NCERT Solutions for Class 9 Maths Chapter 2 Polynomials Ex 2.5

Question 1.

Use suitable identities to find the following products:

(i) $(x + 4) (x + 10)$

(ii) $(x + 8) (x – 10)$

(iii) $(3x + 4) (3x – 5)$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

(v) $(3 – 2x) (3 + 2x)$

Solution:

(i) Using identity,

$(x + a) (x + b) = x^2 + (a + b)x + ab$

In $(x + 4) (x + 10)$, $a = 4$ and $b = 10$

Now,

$(x + 4) (x + 10) = x^2 + (4 + 10)x + (4 * 10)$

$= x^2 + 14x + 40$

(ii) Using identity,

$(x + a) (x + b) = x^2 + (a + b) x + ab$

Here, $a = 8$ and $b = -10$

$(x + 8) (x – 10) = x^2 + \{8 + (-10)\} x + \{8 * (-10)\}$

$= x^2 + (8 – 10) x – 80$

$= x^2 – 2x – 80$

(iii) Using identity,

$(x + a) (x + b) = x^2 + (a + b) x + ab$

Here, x = 3x, a = 4 and b = -5

$(3x + 4) (3x – 5) = (3x)^2 + \{4 + (-5)\} 3x + \{4 * (-5)\}$

$= 9x^2 + 3x (4 – 5) – 20$

$= 9x^2 – 3x – 20$

(iv) Using identity,

$(x + y) (x – y) = x^2 – y^2$

Here, x = 3 and y = 2x

$(3 – 2x) (3 + 2x) = 3^2 – (2x)^2$

$= 9 – 4x^2$

(v) Using identity,

$(x + y) (x – y) = x^2 – y^2$

Here, x = 3 and y = 2z

$(3 – 2x) (3 + 2x) = 3^2 – (2x)^2 = 9 – 4x^2$

Question 2.

Evaluate the following products without multiplying directly:

(i) 103 × 107

(ii) 95 × 96s

(iii) 104 × 96

Solution:

(i) Using identity,

$(x + a) (x + b) = x^2 + (a + b) x + ab$

Here, $x = 100$, $a = 3$ and $b = 7$

$103 * 107 = (100 + 3) (100 + 7)$

$= (100)^2 + (3 + 7) 10 + (3 * 7)$

$= 10000 + 100 + 21$

$= 10121$

(ii) Using identity,

$(x + a) (x + b) = x^2 + (a + b) x + ab$

Here, $x = 90$, $a = 5$ and $b = 4$

$95 * 96 = (90 + 5) (90 + 4)$

$= (90)^2 + (3 + 7) 90 + (5 * 6)$

$= 8100 + 990 + 30$

$= 9120$

(iii) Using identity,

$(x + y) (x – y) = x^2 – y^2$

Here, $x = 100$ and $y = 4$

$104 * 96 = (100 + 4) (100 – 4)$

$= (100)^2 – (4)^2$

$= 10000 – 16$

$= 9984$

Question 3.

Factorize the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Solution:

(i) $9x^2 + 6xy + y^2 = (3x)^2 + (2 * 3x * y) + y^2$

Using identity, $(a + b)^2 = a^2 + 2ab + b^2$

Here, a = 3x and b = y

$9x^2 + 6xy + y^2 = (3x)^2 + (2 * 3 * y) + y^2$

$= (3x + y)^2$

$= (3x + y) (3x + y)$

(ii) $4y^2 - 4y + 1 = (4y)^2 - (2 * 2y * 1) + 1^2$

Using identity, $(a - b)^2 = a^2 - 2ab + b^2$

Here, a = 2y and b = 1

$4y^2 - 4y + 1 = (2y)^2 - (2 * 2y * 1) + 1^2$

$= (2y - 1)^2$

$= (2y - 1)(2y - 1)$

(iii) Using identity,

$a^2 - b^2 = (a + b)(a - b)$

Here, $a = x$ and $b = (y/10)$

$x^2 - y*y/100 = x^2 - (y/10)^2$

$= (x - y/10)(x + y/10)$

Question 4.

Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Solution:

(i) Using identity,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = x$, $b = 2y$ and $c = 4z$

$(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + (2 * x * 2y) + (2 * 2y * 4z) + (2 * 4z * x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) Using identity,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = 2x$, $b = -y$ and $c = z$

$(2x – y + z)^2 = (2x)^2 + (-y)^2 + z^2 + (2 * 2x * -y) + (2 * y * z) + (2 * z * 2x)$

$= 4x^2 + y^2 + z^2 – 4xy – 2yz + 4xz$

(iii) Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = -2x$, $b = 3y$ and $c = 2z$

$(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + (2 * 2x * 3y) + (2 * 3y * 2z) + (2 * 2z * 2x)$

$= 4x^2 + 9yz + 4z^2 – 12xy + 12yz – 8xz$

(iv) Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = 3a$, $b = -7b$ and $c = -c$

$(3a – 7b – c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + (2 *3a * -7b) + (2 * -7b * -c) + (2 * -c * 3a)$

$= 9a^2 + 49b^2 + c^2 – 42ab + 14bc – 6ac$

(v) Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, a = -2x, b = -5y and c = -3z

$(-2x + 5y – 3z)^2 = (-2x)^2 + (-5y)^2 + (-3z)^2 + (2 * -2x * 5y) + (2 * 5y * -3z) + (2 * -3z * -2x)$

$= 4x^2 + 25y^2 + 9z^2 – 20xy -30yz + 12xz$

(vi) Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, a = 1/4a, b = –1/2b and c = 1

$(1/4a – 1/2b + 1)^2 = (1/4a)^2 + (-1/2b)^2 + (1)^2 + (2 * 1/4a * -1/2b) + (2 * -1/2b * 1) + (2 * 1 * 1/4a)$

$= 1/16a^2 + 1/4b^2 + 1 – 1/4ab - b + 1/2a$

Question 5.

Factorize:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24xy - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Solution:

(i) Using identity,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$4x^2 + 9y^2 + 16z^2 + 12xy – 24yz – 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + (2 * 2 * 3y) + (2 * 3y * -4z) + (2 * -4z * 2x) += (2x + 3y – 4z)$

$= (2x + 3y – 4z)^2$

$= (2x + 3y – 4z) (2x + 3y – 4z)$

(ii) Using identity,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz – 8xz$

$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + (2 * -\sqrt{2}x * y) + (2 * y * 2\sqrt{2}z) + (2 * 2\sqrt{2}z * -\sqrt{2}x)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

Question 6.

Write the following cubes in expanded form:

(i) $(2x + 1)^3$

(ii) $(2a – 3b)^3$

(iii) $\left[\frac{3}{2}x + 1\right]^3$

(iv) $\left[x - \frac{2}{3}y\right]^3$

Solution:

(i) $(2x + 1)^3$

Using identity,

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$(2x + 1)^3 = (2x)^3 + (1)^3 + (3 * 2 * 1)(2x + 1)$

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) Using identity,

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$(2a - 3b)^3 = (2a)^3 - (3b)^3 - (3 * 2a * 3b)(2a - 3b)$

$= 8a^3 - 27b^3 - 18ab(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) Using identity,

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$(3/2x + 1)^3 = (3/2x)^3 + 1^3 + (3 * 3/2x * 1)(3/2x + 1)$

$= 27/8\, x^3 + 1 + 9/2\, x(3/2x + 1)$

$= 27/8x^3 + 1 + 27/4\, x^2 + 9/2x$

$= 27/8\, x^3 + 27/4\, x^2 + 9/2\, x + 1$

(iv) Using identity,

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$(x - 2/3y)^3 = (x)^3 - (2/3y)^3 - (3 * x * 2/3y)(x - 2/3y)$

$= x^3 - 8/27\, y^3 - 2xy(x - 2/3y)$

$= x^3 - 8/27\, y^3 - 2x^2y + 4/3xy^2$

Question 7.

Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Solution:

(i) $(99)^3 = (100 - 1)^3$

Using identity,

$(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$

$(100 - 1)^3 = (100)^3 - 1^3 - (3 * 100 * 1)\,(100 - 1)$

$= 1000000 - 1 - 300\,(100 - 1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3 = (100 + 2)^3$

Using identity,

$(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$

$(100 + 2)^3 = (100)^3 + 2^3 + (3 * 100 * 2)\,(100 + 2)$

$= 1000000 + 8 + 600\,(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3 = (1000 - 2)^3$

Using identity,

$(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$

$(1000 - 2)^3 = (1000)^3 - 2^3 - (3 * 1000 * 2)\,(1000 - 2)$

$= 100000000 - 8 - 6000\,(1000 - 2)$

$= 100000000 - 8 - 6000000 + 12000$

= 994011992

Question 8.

Factorize each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Solution:

(i) Using identity,

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + b^3 + 3\ (2a)\ (2b) + 3\ (2a)\ (b)^2$

$= (2a + b)^3$

$= (2a + b)\ (2a + b)\ (2a + b)$

(ii) Using identity,

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - b^3 - 3\ (2a)^2b + 3\ (2a)\ (b)^2$

$= (2a - b)^3$

$= (2a – b)\ (2a – b)\ (2a – b)$

(iii) Using identity,

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$27 – 125a^3 – 135a + 225a^2$

$= 3^3 – (5a)^3 – 3\ (3)^2(5a) + 3\ (3)\ (5a)^2$

$= (3 – 5a)^3$

$= (3 – 5a)\ (3 – 5a)\ (3 – 5a)$

(iv) Using identity,

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$64a^3 – 27b^3 – 144a^2b + 108ab^2$

$= (4a)^3 – (3b)^3 – 3\ (4a)^2\ (3b) + 3\ (4a)\ (3b)^2$

$= (4a – 3b)^2$

$= (4a – 3b)\ (4a – 3b)\ (4a – 3b)$

(v) Using identity,

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$27p^3 – 1/216 – 9/2p^2 + 1/4p$

$= (3p)^3 – (1/6)^3 – 3\ (3p)^2\ (1/6) + 3\ (3p)\ (1/6k)^2$

$= (3p – 1/6)^3$

$= (3p – 1/6)\ (3p - 1/6)\ (3p - 1/6)$

Question 9.

Verify:

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Solution:

(i) We know that,

$(x + y)^3 = x^3 + y^3 + 3xy\ (x + y)$

$= x^3 + y^3 = (x + y)^3 – 3xy\ (x + y)$

$= x^3 + y^3 = (x + y)\ [(x + y)^2 – 3xy)$

{Taking (x + y) common}

$= x^3 + y^3 = (x + y)\ [(x^2 + y^2 + 2xy) – 3xy]$

$= x^3 + y^3 = (x + y)\ (x^2 + y^2 – xy)$

(ii) We know that,

$(x - y)^3 = x^3 - y^3 - 3xy\ (x - y)$

$= x^3 - y^3 = (x - y)^3 + 3xy\ (x - y)$

$= x^3 + y^3 = (x - y)\ [(x - y)^2 + 3xy)$

{Taking (x + y) common}

$= x^3 + y^3 = (x - y)\ [(x^2 + y^2 - 2xy) + 3xy]$

$= x^3 - y^3 = (x - y)\ (x^2 + y^2 + xy)$

Question 10. Factorize each of the following:

[Hint: See Question 9]

(i) $27x^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Solution:

(i) Using identity,

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)\{(3y)^2 - (3y)(5z) + (5z)^2$

$= (3y + 5z)(9y^2 - 15yz + 25z)^2$

(ii) Using identity,

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$= (4m + 7n)\{(4m)^2 + (4m)(7n) + (7n)^2\}$

$= (4m + 7n)(16m^2 + 28mn + 49n)^2$

Question 11.

