 11$ ……..(iii) ($\because$ x and y cannot be negative)

On adding (ii) and (iii), we get:

$2x = 7 + 11 = 18$

$\Rightarrow x = 9$

On substituting x = 9in (ii) we get

$9 – y = 7$

$\Rightarrow y = (9 – 7) = 2$

$\therefore$ Number = $(10x + y) = 10 \times 9 + 2 = 90 + 2 = 92$

Hence, the required number is 92.

49. The sum of a two-digit number and the number obtained by reversing the order of its digits is 121, and the two digits differ by 3. Find the number,

Sol:

Let x be the ones digit and y be the tens digit. Then

Two digit number before reversing = $10y + x$

Two digit number after reversing = $10x + y$

As per the question

$(10y + x) + (10x + y) = 121$

$\Rightarrow 11x + 11y = 121$

$\Rightarrow x + y = 11$ …….(i)

Since the digits differ by 3, so

$x – y = 3$ ………(ii)

Adding (i) and (ii), we get

$2x = 14 \Rightarrow x = 7$

Putting x = 7 in (i), we get

$7 + y = 11 \Rightarrow y = 4$

Changing the role of x and y, x = 4 and y = 7

Hence, the two-digit number is 74 or 47.

50. The sum of the numerator and denominator of a fraction is 8. If 3 is added to both of the numerator and the denominator, the fraction becomes $\frac{3}{4}$. Find the fraction.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

$x + y = 8$ ……(i)

And, $\frac{x+3}{y+3} = \frac{3}{4}$

$\Rightarrow 4(x + 3) = 3(y + 3)$

$\Rightarrow 4x + 12 = 3y + 9$

$\Rightarrow$ 4x – 3y = -3 ……(ii)
On multiplying (i) by 3, we get:
3x + 3y = 24
On adding (ii) and (iii), we get:
7x = 21
$\Rightarrow$ x = 3
On substituting x = 3 in (i), we get:
3 + y = 8
$\Rightarrow$ y = (8 – 3) = 5
$\therefore$ x = 3 and y = 5
Hence, the required fraction is $\frac{3}{5}$.

51. If 2 is added to the numerator of a fraction, it reduces to $\left(\frac{1}{2}\right)$ and if 1 is subtracted from the denominator, it reduces to $\left(\frac{1}{3}\right)$. Find the fraction.

Sol:
Let the required fraction be $\frac{x}{y}$.
Then, we have:
$\frac{x+2}{y} = \frac{1}{2}$
$\Rightarrow$ 2(x + 2) = y
$\Rightarrow$ 2x + 4 = y
$\Rightarrow$2x - y = -4 ……(i)
Again, $\frac{x}{y-1} = \frac{1}{3}$
$\Rightarrow$3x = 1(y – 1)
$\Rightarrow$3x – y = -1 ……(ii)
On subtracting (i) from (ii), we get:
x = (-1 + 4) = 3
On substituting x = 3 in (i), we get:
2 × 3 – y = -4
$\Rightarrow$ 6 – y = -4
$\Rightarrow$ y = (6 + 4) = 10
$\therefore$ x = 3 and y = 10
Hence, the required fraction is $\frac{3}{10}$.

52. The denominator of a fraction is greater than its numerator by 11. If 8 is added to both its numerator and denominator, it becomes $\frac{3}{4}$. Find the fraction.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

y = x + 11

⇒ y – x = 11 ……(i)

Again, $\frac{x+8}{y+8} = \frac{3}{4}$

⇒4(x + 8) = 3(y + 8)

⇒4x + 32 = 3y + 24

⇒ 4x – 3y = -8 ……(ii)

On multiplying (i) by 4, we get:

4y – 4x = 44

On adding (ii) and (iii), we get:

y = (-8 + 44) = 36

On substituting y = 36 in (i), we get:

36 – x = 11

⇒ x = (36 – 11) = 25

∴ x = 25 and y = 36

Hence, the required fraction is $\frac{25}{36}$.

