Exercise 20(D)

1. The perimeter of a triangle is 450 cm and its side are in the ratio 12 : 5 : 13. Find the area of the triangle.
Solution:

Let's assume the sides of the triangle to be
a = 12x
b = 5x
c = 13x
And, given that the perimeter of the triangle = 450 cm
So, 12x + 5x + 13x = 450
⇒ 30x = 450
⇒ x = 15
Thus, the sides of a triangle are
a = 12x = 12(15) = 180 cm
b = 5x = 5(15) = 75 cm
c = 13x = 13(15) = 195 cm

Now,
The semi-perimeter of the triangle, s = (a + b + c)/2
= (180 + 75 + 195)/2
= 450/2
= 225 cm
Hence, the area of the triangle is given by
Area = $\sqrt{[s(s - a)(s - b)(s - c)]}$
= $\sqrt{[225(225 – 180)(225 – 75)(225 – 195)]}$
= $\sqrt{(225 \times 45 \times 150 \times 30)}$
= 15 $\sqrt{(9 \times 5 \times 5 \times 30 \times 30)}$
= 15 x 3 x 5 x 30
= 6750 cm^2

2. A triangle and a parallelogram have the same base and the same area. If the side of the triangle are 26 cm, 28 cm and 30 cm and the parallelogram stands on the base 28 cm, find the height of the parallelogram.
Solution:

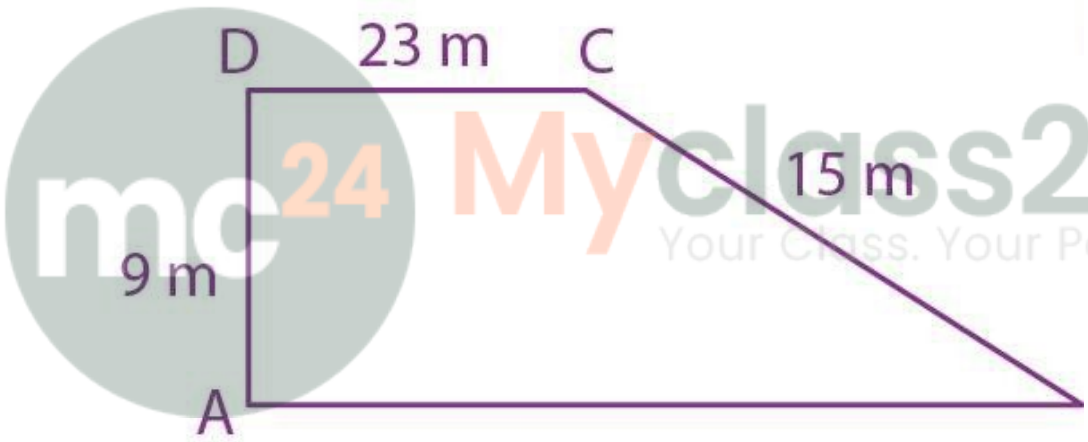
Let's assume the sides of the triangle to be
a = 26 cm, b = 28 cm and c = 30 cm
Now,
The semi-perimeter of the triangle, s = (a + b + c)/2
= (26 + 28 + 30)/2
= 84/2
= 42 cm
Hence, the area of the triangle is given by
Area = $\sqrt{[s(s - a)(s - b)(s - c)]}$
= $\sqrt{[42(42 – 26)(42 – 28)(42 – 30)]}$

$= \sqrt{(42 \times 16 \times 14 \times 12)}$
$= \sqrt{(7 \times 6 \times 4^2 \times 7 \times 2 \times 6 \times 2)}$
$= 7 \times 6 \times 4 \times 2$
$= 336\ cm^2$

Given, the base of the parallelogram = 28 cm
And, area of the parallelogram = area of the triangle
So,
Base x Height = 336
28 x Height = 336
Height = 336/28
= 12 cm

Therefore, the height of the parallelogram is 12 cm

3.
4. Using the information in the following figure, find its area.