Factorize: $27x^3 + y^3 + z^3 - 9xyz$

Solution:

$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3 * 3xyz$

Using identity,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$27x^3 + y^3 + z^3 - 9xyz$

$= (3x + y + z) \{(3x)^2 + y^2 + z^2 - 3xy - yz - 3xz\}$

$= (3x + y + z) (9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

Question 12.

Verify that:

$$x^3 + y^3 + = z^3 - 3xyz = \frac{1}{2}(x + y + z)$$
$$\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]$$

Solution:

We know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - xz)$

$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2} * (x + y + z) 2 (x^2 + y^2 + z^2 - xy - yz - xz)$

$= \frac{1}{2} (x + y + z) (2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz)$

$= \frac{1}{2} (x + y + z) [(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (x^2 + z^2 - 2xz)]$

$= \frac{1}{2} (x + y + z) [(x - y)^2 + (y - z)^2 + (z - x)^2]$

Question 13.

If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

Solution:

We know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - xz)$

Now, put $(x + y + z) = 0$

$x^3 + y^3 + z^3 - 3xyz = (0) (x^2 + y^2 + z^2 - xy - yz - xz)$

$x^3 + y^3 + z^3 - 3xyz = 0$

Question 14.

Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Solution:

(i) Let x = -12, y = 7 and z = 5

We observed that,

x + y + z = -12 + 7 + 5 = 0

We know that if,

x + y + z = 0

Then,

$x^3 + y^3 + z^3 = 3xyz$

$(-12)^3 + (7)^3 + (5)^3 = 3\ (-12)\ (7)\ (5)$

= -1260

(ii) Let x = 28, y = -15 and z = -13

We observed that,

x + y + z = 28 – 15 – 13 = 0

We know that if,

x + y + z = 0

Then,

$x^3 + y^3 + z^3 = 3xyz$

$(28)^3 + (-15)^3 + (-13)^3 = 3\ (28)\ (-15)\ (-13)$

$= 16380$

Question 15.

Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2 – 35a + 12$

(ii) Area : $35y^2 + 13y – 12$

Solution:

(i) Area: $25a^2 – 35a + 12$

Since, area is product of length and breadth therefore by factorizing the given area, we can know the length and breadth of rectangle

$25a^2 – 35a + 12$

$= 25a^2 – 15a – 20a + 12$

$= 5a\ (5a – 3) – 4\ (5a – 3)$

$= (5a – 4)\ (5a – 3)$

Possible expression for length = $5a – 4$

Possible expression for breadth = $5a – 3$

(ii) Area: $35y^2 + 13y – 12$

$35y^2 + 13y – 12$

$= 35y^2 – 15y + 28y – 12$

$= 5y\ (7y – 3) + 4\ (7y – 3)$

$= (5y + 4)\ (7y – 3)$

Possible expression for length = $(5y + 4)$

Possible expression for breadth = $(7y - 3)$

Question 16.

What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2 - 12x$

(ii) Volume : $12ky^2 + 8ky - 20k$

Solution:

(i) Volume: $3x^2 - 12x$

Since, volume is product of length, breadth and height therefore by factorizing the given volume we can know the length, breadth and height of the cuboid

$3x^2 - 12x$

$= 3x (x - 4)$

Possible expression for length = 3

Possible expression for breadth = x

Possible expression for height = $(x - 4)$

(ii) Volume: $12ky^2 + 8ky - 20k$

Since, volume is product of length, breadth and height therefore by factorizing the given volume, we can know the length, breadth and height of the cuboid

$12ky^2 + 8ky - 20k$

$= 4k\ (3y^2 + 2y - 5)$

$= 4k\ (3y^2 + 5y - 3y - 5)$

$= 4k\ [y\ (3y + 5) - 1\ (3y + 5)]$

$= 4k\ (3y + 5)\ (y - 1)$

Possible expression for length = $4k$

Possible expression for breadth = $(3y + 5)$

Possible expression for height = $(y - 1)$