53. Find a fraction which becomes $\left(\frac{1}{2}\right)$ when 1 is subtracted from the numerator and 2 is added to the denominator, and the fraction becomes $\left(\frac{1}{3}\right)$ when 7 is subtracted from the numerator and 2 is subtracted from the denominator.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

$\frac{x-1}{y+2} = \frac{1}{2}$

⇒ 2(x – 1) = 1(y + 2)

⇒2x – 2 = y + 2

⇒2x – y = 4 ……(i)

Again, $\frac{x-7}{y-2} = \frac{1}{3}$

⇒3(x – 7) = 1(y – 2)

$\Rightarrow 3x - 21 = y - 2$

$\Rightarrow 3x - y = 19$ ……(ii)

On subtracting (i) from (ii), we get:

$x = (19 - 4) = 15$

On substituting x = 15 in (i), we get:

$2 \times 15 - y = 4$

$\Rightarrow 30 - y = 4$

$\Rightarrow y = 26$

$\therefore$ x = 15 and y = 26

Hence, the required fraction is $\frac{15}{26}$.

54. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3. They are in the ratio of 2: 3. Determine the fraction.

Sol:

Let the required fraction be $\frac{x}{y}$.

As per the question

$x + y = 4 + 2x$

$\Rightarrow y - x = 4$ ……(i)

After changing the numerator and denominator

New numerator = x + 3

New denominator = y + 3

Therefore

$\frac{x+3}{y+3} = \frac{2}{3}$

$\Rightarrow 3(x + 3) = 2(y + 3)$

$\Rightarrow 3x + 9 = 2y + 6$

$\Rightarrow 2y - 3x = 3$ ……(ii)

Multiplying (i) by 3 and subtracting (ii), we get:

$3y - 2y = 12 - 3$

$\Rightarrow y = 9$

Now, putting y = 9 in (i), we get:

$9 - x = 4 \Rightarrow x = 9 - 4 = 5$

Hence, the required fraction is $\frac{5}{9}$.

55. The sum of two numbers is 16 and the sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x + y = 16$ ……(i)

And, $\frac{1}{x} + \frac{1}{y} = \frac{1}{3}$ ……(ii)

$\Rightarrow 3(x + y) = xy$

$\Rightarrow 3 \times 16 = xy$ [Since from (i), we have: $x + y = 16$]

$\therefore xy = 48$ …….(iii)

We know:

$(x - y)^2 = (x + y)^2 - 4xy$

$(x - y)^2 = (16)^2 - 4 \times 48 = 256 - 192 = 64$

$\therefore (x - y) = \pm\sqrt{64} = \pm 8$

Since x is larger and y is smaller, we have:

$x - y = 8$ ………(iv)

On adding (i) and (iv), we get:

$2x = 24$

$\Rightarrow x = 12$

On substituting $x = 12$ in (i), we get:

$12 + y = 16 \Rightarrow y = (16 - 12) = 4$

Hence, the required numbers are 12 and 4.

56. There are two classrooms A and B. If 10 students are sent from A to B, the number of students in each room becomes the same. If 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in each room.

Sol:

Let the number of students in classroom A be x

Let the number of students in classroom B be y.

If 10 students are transferred from A to B, then we have:

$x - 10 = y + 10$

$\Rightarrow x - y = 20$ …..(i)

If 20 students are transferred from B to A, then we have:

$2(y - 20) = x + 20$

$\Rightarrow 2y - 40 = x + 20$

$\Rightarrow -x + 2y = 60$ …..(ii)

On adding (i) and (ii), we get:
y = (20 + 60) = 80
On substituting y = 80 in (i), we get:
x – 80 = 20
⇒x = (20 + 80) = 100
Hence, the number of students in classroom A is 100 and the number of students in classroom B is 80.

57. Taxi charges in a city consist of fixed charges per day and the remaining depending upon the distance travelled in kilometers. If a person travels 80km, he pays Rs. 1330, and for travelling 90km, he pays Rs. 1490. Find the fixed charges per day and the rate per km.
Sol:
Let fixed charges be Rs.x and rate per km be Rs.y.
Then as per the question
$x + 80y = 1330$ ………(i)
$x + 90y = 1490$ ……..(ii)
Subtracting (i) from (ii), we get
$10y = 160 \Rightarrow y = \frac{160}{10} = 16$
Now, putting y = 16, we have
$x + 80 \times 16 = 1330$
$\Rightarrow x = 1330 - 1280 = 50$
Hence, the fixed charges be Rs.50 and the rate per km is Rs.16.