Solution:

Let's make a construction of drawing CM ⊥ AB

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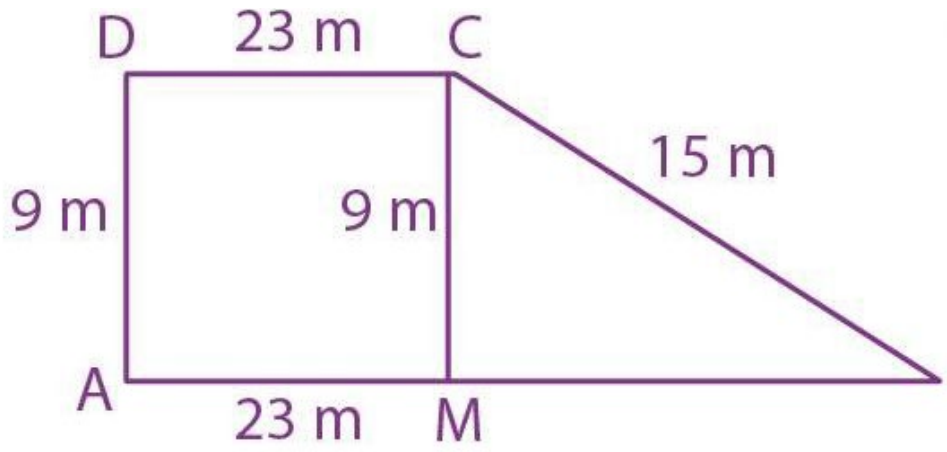

Now,
In right-angled triangle CMB, we have
$BM^2 = BC^2 - CM^2$ [By Pythagoras Theorem]
$= (15)^2 - (9)^2$
$= 225 - 81$
$= 144$ m
On taking square root on both sides, we get
BM = 12 m
Now,
AB = AM + BM
= 23 + 12
= 35 m

Hence, the area of the trapezium = ½ x (sum of parallel sides) x Height
= ½ x (AB + CD) x AD
= ½ x (23 + 35) x 9
= ½ x 58 x 9
= 261 m^2

5. Sum of the areas or two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares.
Solution:

Let's assume the sides of the two squares to be a and b respectively
Then,
Area of one square, $S_1 = a^2$
And, area of the second square, $S_2 = b^2$
According to the question, we have
$S_1 + S_2 = 400\ cm^2$
$\Rightarrow a^2 + b^2 = 400\ cm^2$ … (1)
Also given, the difference in their perimeters = 16 cm
$\Rightarrow 4a - 4b = 16$ cm

a - b = 4
a = (4 + b)
Substituting the value of 'a' in (1), we get
$(4 + b)^2 + b^2 = 400$
$16 + 8b + b^2 + b^2 = 400$
$2b^2 + 8b - 384 = 0$
$b^2 + 4b - 192 = 0$
$b^2 + 16b - 12b - 192 = 0$
b(b + 16) - 12(b + 16) = 0
(b +16) (b - 12) = 0
b + 16 = 0 or b - 12 = 0
⇒ b = -16 or b = 12
As, the side of a square cannot be negative, we neglect the value -16.
Hence, b = 12
And,
a = 4 + b = 4 + 12 = 16

Therefore, the sides of a square are 16 cm and 12 cm respectively

6. Find the area and the perimeter of a square with diagonal 24 cm. [Take √2 = 1.41]
Solution:

Given, the diagonal of a square = 24 cm
We know that,
Diagonal of a square = √2 times the side of a square
24 = √2 x (side of a square)
So,
Side of the square = 24/√2
= 12√2 cm

Thus,
The perimeter of the square = 4 x side
= 4 x 12√2
= 48√2
= 48 x 1.41
= 67.68 cm
And,
The area of the square = $(side)^2$
= $(12\sqrt{2})^2$
= 144 x 2
= 288 cm^2