58. A part of monthly hostel charges in a college are fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 25days, he has to pay Rs. 4550 as hostel charges whereas a student B, who takes food for 30 days, pays Rs. 5200 as hostel charges. Find the fixed charges and the cost of the food per day.
Sol:
Let the fixed charges be Rs.x and the cost of food per day be Rs.y.
Then as per the question
$x + 25y = 4500$ ………(i)
$x + 30y = 5200$ ……..(ii)
Subtracting (i) from (ii), we get
$5y = 700 \Rightarrow y = \frac{700}{5} = 140$
Now, putting y = 140, we have
$x + 25 \times 140 = 4500$
$\Rightarrow x = 4500 - 3500 = 1000$
Hence, the fixed charges be Rs.1000 and the cost of the food per day is Rs.140.

59. A man invested an amount at 10% per annum simple interest and another amount at 10% per annum simple interest. He received an annual interest of Rs. 1350. But, if he had interchanged the amounts invested, he would have received Rs. 45 less. What amounts did he invest at different rates?

Sol:

Let the amounts invested at 10% and 8% be Rs.x and Rs.y respectively.

Then as per the question

$\frac{x \times 10 \times 1}{100} = \frac{y \times 8 \times 1}{100} = 1350$

$10x + 8y = 135000$ …………..(i)

After the amounts interchanged but the rate being the same, we have

$\frac{x \times 8 \times 1}{100} = \frac{y \times 10 \times 1}{100} = 1350 - 45$

$8x + 10y = 130500$ …………….(ii)

Adding (i) and (ii) and dividing by 9, we get

$2x + 2y = 29500$ ……………(iii)

Subtracting (ii) from (i), we get

$2x - 2y = 4500$

Now, adding (iii) and (iv), we have

$4x = 34000$

$x = \frac{34000}{4} = 8500$

Putting x = 8500 in (iii), we get

$2 \times 8500 + 2y = 29500$

$2y = 29500 - 17000 = 12500$

$y = \frac{12500}{2} = 6250$

Hence, the amounts invested are Rs. 8,500 at 10% and Rs. 6,250 at 8%.

60. The monthly incomes of A and B are in the ratio of 5 : 4 and their monthly expenditures are in the ratio of 7 : 5. If each saves Rs. 9000 per month, find the monthly income of each.

Sol:

Let the monthly income of A and B are Rs.x and Rs.y respectively.

Then as per the question

$\frac{x}{y} = \frac{5}{4}$

$\Rightarrow y = \frac{4x}{5}$

Since each save Rs.9,000, so

Expenditure of A = Rs.(x – 9000)

Expenditure of B = Rs.(y – 9000)

The ratio of expenditures of A and B are in the ratio 7:5.

$\therefore \frac{x-9000}{y-9000} = \frac{7}{5}$

$\Rightarrow 7y - 63000 = 5x - 45000$

$\Rightarrow 7y - 5x = 18000$

From (i), substitute $y = \frac{4x}{5}$ in (ii) to get

$7 \times \frac{4x}{5} - 5x = 18000$

$\Rightarrow 28x - 25x = 90000$

$\Rightarrow 3x = 90000$

$\Rightarrow x = 30000$

Now, putting x = 30000, we get

$y = \frac{4 \times 30000}{5} = 4 \times 6000 = 24000$

Hence, the monthly incomes of A and B are Rs. 30,000 and Rs.24,000.

61. A man sold a chair and a table together for Rs. 1520, thereby making a profit of 25% on chair and 10% on table. By selling them together for Rs. 1535, he would have made a profit of 10% on the chair and 25% on the table. Find the cost price of each.

Sol:

Let the cost price of the chair and table be Rs.x and Rs.y respectively.

Then as per the question

Selling price of chair + Selling price of table = 1520

$\frac{100+25}{100} \times x + \frac{100+10}{100} \times y = 1520$

$\Rightarrow \frac{125}{100} x + \frac{110}{100} y = 1520$

$\Rightarrow 25x + 22y - 30400 = 0$ ……….(i)

When the profit on chair and table are 10% and 25% respectively, then

$\frac{100+10}{100} \times x + \frac{100+25}{100} \times y = 1535$

$\Rightarrow \frac{110}{100} x + \frac{125}{100} y = 1535$

$\Rightarrow 22x + 25y - 30700 = 0$ ……….(ii)

Solving (i) and (ii) by cross multiplication, we get

$\frac{x}{(22)(-30700)-(25)(-30400)} = \frac{y}{(-30400)(22)-(-30700)(25)} = \frac{1}{(25)(25)-(22)(22)}$

$\Rightarrow \frac{x}{7600-6754} = \frac{y}{7675-6688} = \frac{100}{3 \times 47}$

$\Rightarrow \frac{x}{846} = \frac{y}{987} = \frac{100}{3 \times 47}$

$\Rightarrow x = \frac{100 \times 846}{3 \times 47}, y = \frac{100 \times 987}{3 \times 47}$

⇒ x = 600, y = 700

Hence, the cost of chair and table are Rs.600 and Rs.700 respectively.

62. Points A and B are 70 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours. But, if they travel towards each other, they meet in 1 hour. Find the speed of each car.

Sol:

Let X and Y be the cars starting from points A and B, respectively and let their speeds be x km/h and y km/h, respectively.

Then, we have the following cases:

Case I: When the two cars move in the same direction

In this case, let the two cars meet at point M.

Distance covered by car X in 7 hours = 7x km

Distance covered by car Y in 7 hours = 7y km

$\therefore$ AM = (7x) km and BM = (7y) km

$\Rightarrow (AM - BM) = AB$

$\Rightarrow (7x - 7y) = 70$

$\Rightarrow 7(x - y) = 70$

$\Rightarrow (x - y) = 10$ ……..(i)

Case II: When the two cars move in opposite directions

In this case, let the two cars meet at point N.

Distance covered by car X in 1 hour = x km

Distance covered by car Y in 1 hour = y km

$\therefore$ AN = x km and BN = y km

$\Rightarrow AN + BN = AB$

$\Rightarrow x + y = 70$ ………(ii)

On adding (i) and (ii), we get:

$2x = 80$

$\Rightarrow x = 40$

On substituting x = 40 in (i), we get:

$40 - y = 10$

$\Rightarrow y = (40 - 10) = 30$

Hence, the speed of car X is 40km/h and the speed of car Y is 30km/h.

63. A train covered a certain distance at a uniform speed. If the train had been 5 kmph faster, it would have taken 3 hours less than the scheduled time. And, if the train were slower by 4 kmph, it would have taken 3 hours more than the scheduled time. Find the length of the journey.

Sol:

Let the original speed be x kmph and let the time taken to complete the journey be y hours.

$\therefore$ Length of the whole journey = (xy) km

Case I:

When the speed is (x + 5) kmph and the time taken is (y – 3) hrs:

Total journey = (x + 5) (y – 3) km

$\Rightarrow (x + 5)(y - 3) = xy$

$\Rightarrow xy + 5y - 3x - 15 = xy$

$\Rightarrow 5y - 3x = 15$ ....(i)

Case II:

When the speed is (x – 4) kmph and the time taken is (y + 3) hrs:

Total journey = (x – 4) (y + 3) km

$\Rightarrow (x - 4)(y + 3) = xy$

$\Rightarrow xy - 4y + 3x - 12 = xy$

$\Rightarrow 3x - 4y = 12$ ....(ii)

On adding (i) and (ii), we get:

y = 27

On substituting y = 27 in (i), we get:

$5 \times 27 - 3x = 15$

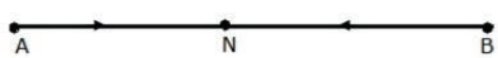
$\Rightarrow 135 - 3x = 15$

$\Rightarrow 3x = 120$

$\Rightarrow x = 40$

$\therefore$ Length of the journey = (xy) km = (40×27) km = 1080 km

64. Abdul travelled 300 km by train and 200 km by taxi taking 5 hours and 30 minutes. But, if he travels 260km by train and 240km by taxi, he takes 6 minutes longer. Find the speed of the train and that of taxi.



Sol:

Let the speed of the train and taxi be x km/h and y km/h respectively.

Then as per the question

$\frac{3}{x} + \frac{2}{y} = \frac{11}{200}$ ....(i)

When the speeds of the train and taxi are 260 km and 240 km respectively, then

$\frac{260}{x} + \frac{240}{y} = \frac{11}{2} + \frac{6}{60}$

$\Rightarrow \frac{13}{x} + \frac{12}{y} = \frac{28}{100}$ ………….(ii)

Multiplying (i) by 6 and subtracting (ii) from it, we get

$\frac{18}{x} - \frac{13}{x} = \frac{66}{200} - \frac{28}{100}$

$\Rightarrow \frac{5}{x} = \frac{10}{200} \Rightarrow$ x = 100

Putting x = 100 in (i), we have

$\frac{3}{100} + \frac{2}{y} = \frac{11}{200}$

$\Rightarrow \frac{2}{y} = \frac{11}{200} - \frac{3}{100} = \frac{1}{40}$

$\Rightarrow$ y = 80

Hence, the speed of the train and that of the taxi are 100 km/h and 80 km/h respectively.

65. Places A and B are 160 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 8 hours. But, if they travel towards each other, they meet in 2 hours. Find the speed of each car.

Sol:

Let the speed of the car A and B be x km/h and y km/h respectively. Let x > y.

Case-1: When they travel in the same direction

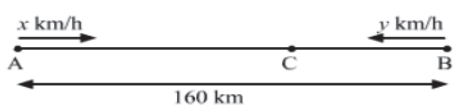
From the figure

AC – BC = 160

$\Rightarrow$ x × 8 – y × 8 = 160

$\Rightarrow$ x – y = 20

Case-2: When they travel in opposite direction



From the figure

AC + BC = 160

$\Rightarrow$ x × 2 + y × 2 = 160

$\Rightarrow$ x + y = 80

Adding (i) and (ii), we get

2x = 100 $\Rightarrow$ x = 50 km/h

Putting x = 50 in (ii), we have

$50 + y = 80 \Rightarrow y = 80 - 50 = 30$ km/h

Hence, the speeds of the cars are 50 km/h and 30 km/h.

66. A sailor goes 8 km downstream in 420 minutes and returns in 1 hour. Find the speed of the sailor in still water and the speed of the current .

Sol:

Let the speed of the sailor in still water be x km/h and that of the current y km/h.

Speed downstream = (x + y) km/h

Speed upstream = (x – y) km/h

As per the question

$(x + y) \times \frac{40}{60} = 8$

$\Rightarrow x + y = 12$ ....(i)

When the sailor goes upstream, then

$(x - y) \times 1 = 8$

$x - y = 8$ ....(ii)

Adding (i) and (ii), we get

$2x = 20 \Rightarrow x = 10$

Putting x = 10 in (i), we have

$10 + y = 12 \Rightarrow y = 2$

Hence, the speeds of the sailor in still water and the current are 10 km/h and 2 km/h respectively.

67. A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream

Sol:

Let the speed of the boat in still water be x km/h and the speed of the stream be y km/h.

Then we have

Speed upstream = (x – y) km/hr

Speed downstream = (x + y) km/hr

Time taken to cover 12 km upstream = $\frac{12}{(x-y)}$ hrs

Time taken to cover 40 km downstream = $\frac{40}{(x+y)}$ hrs

Total time taken = 8 hrs

$\therefore \frac{12}{(x-y)} + \frac{40}{(x+y)} = 8$ ....(i)

Again, we have:

Time taken to cover 16 km upstream = $\frac{16}{(x-y)}$ hrs

Time taken to cover 32 km downstream = $\frac{32}{(x+y)}$ hrs

Total time taken = 8 hrs

$\therefore \frac{16}{(x-y)} + \frac{32}{(x+y)} = 8$ ……….(ii)

Putting $\frac{1}{(x-y)}$ = u and $\frac{1}{(x+y)}$ = v in (i) and (ii), we get:

12u + 40v = 8

3u + 10v = 2 ………(a)

And, 16u + 32v = 8

⇒2u + 4v = 1 ……….(b)

On multiplying (a) by 4 and (b) by 10, we get:

12u + 40v = 8 …….(iii)

And, 20u + 40v = 10 …….(iv)

On subtracting (iii) from (iv), we get:

8u = 2

$\Rightarrow u = \frac{2}{8} = \frac{1}{4}$

On substituting $u = \frac{1}{4}$ in (iii), we get:

40v = 5

$\Rightarrow v = \frac{5}{40} = \frac{1}{8}$

Now, we have:

$u = \frac{1}{4}$

$\Rightarrow \frac{1}{(x-y)} = \frac{1}{4} \Rightarrow x - y = 4$ ……..(v)

$v = \frac{1}{8}$

$\Rightarrow \frac{1}{(x+y)} = \frac{1}{8} \Rightarrow x + y = 8$ ……..(vi)

On adding (v) and (vi), we get:

2x = 12

⇒ x = 6

On substituting x = 6 in (v), we get:

6 – y = 4

y = (6 – 4) = 2

∴ Speed of the boat in still water = 6km/h

And, speed of the stream = 2 km/h

68. 2 men and 5 boys can finish a piece of work in 4 days, while 3 men and 6 boys can finish it in 3 days. Find the time taken by one man alone to finish the work and that taken by one boy alone to finish the work.

Sol:

Let us suppose that one man alone can finish the work in x days and one boy alone can finish it in y days.

$\therefore$ One man's one day's work = $\frac{1}{x}$

And, one boy's one day's work = $\frac{1}{y}$

2 men and 5 boys can finish the work in 4 days.

$\therefore$ (2 men's one day's work) + (5 boys' one day's work) = $\frac{1}{4}$

$\Rightarrow \frac{2}{x} + \frac{5}{y} = \frac{1}{4}$

$\Rightarrow 2u + 5v = \frac{1}{4}$ …….(i) Here, $\frac{1}{x} = u$ and $\frac{1}{y} = v$

Again, 3 men and 6 boys can finish the work in 3days.

$\therefore$ (3 men's one day's work) + (6 boys' one day's work) = $\frac{1}{3}$

$\Rightarrow \frac{3}{x} + \frac{6}{y} = \frac{1}{3}$

$\Rightarrow 3u + 6v = \frac{1}{3}$ …….(ii) Here, $\frac{1}{x} = u$ and $\frac{1}{y} = v$

On multiplying (iii) from (iv), we get:

$3u = \left(\frac{5}{3} - \frac{6}{4}\right) = \frac{2}{12} = \frac{1}{6}$

$\Rightarrow u = \frac{1}{6 \times 3} = \frac{1}{18} \Rightarrow \frac{1}{x} = \frac{1}{18} \Rightarrow x = 18$

On substituting $u = \frac{1}{18}$ in (i), we get:

$2 \times \frac{1}{18} + 5v = \frac{1}{4} \Rightarrow 5v = \left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}$

$\Rightarrow v = \left(\frac{5}{36} \times \frac{1}{5}\right) = \frac{1}{36} \Rightarrow \frac{1}{y} = \frac{1}{36} \Rightarrow y = 36$

Hence, one man alone can finish the work is 18days and one boy alone can finish the work in 36 days.

69. The length of a room exceeds its breadth by 3 meters. If the length is increased by 3 meters and the breadth is decreased by 2 meters, the area remains the same. Find the length and the breadth of the room.

Sol:

Let the length of the room be x meters and he breadth of the room be y meters.

Then, we have:

Area of the room = xy

According to the question, we have:

$x = y + 3$

$\Rightarrow x - y = 3$ ....(i)

And, $(x + 3)(y - 2) = xy$

$\Rightarrow xy - 2x + 3y - 6 = xy$

$\Rightarrow 3y - 2x = 6$ ....(ii)

On multiplying (i) by 2, we get:

$2x - 2y = 6$ ....(iii)

On adding (ii) and (iii), we get:

$y = (6 + 6) = 12$

On substituting y = 12 in (i), we get:

$x - 12 = 3$

$\Rightarrow x = (3 + 12) = 15$

Hence, the length of the room is 15 meters and its breadth is 12 meters.

70. The area of a rectangle gets reduced by $8m^2$, when its length is reduced by 5m and its breadth is increased by 3m. If we increase the length by 3m and breadth by 2m, the area is increased by $74m^2$. Find the length and the breadth of the rectangle.

Sol:

Let the length and the breadth of the rectangle be x m and y m, respectively.

$\therefore$ Area of the rectangle = (xy) sq.m

Case 1:

When the length is reduced by 5m and the breadth is increased by 3 m:

New length = (x – 5) m

New breadth = (y + 3) m

$\therefore$ New area = (x – 5) (y + 3) sq.m

$\therefore xy - (x - 5)(y + 3) = 8$

$\Rightarrow xy - [xy - 5y + 3x - 15] = 8$

$\Rightarrow xy - xy + 5y - 3x + 15 = 8$

$\Rightarrow 3x - 5y = 7$ ....(i)

Case 2:

When the length is increased by 3 m and the breadth is increased by 2 m:

New length = (x + 3) m

New breadth = (y + 2) m

$\therefore$ New area = (x + 3) (y + 2) sq.m

$\Rightarrow (x + 3)(y + 2) - xy = 74$

$\Rightarrow [xy + 3y + 2x + 6] - xy = 74$

$\Rightarrow 2x + 3y = 68$ ....(ii)

On multiplying (i) by 3 and (ii) by 5, we get:
$9x - 15y = 21$ ……….(iii)
$10x + 15y = 340$ ………(iv)
On adding (iii) and (iv), we get:
$19x = 361$

$\Rightarrow x = 19$
On substituting x = 19 in (iii), we get:

$9 \times 19 - 15y = 21$

$\Rightarrow 171 - 15y = 21$

$\Rightarrow 15y = (171 - 21) = 150$

$\Rightarrow y = 10$
Hence, the length is 19m and the breadth is 10m.

71. The area of a rectangle gets reduced by 67 square meters, when its length is increased by 3m and the breadth is decreased by 4m. If the length is reduced by 1m and breadth is increased by 4m, the area is increased by 89 square meters, Find the dimension of the rectangle.
Sol:
Let the length and the breadth of the rectangle be x m and y m, respectively.
Case 1: When length is increased by 3m and the breadth is decreased by 4m:
$xy - (x + 3)(y - 4) = 67$

$\Rightarrow xy - xy + 4x - 3y + 12 = 67$

$\Rightarrow 4x - 3y = 55$ ………(i)
Case 2: When length is reduced by 1m and breadth is increased by 4m:
$(x - 1)(y + 4) - xy = 89$

$\Rightarrow xy + 4x - y - 4 - xy = 89$

$\Rightarrow 4x - y = 93$ ………(ii)
Subtracting (i) and (ii), we get:

$2y = 38 \Rightarrow y = 19$
On substituting y = 19 in (ii), we have
$4x - 19 = 93$

$\Rightarrow 4x = 93 + 19 = 112$

$\Rightarrow x = 28$
Hence, the length = 28m and breadth = 19m.

72. A railway half ticket costs half the full fare and the reservation charge is the some on half ticket as on full ticket. One reserved first class ticket from Mumbai to Delhi costs ₹4150 while one full and one half reserved first class ticket cost ₹ 6255. What is the basic first class full fare and what is the reservation charge?

Sol:

Let the basic first class full fare be Rs.x and the reservation charge be Rs.y.

Case 1: One reservation first class full ticket cost Rs.4, 150

$x + y = 4150$ ....(i)

Case 2: One full and one and half reserved first class tickets cost Rs.6,255

$(x + y) + \left(\frac{1}{2}x + y\right) = 6255$

$\Rightarrow 3x + 4y = 12510$ ....(ii)

Substituting $y = 4150 - x$ from (i) in (ii), we get

$3x + 4(4150 - x) = 12510$

$\Rightarrow 3x - 4x + 16600 = 12510$

$\Rightarrow x = 16600 - 12510 = 4090$

Now, putting $x = 4090$ in (i), we have

$4090 + y = 4150$

$\Rightarrow y = 4150 - 4090 = 60$

Hence, cost of basic first class full fare = Rs.4,090 and reservation charge = Rs.60.

73. Five years hence, a man's age will be three times the sum of the ages of his son. Five years ago, the man was seven times as old as his son. Find their present ages

Sol:

Let the present age of the man be x years and that of his son be y years.

After 5 years man's age = $x + 5$

After 5 years ago son's age = $y + 5$

As per the question

$x + 5 = 3(y + 5)$

$\Rightarrow x - 3y = 10$ ....(i)

5 years ago man's age = $x - 5$

5 years ago son's age = $y - 5$

As per the question

$x - 5 = 7(y - 5)$

$\Rightarrow x - 7y = -30$ ....(ii)

Subtracting (ii) from (i), we have

$4y = 40 \Rightarrow y = 10$

Putting $y = 10$ in (i), we get

$x - 3 \times 10 = 10$

$\Rightarrow x = 10 + 30 = 40$

Hence, man's present age = 40 years and son's present age = 10 years.

74. The present age of a man is 2 years more than five times the age of his son. Two years hence, the man's age will be 8 years more than three times the age of his son. Find their present ages.

Sol:

Let the man's present age be x years.
Let his son's present age be y years.
According to the question, we have:
Two years ago:
Age of the man = Five times the age of the son

$\Rightarrow (x-2) = 5(y-2)$

$\Rightarrow x - 2 = 5y - 10$

$\Rightarrow x - 5y = -8$ …….(i)

Two years later:
Age of the man = Three times the age of the son + 8

$\Rightarrow (x+2) = 3(y+2) + 8$

$\Rightarrow x + 2 = 3y + 6 + 8$

$\Rightarrow x - 3y = 12$ ………..(ii)

Subtracting (i) from (ii), we get:

$2y = 20$

$\Rightarrow y = 10$

On substituting y = 10 in (i), we get:

$x - 5 \times 10 = -8$

$\Rightarrow x - 50 = -8$

$\Rightarrow x = (-8 + 50) = 42$

Hence, the present age of the man is 42 years and the present age of the son is 10 years.

75. If twice the son's age in years is added to the mother's age, the sum is 70 years. But, if twice the mother's age is added to the son's age, the sum is 95 years. Find the age of the mother and that of the son.

Sol:

Let the mother's present age be x years.
Let her son's present age be y years.
Then, we have:

$x + 2y = 70$ …….(i)

And, $2x + y = 95$ ……(ii)

On multiplying (ii) by 2, we get:
$4x + 2y = 190$ ……..(iii)
On subtracting (i) from (iii), we get:
$3x = 120$
$\Rightarrow x = 40$
On substituting x = 40 in (i), we get:
$40 + 2y = 70$
$\Rightarrow 2y = (70 - 40) = 30$
$\Rightarrow y = 15$
Hence, the mother's present age is 40 years and her son's present age is 15 years.

76. The present age of a woman is 3 years more than three times the age of her daughter. Three years hence, the woman's age will be 10 years more than twice the age of her daughter. Find their present ages.
Sol:
Let the woman's present age be x years.
Let her daughter's present age be y years.
Then, we have:
$x = 3y + 3$
$\Rightarrow x - 3y = 3$ …….(i)

After three years, we have:
$(x + 3) = 2(y + 3) + 10$
$\Rightarrow x + 3 = 2y + 6 + 10$
$\Rightarrow x - 2y = 13$ ……(ii)
Subtracting (ii) from (i), we get:
$-y = (3 - 13) = -10$
$\Rightarrow y = 10$
On substituting y = 10 in (i), we get:
$x - 3 \times 10 = 3$
$\Rightarrow x - 30 = 3$
$\Rightarrow x = (3 + 30) = 33$
Hence, the woman's present age is 33 years and her daughter's present age is 10 years.

77. On selling a tea-set at 5% loss and a lemon-set at 15% gain, a shopkeeper gains Rs. 7. However, if he sells the tea-set at 5% gain and the lemon-set at 10% gain, he gains Rs. 14. Find the price of the tea-set and that of the lemon-set paid by the shopkeeper.
Sol: