

ML Aggarawal Solutions for Class 10 Maths

Chapter 18: Trigonometry Identities

Exercise 18

1. If A is an acute angle and $\sin A = 3/5$, find all other trigonometric ratios of angle A (using trigonometric identities).

Solution:

Given,

$\sin A = 3/5$ and A is an acute angle

So, in $\triangle ABC$ we have $\angle B = 90^\circ$

And,

$AC = 5$ and $BC = 3$

By Pythagoras theorem,

$$\begin{aligned} AB &= \sqrt{AC^2 - BC^2} \\ &= \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} \\ &= 4 \end{aligned}$$

Now,

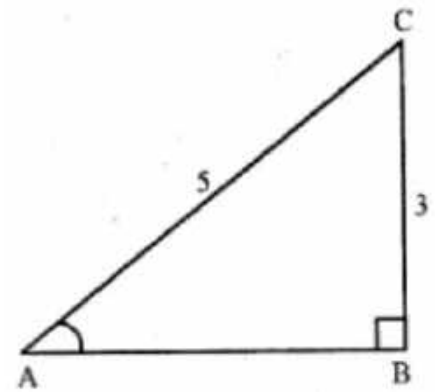
$$\cos A = AB/AC = 4/5$$

$$\tan A = BC/AB = 3/4$$

$$\cot A = 1/\tan \theta = 4/3$$

$$\sec A = 1/\cos \theta = 5/4$$

$$\operatorname{cosec} A = 1/\sin \theta = 5/3$$



2. If A is an acute angle and $\sec A = 17/8$, find all other trigonometric ratios of angle A (using trigonometric identities).

Solution:

Given,

$\sec A = 17/8$ and A is an acute angle

So, in $\triangle ABC$ we have $\angle B = 90^\circ$

And,

$AC = 17$ and $AB = 8$

By Pythagoras theorem,

$$\begin{aligned} BC &= \sqrt{AC^2 - AB^2} \\ &= \sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225} \\ &= 15 \end{aligned}$$

Now,

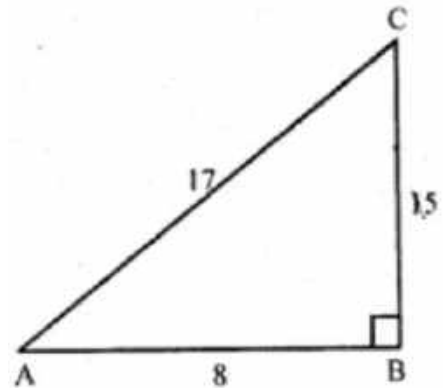
$$\sin A = BC/AC = 15/17$$

$$\cos A = 1/\sec A = 8/17$$

$$\tan A = BC/AB = 15/8$$

$$\cot A = 1/\tan A = 8/15$$

$$\operatorname{cosec} A = 1/\sin A = 17/15$$



3. Express the ratios $\cos A$, $\tan A$ and $\sec A$ in terms of $\sin A$.

Solution:

We know that,

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$$\sin^2 A + \cos^2 A = 1$$

So,

$$\cos A = \sqrt{1 - \sin^2 A}$$

$$\tan A = \sin A / \cos A = \sin A / \sqrt{1 - \sin^2 A}$$

$$\sec A = 1 / \cos A = 1 / \sqrt{1 - \sin^2 A}$$

4. If $\tan A = 1/\sqrt{3}$, find all other trigonometric ratios of angle A.

Solution:

Given, $\tan A = 1/\sqrt{3}$

In right ΔABC ,

$$\tan A = BC/AB = 1/\sqrt{3}$$

So,

$$BC = 1 \text{ and } AB = \sqrt{3}$$

By Pythagoras theorem,

$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2} = \sqrt{(\sqrt{3})^2 + (1)^2} \\ &= \sqrt{3 + 1} = \sqrt{4} = 2 \end{aligned}$$

Hence,

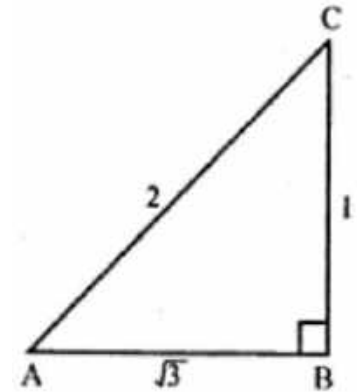
$$\sin A = BC/AC = 1/2$$

$$\cos A = AB/AC = \sqrt{3}/2$$

$$\cot A = 1/\tan A = \sqrt{3}$$

$$\sec A = 1/\cos A = 2/\sqrt{3}$$

$$\operatorname{cosec} A = 1/\sin A = 2/1 = 2$$



5. If $12 \operatorname{cosec} \theta = 13$, find the value of $(2 \sin \theta - 3 \cos \theta) / (4 \sin \theta - 9 \cos \theta)$

Solution:

Given,

$$12 \operatorname{cosec} \theta = 13$$

$$\Rightarrow \operatorname{cosec} \theta = 13/12$$

In right ΔABC ,

$$\angle A = \theta$$

$$\text{So, } \operatorname{cosec} \theta = AC/BC = 13/12$$

$$AC = 13 \text{ and } BC = 12$$

By Pythagoras theorem,

$$\begin{aligned} AB &= \sqrt{AC^2 - BC^2} \\ &= \sqrt{(13)^2 - (12)^2} \\ &= \sqrt{169 - 144} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Now,

$$\sin \theta = BC/AC = 12/13$$

$$\cos \theta = AB/AC = 5/13$$

Hence,

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$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} - 9 \times \frac{5}{13}}$$

$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}} = \frac{\frac{9}{13}}{\frac{3}{13}} = \frac{9}{13} \times \frac{13}{3} = 3$$

Without using trigonometric tables, evaluate the following (6 to 10):

6. (i) $\cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + (\tan 36^\circ / \cot 54^\circ)$

(ii) $(\sec 17^\circ / \operatorname{cosec} 73^\circ) + (\tan 68^\circ / \cot 22^\circ) + \cos^2 44^\circ + \cos^2 46^\circ$

Solution:

Given,

(i) $\cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + (\tan 36^\circ / \cot 54^\circ)$

$$= \cos^2 26^\circ + \cos (90^\circ - 16^\circ) \sin 26^\circ + [\tan 36^\circ / \cot (90^\circ - 54^\circ)]$$

$$= [\cos^2 26^\circ + \sin^2 26^\circ] + (\tan 36^\circ / \tan 36^\circ)$$

$$= 1 + 1 = 2$$

(ii) $(\sec 17^\circ / \operatorname{cosec} 73^\circ) + (\tan 68^\circ / \cot 22^\circ) + \cos^2 44^\circ + \cos^2 46^\circ$

$$= [\sec 17^\circ / \operatorname{cosec} (90^\circ - 73^\circ)] + [(\tan 90^\circ - 22^\circ) / \cot 22^\circ] + \cos^2 (90^\circ - 44^\circ) + \cos^2 46^\circ$$

$$= [\sec 17^\circ / \sec 17^\circ] + [\cot 22^\circ / \cot 22^\circ] + [\sin^2 46^\circ + \cos^2 46^\circ]$$

$$= 1 + 1 + 1$$

$$= 3$$

7. (i) $(\sin 65^\circ / \cos 25^\circ) + (\cos 32^\circ / \sin 58^\circ) - \sin 28^\circ \sec 62^\circ + \operatorname{cosec}^2 30^\circ$

(ii) $(\sin 29^\circ / \operatorname{cosec} 61^\circ) + 2 \cot 8^\circ \cot 17^\circ \cot 45^\circ \cot 73^\circ \cot 82^\circ - 3(\sin^2 38^\circ + \sin^2 52^\circ)$.

Solution:

Given,

(i) $(\sin 65^\circ / \cos 25^\circ) + (\cos 32^\circ / \sin 58^\circ) - \sin 28^\circ \sec 62^\circ + \operatorname{cosec}^2 30^\circ$

$$= (\sin 65^\circ / \cos (90^\circ - 65^\circ)) + (\cos 32^\circ / \sin (90^\circ - 32^\circ)) - \sin 28^\circ \sec (90^\circ - 28^\circ) + 2^2$$

$$= (\sin 65^\circ / \sin 65^\circ) + (\cos 32^\circ / \cos 32^\circ) - [\sin 28^\circ \times \operatorname{cosec} 28^\circ] + 4$$

$$= 1 + 1 - 1 + 4$$

$$= 5$$

(ii) $(\sin 29^\circ / \operatorname{cosec} 61^\circ) + 2 \cot 8^\circ \cot 17^\circ \cot 45^\circ \cot 73^\circ \cot 82^\circ - 3(\sin^2 38^\circ + \sin^2 52^\circ)$.

$$= (\sin 29^\circ / \operatorname{cosec} (90^\circ - 29^\circ)) + [2 \cot 8^\circ \cot 17^\circ \cot 45^\circ \cot (90^\circ - 17^\circ) \cot (90^\circ - 8^\circ)] - 3(\sin^2 38^\circ + \sin^2 (90^\circ - 38^\circ))$$

$$= (\sin 29^\circ / \sin 29^\circ) + [2 \cot 8^\circ \cot 17^\circ \cot 45^\circ \tan 17^\circ \tan 8^\circ] - 3(\sin^2 38^\circ + \cos^2 38^\circ)$$

$$= 1 + 2[(\cot 8^\circ \tan 8^\circ) (\cot 17^\circ \tan 17^\circ) \cot 45^\circ] - 3(1)$$

$$= 1 + 2[1 \times 1 \times 1] - 3$$

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$$= 1 + 2 - 3$$

$$= 0$$

8. (i) $(\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ) / (\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ)$
 (ii) $\sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$

Solution:

Given,

(i) $(\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ) / (\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ)$

$$= \frac{\sin 35^\circ \cos (90^\circ - 35^\circ) + \cos 35^\circ \sin (90^\circ - 35^\circ)}{\operatorname{cosec}^2 10^\circ - \tan^2 (90^\circ - 10^\circ)}$$

$$= \frac{\sin 35^\circ \sin 35^\circ + \cos 35^\circ \cos 35^\circ}{\operatorname{cosec}^2 10^\circ - \cot^2 10^\circ}$$

$$= \frac{\sin^2 35^\circ + \cos^2 35^\circ}{\operatorname{cosec}^2 10^\circ - \cot^2 10^\circ} = \frac{1}{1} = 1$$

(ii) $\sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$

$$= \sin^2 34^\circ + \sin^2 (90^\circ - 34^\circ) + 2 \tan 18^\circ \tan (90^\circ - 18^\circ) - \cot^2 30^\circ$$

$$= [\sin^2 34^\circ + \cos^2 34^\circ] + 2 \tan 18^\circ \cot 18^\circ - \cot^2 30^\circ$$

$$= 1 + 2 \times 1 - (\sqrt{3})^2$$

$$= 1 + 2 - 3$$

$$= 0$$

9. (i) $(\tan 25^\circ / \operatorname{cosec} 65^\circ)^2 + (\cot 25^\circ / \sec 65^\circ)^2 + 2 \tan 18^\circ \tan 45^\circ \tan 75^\circ$

(ii) $(\cos^2 25^\circ + \cos^2 65^\circ) + \operatorname{cosec} \theta \sec (90^\circ - \theta) - \cot \theta \tan (90^\circ - \theta)$

Solution:

Given,

(i) $(\tan 25^\circ / \operatorname{cosec} 65^\circ)^2 + (\cot 25^\circ / \sec 65^\circ)^2 + 2 \tan 18^\circ \tan 45^\circ \tan 75^\circ$

$$= \left(\frac{\tan 25^\circ}{\operatorname{cosec} (90^\circ - 25^\circ)} \right)^2 + \left(\frac{\cot 25^\circ}{\sec (90^\circ - 25^\circ)} \right)^2 + 2 \tan 18^\circ \tan (90^\circ - 18^\circ) \tan 45^\circ$$

$$= \left(\frac{\tan 25^\circ}{\sec 25^\circ} \right)^2 + \left(\frac{\cot 25^\circ}{\operatorname{cosec} 25^\circ} \right)^2 + 2 \tan 18^\circ \cot 18^\circ \tan 45^\circ$$

$$= \left(\frac{\sin 25^\circ \times \cos 25^\circ}{\cos 25^\circ \times 1} \right)^2 + \left(\frac{\cos 25^\circ \times \sin 25^\circ}{\sin 25^\circ \times 1} \right) + 2 \times 1 \times 1$$

$$= \sin^2 25^\circ + \cos^2 25^\circ + 2$$

$$= 1 + 2 = 3$$

$$\left\{ \begin{array}{l} \because \sin^2 \theta + \cos^2 \theta = 1 \\ \tan \theta \cot \theta = 1 \end{array} \right\}$$

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$$\begin{aligned}
 & \text{(ii) } (\cos^2 25^\circ + \cos^2 65^\circ) + \operatorname{cosec} \theta \sec (90^\circ - \theta) - \cot \theta \tan (90^\circ - \theta) \\
 & = \cos^2 25^\circ + \cos^2 (90^\circ - 25^\circ) + \operatorname{cosec} \theta \sec (90^\circ - \theta) - \cot \theta \cdot \cot \theta \\
 & = (\cos^2 25^\circ + \sin^2 25^\circ) + (\operatorname{cosec}^2 \theta - \cot^2 \theta) \\
 & = 1 + 1 = 2
 \end{aligned}$$

$$\begin{aligned}
 & \text{10. (i) } 2(\sec^2 35^\circ - \cot^2 55^\circ) - \frac{\cos 28^\circ \operatorname{cosec} 62^\circ}{\tan 18^\circ \tan 36^\circ \tan 30^\circ \tan 54^\circ \tan 72^\circ} \\
 & \text{(ii) } \frac{\operatorname{cosec}^2 (90^\circ - \theta) - \tan^2 \theta}{2(\cos^2 48^\circ + \cos^2 42^\circ)} - \frac{2 \tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{\operatorname{cosec}^2 70^\circ - \tan^2 20^\circ}
 \end{aligned}$$

Solution:

Given,

$$\begin{aligned}
 & \text{(i) } 2(\sec^2 35^\circ - \cot^2 55^\circ) - \frac{\cos 28^\circ \operatorname{cosec} 62^\circ}{\tan 18^\circ \tan 36^\circ \tan 30^\circ \tan 54^\circ \tan 72^\circ} \\
 & = 2[\sec^2 35^\circ - \cot^2 (90^\circ - 35^\circ)] - \frac{\cos 28^\circ \operatorname{cosec} (90^\circ - 28^\circ)}{\tan 18^\circ \tan (90^\circ - 18^\circ) \tan 36^\circ \tan (90^\circ - 36^\circ) \tan 30^\circ} \\
 & = 2[\sec^2 35^\circ - \tan^2 35^\circ] - \frac{\cos 28^\circ \sec 28^\circ}{\tan 18^\circ \cot 18^\circ \tan 36^\circ \cot 36^\circ \tan 30^\circ} \left\{ \begin{array}{l} \because \sec^2 \theta - \tan^2 \theta = 1 \\ \tan \theta \cot \theta = 1 \\ \cos \theta \sec \theta = 1 \end{array} \right\} \\
 & = 2(1) - \frac{1}{1 \times 1 \times \frac{1}{\sqrt{3}}} = 2 - \frac{\sqrt{3}}{1} = 2 - \sqrt{3} \\
 & \text{(ii) } \frac{\operatorname{cosec}^2 (90^\circ - \theta) - \tan^2 \theta}{2(\cos^2 48^\circ + \cos^2 42^\circ)} - \frac{2(\tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ)}{\operatorname{cosec}^2 70^\circ - \tan^2 20^\circ} \\
 & = \frac{\sec^2 \theta - \tan^2 \theta}{2(\cos^2 48^\circ + \cos^2 (90^\circ - 48^\circ))} - \frac{2[\tan^2 30^\circ \sec^2 52^\circ \sin^2 (90^\circ - 52^\circ)]}{\operatorname{cosec}^2 70^\circ - \tan^2 (90^\circ - 70^\circ)} \\
 & = \frac{1}{2[\cos^2 48^\circ + \sin^2 48^\circ]} - \frac{2\left[\left(\frac{1}{\sqrt{3}}\right)^2 \sec^2 52^\circ \cos^2 52^\circ\right]}{\operatorname{cosec}^2 70^\circ - \cot^2 70^\circ} \left\{ \begin{array}{l} \because \sin^2 \theta + \cos^2 \theta = 1 \\ \sec^2 \theta - \tan^2 \theta = 1 \\ \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \end{array} \right\} \\
 & = \frac{1}{2 \times 1} - \frac{2\left[\frac{1}{3} \times 1\right]}{1} \\
 & = \frac{1}{2} - \frac{2}{3} = \frac{3-4}{6} = \frac{-1}{6}
 \end{aligned}$$

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11. Prove that following:

(i) $\cos \theta \sin (90^\circ - \theta) + \sin \theta \cos (90^\circ - \theta) = 1$

(ii) $\tan \theta / \tan (90^\circ - \theta) + \sin (90^\circ - \theta) / \cos \theta = \sec^2 \theta$

(iii) $(\cos (90^\circ - \theta) \cos \theta) / \tan \theta + \cos^2 (90^\circ - \theta) = 1$

(iv) $\sin (90^\circ - \theta) \cos (90^\circ - \theta) = \tan \theta / (1 + \tan^2 \theta)$

Solution:

(i) L.H.S. = $\cos \theta \sin (90^\circ - \theta) + \sin \theta \cos (90^\circ - \theta)$
= $\cos \theta \times \cos \theta + \sin \theta \times \sin \theta$
= $\cos^2 \theta + \sin^2 \theta$
= $1 = \text{R.H.S.}$

(ii) L.H.S. = $\tan \theta / \tan (90^\circ - \theta) + \sin (90^\circ - \theta) / \cos \theta$
= $\tan \theta / \cot \theta + \cos \theta / \cos \theta$
= $\tan \theta / (1/\tan \theta) + 1$
= $\tan^2 \theta + 1 = \sec^2 \theta = \text{R.H.S.}$

(iii) L.H.S. = $(\cos (90^\circ - \theta) \cos \theta) / \tan \theta + \cos^2 (90^\circ - \theta)$
= $(\sin \theta \cos \theta) / \tan \theta + \sin^2 \theta$
= $(\sin \theta \cos \theta) / (\sin \theta / \cos \theta) + \sin^2 \theta$
= $\cos^2 \theta + \sin^2 \theta$
= $1 = \text{R.H.S.}$

(iv) $\sin (90^\circ - \theta) \cos (90^\circ - \theta) = \frac{\tan \theta}{1 + \tan^2 \theta}$

L.H.S. = $\sin (90^\circ - \theta) \cos (90^\circ - \theta)$ $\left\{ \begin{array}{l} \because \sin (90^\circ - \theta) = \cos \theta \\ \sin^2 \theta + \cos^2 \theta = 1 \end{array} \right\}$
= $\cos \theta \sin \theta$

R.H.S. = $\frac{\tan \theta}{1 + \tan^2 \theta} = \frac{\frac{\sin \theta}{\cos \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}$
= $\frac{\frac{\sin \theta}{\cos \theta}}{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}}$
= $\frac{\frac{\sin \theta}{\cos \theta}}{1} = \frac{\sin \theta}{\cos \theta} \times \cos^2 \theta = \sin \theta \cos \theta$

$\therefore \text{L.H.S.} = \text{R.H.S.}$

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Prove that following (12 to 30) identities, where the angles involved are acute angles for which the trigonometric ratios as defined:

12. (i) $(\sec A + \tan A) (1 - \sin A) = \cos A$

(ii) $(1 + \tan^2 A) (1 - \sin A) (1 + \sin A) = 1$.

Solution:

Given,

(i) L.H.S = $(\sec A + \tan A) (1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) (1 - \sin A)$$

$$= \frac{(1 + \sin A) \times (1 - \sin A)}{\cos A} = \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} = \cos A = \text{R.H.S.}$$

$$(1 - \sin^2 A = \cos^2 A)$$

(ii) L.H.S. = $(1 + \tan^2 A) (1 - \sin A) (1 + \sin A)$

$$= \left(1 + \frac{\sin^2 A}{\cos^2 A} \right) (1 - \sin^2 A)$$

$$= \frac{\cos^2 A + \sin^2 A}{\cos^2 A} \times \cos^2 A \quad \left\{ \begin{array}{l} \because 1 - \sin^2 A = \cos^2 A \\ \sin^2 A + \cos^2 A = 1 \end{array} \right.$$

$$= \frac{1}{\cos^2 A} \times \cos^2 A = 1 = \text{R.H.S.}$$

13. (i) $\tan A + \cot A = \sec A \operatorname{cosec} A$

(ii) $(1 - \cos A) (1 + \sec A) = \tan A \sin A$.

Solution:

(i) L.H.S. = $\tan A + \cot A$

$$= \sin A / \cos A + \cos A / \sin A$$

$$= (\sin^2 A + \cos^2 A) / (\sin A \cos A)$$

$$= 1 / (\sin A \cos A)$$

$$= \sec A \operatorname{cosec} A$$

$$= \text{R.H.S}$$

(ii) L.H.S. = $(1 - \cos A) (1 + \sec A)$

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$$\begin{aligned} &= (1 - \cos A) \left(1 + \frac{1}{\cos A} \right) \\ &= (1 - \cos A) \frac{(\cos A + 1)}{\cos A} \\ &= \frac{(1 - \cos A)(1 + \cos A)}{\cos A} = \frac{1 - \cos^2 A}{\cos A} = \frac{\sin^2 A}{\cos A} \\ &= \frac{\sin^2 A}{\cos A} = \sin A \times \frac{\sin A}{\cos A} \\ &\qquad\qquad\qquad \{ 1 - \cos^2 A = \sin^2 A \} \\ &= \tan A \sin A = \text{R.H.S.} \end{aligned}$$

14. (i) $1/(1 + \cos A) + 1/(1 - \cos A) = 2 \operatorname{cosec}^2 A$

(ii) $1/(\sec A + \tan A) + 1/(\sec A - \tan A) = 2 \sec A$

Solution:

(i) L.H.S. = $1/(1 + \cos A) + 1/(1 - \cos A)$

$$\begin{aligned} &= \frac{1 - \cos A + 1 + \cos A}{(1 + \cos A)(1 - \cos A)} \\ &= \frac{2}{1 - \cos^2 A} = \frac{2}{\sin^2 A} \\ &\qquad\qquad\qquad (\because 1 - \cos^2 A = \sin^2 A) \\ &= 2 \operatorname{cosec}^2 A = \text{R.H.S.} \end{aligned}$$

(ii)

$$\begin{aligned} \text{L.H.S.} &= \frac{1}{\sec A + \tan A} + \frac{1}{\sec A - \tan A} \\ &= \frac{\sec A - \tan A + \sec A + \tan A}{(\sec A + \tan A)(\sec A - \tan A)} \\ &= \frac{2 \sec A}{\sec^2 A - \tan^2 A} = \frac{2 \sec A}{1} \\ &\qquad\qquad\qquad (\because \sec^2 A - \tan^2 A = 1) \\ &= 2 \sec A = \text{R.H.S.} \end{aligned}$$

15. (i) $\sin A/(1 + \cos A) = (1 - \cos A)/\sin A$

(ii) $(1 - \tan^2 A)/(\cot^2 A - 1) = \tan^2 A$

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(iii) $\sin A / (1 + \cos A) = \operatorname{cosec} A - \cot A$

Solution:

(i) L.H.S. = $\sin A / (1 + \cos A)$

On multiplying and dividing by $(1 - \cos A)$, we have

$$= \frac{\sin A (1 - \cos A)}{1 - \cos^2 A} = \frac{\sin A (1 - \cos A)}{\sin^2 A}$$

($\because 1 - \cos^2 A = \sin^2 A$)

$$= \frac{1 - \cos A}{\sin A} = \text{R.H.S.}$$

(ii) L.H.S. = $\frac{1 - \tan^2 A}{\cot^2 A - 1} = \frac{1 - \frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A} - 1}$

$$= \frac{\frac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A - \sin^2 A}{\sin^2 A}} = \frac{\cos^2 A - \sin^2 A}{\cos^2 A} \times \frac{\sin^2 A}{\cos^2 A - \sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A = \text{R.H.S.}$$

(iii) R.H.S. = $\operatorname{cosec} A - \cot A$

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A} = \frac{1 - \cos A}{\sin A}$$

$$= \frac{(1 - \cos A)(1 + \cos A)}{\sin A(1 + \cos A)} \quad \{\text{Multiplying and dividing by } 1 + \cos A\}$$

$$= \frac{1 - \cos^2 A}{\sin A(1 + \cos A)} = \frac{\sin^2 A}{\sin A(1 + \cos A)} \quad \{\because 1 - \cos^2 A = \sin^2 A\}$$

$$= \frac{\sin A}{1 + \cos A} = \text{L.H.S.}$$

16. (i) $(\sec A - 1)/(\sec A + 1) = (1 - \cos A)/(1 + \cos A)$

(ii) $\tan^2 \theta / (\sec \theta - 1)^2 = (1 + \cos \theta) / (1 - \cos \theta)$

(iii) $(1 + \tan A)^2 + (1 - \tan A)^2 = 2 \sec^2 A$

(iv) $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \cdot \operatorname{cosec}^2 A$

Solution:

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$$\frac{1 - \cos A}{\cos A} = \frac{1 - \cos A}{\cos A} \times \frac{\cos A}{1 + \cos A}$$

$$= \frac{1 - \cos A}{1 + \cos A} = \text{R.H.S.}$$

$$\begin{aligned} \text{(ii) LHS} &= \frac{\tan^2 \theta}{(\sec \theta - 1)^2} = \frac{\tan^2 \theta}{\sec^2 \theta + 1 - 2\sec \theta} \\ &= \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{1}{\cos^2 \theta} + 1 - \frac{2}{\cos \theta}} = \frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{\cos^2 \theta}{1 + \cos^2 \theta - 2\cos \theta} \\ &= \frac{\sin^2 \theta}{(1 - \cos \theta)^2} = \frac{(1 - \cos^2 \theta)}{(1 - \cos \theta)^2} = \frac{(1 + \cos \theta)(1 - \cos \theta)}{(1 - \cos \theta)^2} \\ &= \frac{1 + \cos \theta}{1 - \cos \theta} = \text{R.H.S. Proved.} \end{aligned}$$

$$\begin{aligned} \text{(iii) L.H.S.} &= (1 + \tan A)^2 + (1 - \tan A)^2 \\ &= 1 + 2 \tan A + \tan^2 A + 1 - 2 \tan A + \tan^2 A \\ &= 2 + 2 \tan^2 A \\ &= 2(1 + \tan^2 A) \quad [\text{As } 1 + \tan^2 A = \sec^2 A] \\ &= 2 \sec^2 A \\ &= \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} \text{(iv) L.H.S} &= \sec^2 A + \operatorname{cosec}^2 A \\ &= 1/\cos^2 A + 1/\sin^2 A \\ &= (\sin^2 A + \cos^2 A)/(\sin^2 A \cos^2 A) \\ &= 1/(\sin^2 A \cos^2 A) \\ &= \sec^2 A \operatorname{cosec}^2 A = \text{R.H.S} \end{aligned}$$

17. (i) $(1 + \sin A)/\cos A + \cos A/(1 + \sin A) = 2 \sec A$

(ii) $\tan A/(\sec A - 1) + \tan A/(\sec A + 1) = 2 \operatorname{cosec} A$

Solution:

(i) L.H.S. = $(1 + \sin A)/\cos A + \cos A/(1 + \sin A)$

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$$\begin{aligned}
 &= \frac{(1 + \sin A)(1 + \sin A) + \cos^2 A}{\cos A (1 + \sin A)} \\
 &= \frac{1 + \sin A + \sin A + \sin^2 A + \cos^2 A}{\cos A (1 + \sin A)} \\
 &= \frac{1 + 2 \sin A + 1}{\cos A (1 + \sin A)} = \frac{2 + 2 \sin A}{\cos A (1 + \sin A)} \\
 &= \frac{2(1 + \sin A)}{\cos A (1 + \sin A)} = \frac{2}{\cos A} = 2 \sec A \\
 &= \text{R.H.S.}
 \end{aligned}$$

(ii) $\frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1} = 2 \operatorname{cosec} A$

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1} \\
 &= \tan A \left(\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} \right) \\
 &= \tan A \left(\frac{\sec A + 1 + \sec A - 1}{(\sec A - 1)(\sec A + 1)} \right) \\
 &= \frac{\tan A \times 2 \sec A}{\sec^2 A - 1} = \frac{2 \sec A \tan A}{\tan^2 A} = \frac{2 \sec A}{\tan A} \\
 &= \frac{2 \times 1 \times \cos A}{\cos A \times \sin A} \\
 &= \frac{2}{\sin A} = 2 \operatorname{cosec} A = \text{R.H.S.}
 \end{aligned}$$

18. (i) $\operatorname{cosec} A / (\operatorname{cosec} A - 1) + \operatorname{cosec} A / (\operatorname{cosec} A + 1) = 2 \sec^2 A$

(ii) $\cot A - \tan A = (2 \cos^2 A - 1) / (\sin A - \cos A)$

(iii) $(\cot A - 1) / (2 - \sec^2 A) = \cot A / (1 + \tan A)$

Solution:

(i) L.H.S. = $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1}$

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$$\begin{aligned}
 &= \operatorname{cosec} A \left[\frac{1}{\operatorname{cosec} A - 1} + \frac{1}{\operatorname{cosec} A + 1} \right] \\
 &= \operatorname{cosec} A \left[\frac{\operatorname{cosec} A + 1 + \operatorname{cosec} A - 1}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)} \right] \\
 &= \frac{\operatorname{cosec} A \times 2 \operatorname{cosec} A}{\operatorname{cosec}^2 A - 1} = \frac{2 \operatorname{cosec}^2 A}{\cot^2 A} \\
 &= \frac{2 \times \sin^2 A}{\sin^2 A \times \cos^2 A} = \frac{2}{\cos^2 A} \\
 &= 2 \sec^2 A = \text{R.H.S.}
 \end{aligned}$$

(ii) L.H.S. = $\cot A - \tan A$

$$\begin{aligned}
 &= \frac{\cos A}{\sin A} - \frac{\sin A}{\cos A} = \frac{\cos^2 A - \sin^2 A}{\sin A \cos A} \\
 &= \frac{\cos^2 A - (1 - \cos^2 A)}{\sin A \cos A} \\
 &= \frac{\cos^2 A - 1 + \cos^2 A}{\sin A \cos A} \\
 &= \frac{2 \cos^2 A - 1}{\sin A \cos A} = \text{R.H.S.}
 \end{aligned}$$

(iii) L.H.S. = $\frac{\cot A - 1}{2 - \sec^2 A} = \frac{\frac{\cos A}{\sin A} - 1}{2 - \frac{1}{\cos^2 A}}$

$$\begin{aligned}
 &= \frac{\frac{\cos A - \sin A}{\sin A}}{\frac{2 \cos^2 A - 1}{\cos^2 A}} \\
 &= \frac{\cos A - \sin A}{\sin A} \times \frac{\cos^2 A}{2 \cos^2 A - 1} \\
 &= \frac{\cos^2 A (\cos A - \sin A)}{\sin A (2 \cos^2 A - 1)} \\
 &= \frac{\cos^2 A (\cos A - \sin A)}{\sin A [2 \cos^2 A - (\sin^2 A + \cos^2 A)]} \\
 &= \frac{\cos^2 A (\cos A - \sin A)}{\sin A [2 \cos^2 A - \sin^2 A - \cos^2 A]} \\
 &= \frac{\cos^2 A (\cos A - \sin A)}{\sin A (\cos^2 A - \sin^2 A)} \\
 &= \frac{\cos^2 A (\cos A - \sin A)}{\sin A (\cos A + \sin A)(\cos A - \sin A)} \\
 &= \frac{\cos^2 A}{\sin A (\cos A + \sin A)}
 \end{aligned}$$



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$$\begin{aligned} \text{R.H.S.} &= \frac{\cot A}{1 + \tan A} = \frac{\frac{\cos A}{\sin A}}{1 + \frac{\sin A}{\cos A}} \\ &= \frac{\frac{\cos A}{\sin A}}{\frac{\cos A + \sin A}{\cos A}} \\ &= \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A + \sin A} \\ &= \frac{\cos^2 A}{\sin A (\cos A + \sin A)} \end{aligned}$$

∴ L.H.S. = R.H.S.

19. (i) $\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$

(ii) $\cos \theta / (1 - \tan \theta) - \sin^2 \theta / (\cos \theta - \sin \theta) = \cos \theta + \sin \theta$

Solution:

(i) L.H.S = $\tan^2 \theta - \sin^2 \theta$

$$\begin{aligned} &= \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta \\ &= \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta} = \sin^2 \theta \times \frac{\sin^2 \theta}{\cos^2 \theta} \end{aligned}$$

$\sin^2 \theta \times \tan^2 \theta = \tan^2 \theta \sin^2 \theta = \text{R.H.S.}$

(ii) $\frac{\cos \theta}{1 - \tan \theta} - \frac{\sin^2 \theta}{\cos \theta - \sin \theta} = \cos \theta + \sin \theta$

$$\begin{aligned} \text{L.H.S.} &= \frac{\cos \theta}{1 - \tan \theta} - \frac{\sin^2 \theta}{\cos \theta - \sin \theta} \\ &= \frac{\cos \theta}{1 - \frac{\sin \theta}{\cos \theta}} - \frac{\sin^2 \theta}{\cos \theta - \sin \theta} \\ &= \frac{\cos \theta}{\frac{\cos \theta - \sin \theta}{\cos \theta}} - \frac{\sin^2 \theta}{\cos \theta - \sin \theta} \\ &= \frac{\cos \theta \times \cos \theta}{\cos \theta - \sin \theta} - \frac{\sin^2 \theta}{\cos \theta - \sin \theta} \end{aligned}$$

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$$\begin{aligned}
 &= \frac{\cos^2 \theta}{\cos \theta - \sin \theta} - \frac{\sin^2 \theta}{\cos \theta - \sin \theta} \\
 &= \frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta - \sin \theta} \\
 &= \frac{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)}{\cos \theta - \sin \theta} \\
 &= \cos \theta + \sin \theta = \text{R.H.S.}
 \end{aligned}$$

20. (i) $\operatorname{cosec}^4 \theta - \operatorname{cosec}^2 \theta = \cot^4 \theta + \cot^2 \theta$

(ii) $2 \sec^2 \theta - \sec^4 \theta - 2 \operatorname{cosec}^2 \theta + \operatorname{cosec}^4 \theta = \cot^4 \theta - \tan^4 \theta$.

Solution:

(i) L.H.S. = $\operatorname{cosec}^4 \theta - \operatorname{cosec}^2 \theta$
 $= \operatorname{cosec}^2 \theta (\operatorname{cosec}^2 \theta - 1)$
 $= \operatorname{cosec}^2 \theta \cot^2 \theta$ [$\operatorname{cosec}^2 \theta - 1 = \cot^2 \theta$]
 $= (\cot^2 \theta + 1) \cot^2 \theta$
 $= \cot^4 \theta + \cot^2 \theta$
 $= \text{R.H.S.}$

(ii) L.H.S. = $2 \sec^2 \theta - \sec^4 \theta - 2 \operatorname{cosec}^2 \theta + \operatorname{cosec}^4 \theta$
 $= 2 (\tan^2 \theta + 1) - (\tan^2 \theta + 1)^2 - 2 (1 + \cot^2 \theta) + (1 + \cot^2 \theta)^2$
{ $\because \sec^2 \theta = \tan^2 \theta + 1$ }
{ $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$ }
 $= 2 \tan^2 \theta + 2 - (\tan^4 \theta + 2 \tan^2 \theta + 1) - 2 - 2 \cot^2 \theta + (1 + 2 \cot^2 \theta + \cot^4 \theta)$
 $= 2 \tan^2 \theta + 2 - \tan^4 \theta - 2 \tan^2 \theta - 1 - 2 - 2 \cot^2 \theta + 1 + 2 \cot^2 \theta + \cot^4 \theta$
 $= \cot^4 \theta - \tan^4 \theta = \text{R.H.S.}$

21. (i) $\frac{1 + \cos \theta - \sin^2 \theta}{\sin \theta (1 + \cos \theta)} = \cot \theta$

(ii) $(\tan^3 \theta - 1) / (\tan \theta - 1) = \sec^2 \theta + \tan \theta$

Solution:

(i) L.H.S. = $\frac{1 + \cos \theta - \sin^2 \theta}{\sin \theta (1 + \cos \theta)} = \frac{\cos \theta + \cos^2 \theta}{\sin \theta (1 + \cos \theta)}$ { $\because 1 - \sin^2 \theta = \cos^2 \theta$ }
 $= \frac{\cos \theta (1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} = \frac{\cos \theta}{\sin \theta} = \cot \theta$
 $= \text{R.H.S.}$

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$$(ii) \frac{\tan^3 \theta - 1}{\tan \theta - 1} = \sec^2 \theta + \tan \theta$$

$$\begin{aligned} \text{L.H.S.} &= \frac{(\tan \theta - 1)}{\tan \theta - 1} (\tan^2 \theta + \tan \theta + 1) \quad \{\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)\} \\ &= \tan^2 \theta + \tan \theta + 1 = \tan^2 \theta + 1 + \tan \theta \\ &= \sec^2 \theta + \tan \theta \quad \{\because \sec^2 \theta = \tan^2 \theta + 1\} \\ &= \text{R.H.S.} \end{aligned}$$

22. (i) $(1 + \operatorname{cosec} A) / \operatorname{cosec} A = \cos^2 A / (1 - \sin A)$

(ii) $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A}$

Solution:

(i) $\frac{1 + \operatorname{cosec} A}{\operatorname{cosec} A} = \frac{\cos^2 A}{1 - \sin A}$
L.H.S. = $\frac{1 + \operatorname{cosec} A}{\operatorname{cosec} A}$

$$\text{L.H.S.} = \frac{1 + \operatorname{cosec} A}{\operatorname{cosec} A} = \frac{1 + \frac{1}{\sin A}}{\frac{1}{\sin A}} = \frac{\sin A + 1}{\sin A} \times \frac{\sin A}{1}$$

$$= \sin A + 1$$

$$\text{R.H.S.} = \frac{\cos^2 A}{1 - \sin A} = \frac{1 - \sin^2 A}{1 - \sin A}$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{1 - \sin A} = 1 + \sin A = \sin A + 1$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

(ii) $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$

$$\text{L.H.S.} = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Rationalising the denominator

$$= \sqrt{\frac{(1 - \cos A)(1 - \cos A)}{(1 + \cos A)(1 - \cos A)}}$$

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$$\begin{aligned}
 &= \sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}} = \sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}} \\
 &= \frac{1 - \cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A} \\
 &= \operatorname{cosec} A - \cot A = \text{R.H.S.}
 \end{aligned}$$

23. (i) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \tan A + \sec A$

(ii) $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$

Solution:

(i) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \tan A + \sec A$

L.H.S. = $\sqrt{\frac{1 + \sin A}{1 - \sin A}}$

L.H.S. = $\sqrt{\frac{1 + \sin A}{1 - \sin A}}$

Rationalising the denominator,

$$= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$$

$$= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} = \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}}$$

$$= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A = \tan A + \sec A = \text{R.H.S.}$$

(ii) $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$

L.H.S. = $\sqrt{\frac{1 - \cos A}{1 + \cos A}}$

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Rationalising the denominator,

$$\begin{aligned}
 &= \sqrt{\frac{(1 - \cos A)(1 - \cos A)}{(1 + \cos A)(1 - \cos A)}} \\
 &= \sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}} = \sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}} = \frac{1 - \cos A}{\sin A} \\
 &= \frac{1}{\sin A} - \frac{\cos A}{\sin A} = \operatorname{cosec} A - \cot A = \text{R.H.S.}
 \end{aligned}$$

24. (i) $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$
 (ii) $\cos A \cot A / (1 - \sin A) = 1 + \operatorname{cosec} A$

Solution:

(i) $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$

L.H.S. = $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}}$

L.H.S. = $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}}$

= $\frac{\sqrt{\sec A - 1}}{\sqrt{\sec A + 1}} + \frac{\sqrt{\sec A + 1}}{\sqrt{\sec A - 1}}$

= $\frac{\sec A - 1 + \sec A + 1}{\sqrt{(\sec A + 1)(\sec A - 1)}} = \frac{2 \sec A}{\sqrt{\sec^2 A - 1}}$

{ $\because \sec^2 A - 1 = \tan^2 A$ }

= $\frac{2 \sec A}{\sqrt{\tan^2 A}} = \frac{2 \sec A}{\tan A}$

= $\frac{2 \times \cos A}{\cos A \times \sin A} = \frac{2}{\sin A}$

= $2 \operatorname{cosec} A = \text{R.H.S.}$

(ii) $\frac{\cos A \cot A}{1 - \sin A} = 1 + \operatorname{cosec} A$

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$$\begin{aligned} \text{L.H.S.} &= \frac{\cos A \cot A}{1 - \sin A} = \frac{\cos A \cos A}{\sin A (1 - \sin A)} \left\{ \cos A = \frac{\cos A}{\sin A} \right\} \\ &= \frac{\cos^2 A}{\sin A (1 - \sin A)} = \frac{1 - \sin^2 A}{\sin A (1 - \sin A)} \\ &\quad \{\because \cos^2 A = 1 - \sin^2 A\} \\ &= \frac{(1 + \sin A)(1 - \sin A)}{\sin A (1 - \sin A)} = \frac{1 + \sin A}{\sin A} \\ &= \frac{1}{\sin A} + \frac{\sin A}{\sin A} = \operatorname{cosec} A + 1 \\ &= 1 + \operatorname{cosec} A = \text{R.H.S.} \end{aligned}$$

25. (i) $(1 + \tan A)/\sin A + (1 + \cot A)/\cos A = 2(\sec A + \operatorname{cosec} A)$

(ii) $\sec^4 A - \tan^4 A = 1 + 2 \tan^2 A$

Solution:

$$\begin{aligned} \text{(i)} \quad & \frac{1 + \tan A}{\sin A} + \frac{1 + \cot A}{\cos A} = 2(\sec A + \operatorname{cosec} A) \\ \text{L.H.S.} &= \frac{1 + \tan A}{\sin A} + \frac{1 + \cot A}{\cos A} \\ &= \frac{1 + \frac{\sin A}{\cos A}}{\sin A} + \frac{1 + \frac{\cos A}{\sin A}}{\cos A} \\ &= \frac{\cos A + \sin A}{\cos A \times \sin A} + \frac{\sin A + \cos A}{\cos A \times \sin A} \\ &= 2 \left[\frac{\cos A + \sin A}{\cos A \sin A} \right] \\ &= 2 \left[\frac{\cos A}{\cos A \sin A} + \frac{\sin A}{\cos A \sin A} \right] \\ &= 2 \left[\frac{1}{\sin A} + \frac{1}{\cos A} \right] \\ &= 2(\operatorname{cosec} A + \sec A) \\ &= 2(\sec A + \operatorname{cosec} A) = \text{R.H.S.} \end{aligned}$$

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(ii) $\sec^4 A - \tan^4 A = 1 + 2 \tan^2 A$

L.H.S. = $\sec^4 A - \tan^4 A$

= $(\sec^2 A - \tan^2 A) (\sec^2 A + \tan^2 A)$

= $(1 + \tan^4 A - \tan^4 A) (1 + \tan^4 A + \tan^4 A)$ [As $\sec^2 A = \tan^2 A + 1$]

= $1 (1 + 2 \tan^2 A)$

= $1 + 2 \tan^2 A = \text{R.H.S.}$

26. (i) $\operatorname{cosec}^6 A - \cot^6 A = 3 \cot^2 A \operatorname{cosec}^2 A + 1$

(ii) $\sec^6 A - \tan^6 A = 1 + 3 \tan^2 A + 3 \tan^4 A$

Solution:

(i) $\operatorname{cosec}^6 A - \cot^6 A = 3 \cot^2 A \operatorname{cosec}^2 A + 1$

L.H.S. = $\operatorname{cosec}^6 A - \cot^6 A$

= $(\operatorname{cosec}^2 A)^3 - (\cot^2 A)^3$

= $(\operatorname{cosec}^2 A - \cot^2 A)^3 + 3 \operatorname{cosec}^2 A \cot^2 A (\operatorname{cosec}^2 A - \cot^2 A)$

= $(1)^3 + 3 \operatorname{cosec}^2 A \cot^2 A \times 1$

= $1 + 3 \cot^2 A \operatorname{cosec}^2 A$

= $3 \cot^2 A \operatorname{cosec}^2 A + 1 = \text{R.H.S.}$

(ii) $\sec^6 A - \tan^6 A = 1 + 3 \tan^2 A + 3 \tan^4 A$

L.H.S. = $\sec^6 A - \tan^6 A$

= $(\sec^2 A)^3 - (\tan^2 A)^3$

= $(\sec^2 A - \tan^2 A)^3 + 3 \sec^2 A \tan^2 A (\sec^2 A - \tan^2 A)$

= $(1)^3 + 3 \sec^2 A \tan^2 A \times 1$

= $1 + 3 \sec^2 A \tan^2 A$

= $1 + 3 [(1 + \tan^2 A) (\tan^2 A)]$

= $1 + 3 [\tan^2 A + \tan^4 A]$

= $1 + 3 \tan^2 A + 3 \tan^4 A = \text{R.H.S.}$

27.

(i) $\frac{\cot\theta - \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1 + \cos\theta}{\sin\theta}$

(ii) $\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$

Solution:

(i)

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$$\begin{aligned}
 \text{L.H.S} &= \frac{\cot\theta - \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} \\
 &= \frac{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} + 1} \\
 &= \frac{\cos\theta + 1 - \sin\theta}{\sin\theta} \times \frac{\sin\theta}{\cos\theta - 1 + \sin\theta} \\
 &= \frac{\cos\theta + 1 - \sin\theta}{\cos\theta - 1 + \sin\theta} \\
 &= \frac{\cos\theta + (1 - \sin\theta)}{\cos\theta - (1 - \sin\theta)} \\
 &= \frac{[\cos\theta + (1 - \sin\theta)][\cos\theta + (1 - \sin\theta)]}{[\cos\theta - (1 - \sin\theta)][\cos\theta + (1 - \sin\theta)]} \\
 &= \frac{[\cos\theta + (1 - \sin\theta)]^2}{\cos^2\theta - (1 - \sin\theta)^2} \\
 &= \frac{(\cos\theta + 1 - \sin\theta)^2}{\cos^2\theta - (1 + \sin^2\theta - 2\sin\theta)} \\
 &= \frac{\cos^2\theta + \sin^2\theta + 1 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{\cos^2\theta - 1 - \sin^2\theta + 2\sin\theta} \\
 &= \frac{1 + 1 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{1 - \sin^2\theta - 1 - \sin^2\theta + 2\sin\theta} \\
 &= \frac{2 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{2\sin\theta - 2\sin^2\theta} \\
 &= \frac{2(1 + \cos\theta) - 2\sin\theta(1 + \cos\theta)}{2\sin\theta(1 - \sin\theta)} \\
 &= \frac{(1 + \cos\theta)2(1 - \sin\theta)}{2\sin\theta(1 - \sin\theta)} \\
 &= \frac{1 + \cos\theta}{\sin\theta} = \text{R.H.S.}
 \end{aligned}$$

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$$(ii) \quad \frac{\sin \theta}{\cot \theta + \operatorname{cosec} \theta} = 2 + \frac{\sin \theta}{\cot \theta - \operatorname{cosec} \theta}$$

$$\begin{aligned} \text{L.H.S.} &= \frac{\sin \theta}{\frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta}} = \frac{\sin \theta}{\frac{\cos \theta + 1}{\sin \theta}} \\ &= \frac{\sin^2 \theta}{1 + \cos \theta} \\ &= \frac{1 - \cos^2 \theta}{1 + \cos \theta} = \frac{(1 + \cos \theta)(1 - \cos \theta)}{1 + \cos \theta} \\ &= 1 - \cos \theta \end{aligned}$$

$$\begin{aligned} \text{R.H.S.} &= 2 + \frac{\sin \theta}{\cot \theta - \operatorname{cosec} \theta} \\ &= 2 + \frac{\sin \theta}{\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}} \\ &= 2 + \frac{\sin^2 \theta}{\cos \theta - 1} \\ &= \frac{2 \cos \theta - 2 + \sin^2 \theta}{\cos \theta - 1} \\ &= \frac{2 \cos \theta - 2 + (1 - \cos^2 \theta)}{\cos \theta - 1} \\ &= \frac{2(\cos \theta - 1) + (1 + \cos \theta)(1 - \cos \theta)}{\cos \theta - 1} \\ &= (\cos \theta - 1)(2 - 1 - \cos \theta) / (\cos \theta - 1) \\ &= 1 - \cos \theta \end{aligned}$$

Hence, L.H.S = R.H.S.

28. (i) $(\sin \theta + \cos \theta)(\sec \theta + \operatorname{cosec} \theta) = 2 + \sec \theta \operatorname{cosec} \theta$

(ii) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) \sec^2 A = \tan A$

Solution:

(i) $(\sin \theta + \cos \theta)(\sec \theta + \operatorname{cosec} \theta) = 2 + \sec \theta \operatorname{cosec} \theta$

L.H.S. = $(\sin \theta + \cos \theta)(\sec \theta + \operatorname{cosec} \theta)$

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$$\begin{aligned}
 &= (\sin \theta + \cos \theta) \left(\frac{1}{\cos \theta} + \frac{1}{\sin \theta} \right) \\
 &= \frac{(\sin \theta + \cos \theta)(\sin \theta + \cos \theta)}{\sin \theta \cos \theta} \\
 &= \frac{\sin^2 \theta + \sin \theta \cos \theta + \sin \theta \cos \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\
 &= \frac{1 + 2\sin \theta \cos \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} + \frac{2\sin \theta \cos \theta}{\sin \theta \cos \theta} \\
 &= \operatorname{cosec} \theta \sec \theta + 2 \\
 &= 2 + \sec \theta \operatorname{cosec} \theta. \\
 &= \text{R.H.S.}
 \end{aligned}$$

(ii)

$$\begin{aligned}
 \text{L.H.S} &= (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \sec^2 A \\
 &= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \frac{1}{\cos^2 A} \\
 &= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \frac{1}{\cos^2 A} \\
 &= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \cdot \frac{1}{\cos^2 A} = \frac{\sin A}{\cos A} = \tan A \\
 &= \text{R.H.S.}
 \end{aligned}$$

29.

(i) $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

(ii) $\frac{\tan^2 A}{1 + \tan^2 A} + \frac{\cot^2 A}{1 + \cot^2 A} = 1$

Solution:

(i) $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

$$\text{L.H.S} = \frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A}$$

$$= \frac{(\sin A + \cos A)(\sin^2 A - \sin A \cos A + \cos^2 A)}{(\sin A + \cos A)} + \frac{(\sin A - \cos A)(\sin^2 A + \sin A \cos A + \cos^2 A)}{(\sin A - \cos A)}$$

$$= (1 - \sin A \cos A) + (1 + \sin A \cos A)$$

$$[\because \sin^2 A + \cos^2 A = 1]$$

$$= 1 - \sin A \cos A + 1 + \sin A \cos A = 2 = \text{R.H.S.}$$

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$$(ii) \frac{\tan^2 A}{1 + \tan^2 A} + \frac{\cot^2 A}{1 + \cot^2 A} = 1$$

$$\begin{aligned} \text{L.H.S.} &= \frac{\tan^2 A}{1 + \tan^2 A} + \frac{\frac{1}{\tan^2 A}}{1 + \frac{1}{\tan^2 A}} \\ &= \frac{\tan^2 A}{1 + \tan^2 A} + \frac{\frac{1}{\tan^2 A}}{\frac{\tan^2 A + 1}{\tan^2 A}} \\ &= \frac{\tan^2 A}{1 + \tan^2 A} + \frac{\tan^2 A}{\tan^2 A (\tan^2 A + 1)} \\ &= \frac{\tan^2 A}{1 + \tan^2 A} + \frac{1}{1 + \tan^2 A} \\ &= \frac{1 + \tan^2 A}{1 + \tan^2 A} = 1 = \text{R.H.S.} \end{aligned}$$

30. (i) $1/(\sec A + \tan A) - 1/\cos A = 1/\cos A - 1/(\sec A - \tan A)$

(ii) $(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A \operatorname{cosec} A)^2$

(iii) $(\tan A + \sin A)/(\tan A - \sin A) = (\sec A + 1)/(\sec A - 1)$

Solution:

$$(i) \frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$$

$$\begin{aligned} \text{L.H.S.} &= \frac{1}{\sec A + \tan A} - \frac{1}{\cos A} \\ &= \frac{1}{\frac{1}{\cos A} + \frac{\sin A}{\cos A}} - \frac{1}{\cos A} \\ &= \frac{\cos A}{1 + \sin A} - \frac{1}{\cos A} \\ &= \frac{\cos^2 A - 1 - \sin A}{\cos A (1 + \sin A)} = \frac{-\sin^2 A - \sin A}{\cos A (1 + \sin A)} \\ &= \frac{-\sin A (1 + \sin A)}{\cos A (1 + \sin A)} = -\tan A \end{aligned}$$

$$\text{R.H.S.} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$$

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$$\begin{aligned}
 &= \frac{1}{\cos A} - \frac{1}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}} \\
 &= \frac{1}{\cos A} - \frac{\cos A}{1 - \sin A} \\
 &= \frac{1 - \sin A - \cos^2 A}{\cos A (1 - \sin A)} = \frac{\sin^2 A - \sin A}{\cos A (1 - \sin A)} \\
 &= \frac{-\sin A + \sin^2 A}{\cos A (1 - \sin A)} = \frac{-\sin A (1 - \sin A)}{\cos A (1 - \sin A)} \\
 &= \frac{-\sin A}{\cos A} = -\tan A
 \end{aligned}$$

∴ L.H.S. = R.H.S.

(ii) $(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A \operatorname{cosec} A)^2$

$$\begin{aligned}
 \text{L.H.S.} &= (\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 \\
 &= \sin^2 A + \sec^2 A + 2\sin A \sec A + \cos^2 A + \operatorname{cosec}^2 A + 2\cos A \operatorname{cosec} A \\
 &= (\sin^2 A + \cos^2 A) + (\sec^2 A + \operatorname{cosec}^2 A) + 2\sin A \times \frac{1}{\cos A} + 2 \times \cos A \times \frac{1}{\sin A} \\
 &= 1 + \left[\frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} \right] + \frac{2\sin^2 A + 2\cos^2 A}{\sin A \cos A} \\
 &= 1 + \left[\frac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A} \right] + \frac{2[\sin^2 A + \cos^2 A]}{\sin A \cos A} \\
 &= 1 + \frac{1}{\cos^2 A \sin^2 A} + \frac{2}{\sin A \cos A} \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\
 &= \left(1 + \frac{1}{\cos A \sin A} \right)^2 \quad [\because (a + b)^2 = a^2 + (b)^2 + 2ab] \\
 &= (1 + \operatorname{cosec} A \sec A)^2 \\
 &= \text{R.H.S.}
 \end{aligned}$$

(iii) $\frac{\tan A + \sin A}{\tan A - \sin A} = \frac{\sec A + 1}{\sec A - 1}$

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\tan A + \sin A}{\tan A - \sin A} = \frac{\frac{\sin A}{\cos A} + \sin A}{\frac{\sin A}{\cos A} - \sin A} = \frac{\frac{\sin A + \sin A \cos A}{\cos A}}{\frac{\sin A - \sin A \cos A}{\cos A}} \\
 &= \frac{\sin A (1 + \cos A)}{\sin A (1 - \cos A)} = \frac{1 + \cos A}{1 - \cos A}
 \end{aligned}$$

On dividing each term by $\cos A$, we have

$$\begin{aligned}
 &= \frac{\frac{1}{\cos A} + 1}{\frac{1}{\cos A} - 1} = \frac{\sec A + 1}{\sec A - 1} = \text{R.H.S.}
 \end{aligned}$$

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31. If $\sin \theta + \cos \theta = \sqrt{2} \sin (90^\circ - \theta)$, show that $\cot \theta = \sqrt{2} + 1$

Solution:

Given, $\sin \theta + \cos \theta = \sqrt{2} \sin (90^\circ - \theta)$

$\sin \theta + \cos \theta = \sqrt{2} \cos \theta$

On dividing by $\sin \theta$, we have

$1 + \cot \theta = \sqrt{2} \cot \theta$

$1 = \sqrt{2} \cot \theta - \cot \theta$

$(\sqrt{2} - 1) \cot \theta = 1$

$\cot \theta = 1/(\sqrt{2} - 1)$

$$= \frac{1 \times (\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)} \quad \text{(Rationalising the denominator)}$$

$$= \frac{(\sqrt{2} + 1)}{(\sqrt{2})^2 - (1)^2} = \frac{\sqrt{2} + 1}{2 - 1} = \frac{\sqrt{2} + 1}{1}$$

$$= \sqrt{2} + 1 = \text{R.H.S.}$$

Hence, $\cot \theta = \sqrt{2} + 1$

32. If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$, $0^\circ \leq \theta \leq 90^\circ$, then find the value of θ .

Solution:

Given,

$7 \sin^2 \theta + 3 \cos^2 \theta = 4$, $0^\circ \leq \theta \leq 90^\circ$

$3 \sin^2 \theta + 3 \cos^2 \theta + 4 \sin^2 \theta = 4$

$3 (\sin^2 \theta + 3 \cos^2 \theta) + 4 \sin^2 \theta = 4$

$3 (1) + 4 \sin^2 \theta = 4$

$4 \sin^2 \theta = 4 - 3$

$\sin^2 \theta = \frac{1}{4}$

Taking square-root on both sides, we get

$\sin \theta = \frac{1}{2}$

Thus, $\theta = 30^\circ$

33. If $\sec \theta + \tan \theta = m$ and $\sec \theta - \tan \theta = n$, prove that $mn = 1$.

Solution:

Given,

$\sec \theta + \tan \theta = m$

$\sec \theta - \tan \theta = n$

Now,

$mn = (\sec \theta + \tan \theta) (\sec \theta - \tan \theta)$

$= \sec^2 \theta - \tan^2 \theta = 1$

Thus, $mn = 1$

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34. If $x = a \sec \theta + b \tan \theta$ and $y = a \tan \theta + b \sec \theta$, prove that $x^2 - y^2 = a^2 - b^2$.

Solution:

Given,

$$x = a \sec \theta + b \tan \theta,$$

$$y = a \tan \theta + b \sec \theta$$

Now,

$$x^2 - y^2 = (a \sec \theta + b \tan \theta)^2 - (a \tan \theta + b \sec \theta)^2$$

$$= (a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta) - (a^2 \tan^2 \theta + b^2 \sec^2 \theta + 2ab \sec \theta \tan \theta)$$

$$= a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta - a^2 \tan^2 \theta - b^2 \sec^2 \theta - 2ab \sec \theta \tan \theta$$

$$= a^2 (\sec^2 \theta - \tan^2 \theta) - b^2 (\sec^2 \theta - \tan^2 \theta)$$

$$= a^2 \times 1 - b^2 \times 1$$

$$\{\sec^2 \theta - \tan^2 \theta = 1\}$$

$$= a^2 - b^2$$

- Hence proved.

35. If $x = h + a \cos \theta$ and $y = k + a \sin \theta$, prove that $(x - h)^2 + (y - k)^2 = a^2$.

Solution:

Given,

$$x = h + a \cos \theta$$

$$y = k + a \sin \theta$$

Now,

$$x - h = a \cos \theta$$

$$y - k = a \sin \theta$$

On squaring and adding we get

$$(x - h)^2 + (y - k)^2 = a^2 \cos^2 \theta + a^2 \sin^2 \theta$$

$$= a^2 (\sin^2 \theta + \cos^2 \theta)$$

$$= a^2 (1)$$

$$[\text{Since, } \sin^2 \theta + \cos^2 \theta = 1]$$

- Hence proved



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Chapter Test

1. (i) If θ is an acute angle and $\operatorname{cosec} \theta = \sqrt{5}$, find the value of $\cot \theta - \cos \theta$.

(ii) If θ is an acute angle and $\tan \theta = 8/15$, find the value of $\sec \theta + \operatorname{cosec} \theta$.

Solution:

Given, θ is an acute angle and $\operatorname{cosec} \theta = \sqrt{5}$

So,

$$\sin \theta = 1/\sqrt{5}$$

$$\text{And, } \cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\cos \theta = \sqrt{1 - (1/\sqrt{5})^2}$$

$$= \sqrt{1 - (1/5)}$$

$$= \sqrt{4/5}$$

$$\cos \theta = 2/\sqrt{5}$$

Now,

$$\cot \theta - \cos \theta = (\cos \theta / \sin \theta) - \cos \theta$$

$$= \sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{1 - \frac{1}{5}} = \sqrt{\frac{5-1}{5}}$$

$$= \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

(ii) Given, θ is an acute angle and $\tan \theta = 8/15$

In fig. we have

$$\tan \theta = BC/AB = 8/15$$

$$\text{So, } BC = 8 \text{ and } AB = 15$$

By Pythagoras theorem, we have

$$AC = \sqrt{(AB^2 + BC^2)} = \sqrt{(5^2 + 8^2)} = \sqrt{(25 + 64)} = \sqrt{89}$$

$$\Rightarrow AC = 17$$

Now,

$$\sec \theta = AC/AB = 17/15$$

$$\operatorname{cosec} \theta = AC/BC = 17/8$$

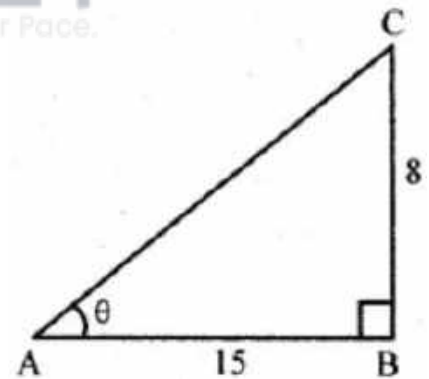
So,

$$\sec \theta + \operatorname{cosec} \theta = 17/15 + 17/8$$

$$= (136 + 255)/120$$

$$= 391/120$$

$$= 3 \frac{31}{120}$$



2. Evaluate the following:

$$(i) 2 \times \left(\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sin^2 25^\circ + \sin^2 65^\circ} \right) - \tan 45^\circ + \tan 13^\circ \tan 23^\circ \tan 30^\circ \tan 67^\circ \tan 77^\circ$$

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(ii) $\frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ$

Solution:

$$\begin{aligned} (i) & 2 \times \left(\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sin^2 25^\circ + \sin^2 65^\circ} \right) - \tan 45^\circ + \tan 13^\circ \tan 23^\circ \tan 30^\circ \tan 67^\circ \tan 77^\circ \\ &= 2 \left[\frac{\cos^2 (90^\circ - 70^\circ) + \cos^2 70^\circ}{\sin^2 25^\circ + \sin^2 (90^\circ - 25^\circ)} \right] - \tan 45^\circ + \tan 13^\circ \tan 77^\circ \tan 23^\circ \tan 67^\circ \tan 30^\circ \\ &= 2 \left[\frac{\sin^2 70^\circ + \cos^2 70^\circ}{\sin^2 25^\circ + \cos^2 25^\circ} \right] - 1 + \tan 13^\circ \tan (90^\circ - 13^\circ) \tan 23^\circ \tan (90^\circ - 23^\circ) \times \frac{1}{\sqrt{3}} \\ &= 2 \left(\frac{1}{1} \right) - 1 + \tan 13^\circ \cot 13^\circ \tan 23^\circ \cot 23^\circ \times \frac{1}{\sqrt{3}} \\ &= 2 - 1 + 1 \times 1 \times \frac{1}{\sqrt{3}} \\ &= 2 - 1 + \frac{1}{\sqrt{3}} = 1 + \frac{1}{\sqrt{3}} = \frac{\sqrt{3} + 1}{\sqrt{3}} \\ &= \frac{(\sqrt{3} + 1)\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{3 + \sqrt{3}}{3} \end{aligned}$$

$$\begin{aligned} (ii) & \frac{\sec 29^\circ}{\operatorname{cosec} 61^\circ} + 2 \cot 8^\circ \cot 17^\circ \cot 45^\circ \cot 73^\circ \cot 82^\circ - 3(\sin^2 38^\circ + \sin^2 52^\circ) \\ &= \frac{\sec 29^\circ}{\operatorname{cosec}(90^\circ - 29^\circ)} + 2 \cot 8^\circ \times \cot (90^\circ - 8^\circ) \times \cot 17^\circ \times \cot (90^\circ - 17^\circ) \cot 45^\circ - \\ & \quad 3[\sin^2 38^\circ + \sin^2 (90^\circ - 38^\circ)] \\ &= \frac{\sec 29^\circ}{\sec 29^\circ} + 2 \cot 8^\circ \tan 8^\circ \times \cot 17^\circ \tan 17^\circ \times 1 - 3(\sin^2 38^\circ + \cos^2 38^\circ) \\ &= 1 + 2 \times 1 \times 1 \times 1 - 3 \times 1 = 1 + 2 - 3 = 0 \end{aligned}$$

$$\begin{aligned} (iii) & \frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ \\ &= \frac{\sin^2 22^\circ + \sin^2 (90^\circ - 22^\circ)}{\cos^2 22^\circ + \cos^2 (90^\circ - 22^\circ)} + \sin^2 63^\circ + \cos 63^\circ \sin (90^\circ - 63^\circ) \\ &= \frac{\sin^2 22^\circ + \cos^2 22^\circ}{\cos^2 22^\circ + \sin^2 22^\circ} + \sin^2 63^\circ + \cos 63^\circ \times \cos 63^\circ \\ &= \frac{1}{1} + (\sin^2 63^\circ + \cos^2 63^\circ) = 1 + 1 = 2 \end{aligned}$$

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3. If $\frac{4}{3}(\sec^2 59^\circ - \cot^2 31^\circ) - \frac{2}{3} \sin 90^\circ + 3 \tan^2 56^\circ \tan^2 34^\circ = x/2$, then find the value of x.

Solution:

Given,

$$\frac{4}{3}(\sec^2 59^\circ - \cot^2 31^\circ) - \frac{2}{3} \sin 90^\circ + 3 \tan^2 56^\circ \tan^2 34^\circ = x/2$$

$$\Rightarrow \frac{4}{3}[\sec^2 59^\circ - \cot^2 (90^\circ - 59^\circ)] - \frac{2}{3} \sin 90^\circ + 3 \tan^2 56^\circ \tan^2 (90^\circ - 56^\circ) = \frac{x}{2}$$

$$\frac{4}{3}[\sec^2 59^\circ - \tan^2 59^\circ] - \frac{2}{3} \times 1 + 3 \tan^2 56^\circ \cot^2 56^\circ = \frac{x}{2}$$

$$\frac{4}{3} \times 1 - \frac{2}{3} + 3 \times 1 = \frac{x}{2}$$

$$\frac{4}{3} - \frac{2}{3} + 3 = \frac{x}{2}$$

$$\frac{4-2+9}{3} = \frac{x}{2}$$

$$\frac{11}{3} = \frac{x}{2} \Rightarrow x = \frac{11 \times 2}{3} = 11$$

$\therefore x = 11$

4. (i) $\cos A / (1 - \sin A) + \cos A / (1 + \sin A) = 2 \sec A$

(ii) $\cos A / (\operatorname{cosec} A + 1) + \cos A / (\operatorname{cosec} A - 1) = 2 \tan A$

Solution:

$$(i) \frac{\cos A}{1 - \sin A} + \frac{\cos A}{1 + \sin A} = 2 \sec A$$

$$\text{L.H.S} = \frac{\cos A}{1 - \sin A} + \frac{\cos A}{1 + \sin A}$$

$$= \cos A \left[\frac{1}{1 - \sin A} + \frac{1}{1 + \sin A} \right]$$

$$= \cos A \left[\frac{1 + \sin A + 1 - \sin A}{(1 - \sin A)(1 + \sin A)} \right]$$

$$= \cos A \left[\frac{2}{1 - \sin^2 A} \right] = \frac{2 \cos A}{\cos^2 A}$$

$$= \frac{2}{\cos A} = 2 \sec A = \text{R.H.S.}$$

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$$(ii) \frac{\cos A}{\operatorname{cosec} A + 1} + \frac{\cos A}{\operatorname{cosec} A - 1} = 2 \tan A$$

$$\text{L.H.S.} = \frac{\cos A}{\operatorname{cosec} A + 1} + \frac{\cos A}{\operatorname{cosec} A - 1}$$

$$= \cos A \left[\frac{1}{\operatorname{cosec} A + 1} + \frac{1}{\operatorname{cosec} A - 1} \right]$$

$$= \cos A \left[\frac{\operatorname{cosec} A - 1 + \operatorname{cosec} A + 1}{(\operatorname{cosec} A + 1)(\operatorname{cosec} A - 1)} \right]$$

$$= \frac{\cos A [2 \operatorname{cosec} A]}{\operatorname{cosec}^2 A - 1} = \frac{2 \cos A}{\sin A (\cot^2 A)}$$

$$= \frac{2 \cot A}{\cot^2 A} = \frac{2}{\cot A} = 2 \tan A = \text{R.H.S.}$$

$$5. (i) \frac{(\cos \theta - \sin \theta)(1 + \tan \theta)}{2 \cos^2 \theta - 1} = \sec \theta$$

$$(ii) (\operatorname{cosec} \theta - \sin \theta) (\sec \theta - \cos \theta) (\tan \theta + \cot \theta) = 1.$$

Solution:

$$(i) \frac{(\cos \theta - \sin \theta)(1 + \tan \theta)}{2 \cos^2 \theta - 1} = \sec \theta$$

$$\text{L.H.S.} = \frac{(\cos \theta - \sin \theta)(1 + \tan \theta)}{2 \cos^2 \theta - 1}$$

$$= \frac{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta (2 \cos^2 \theta - 1)} = \frac{\cos^2 \theta - (1 - \cos^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{\cos^2 \theta - 1 + \cos^2 \theta}{\cos \theta (2 \cos^2 \theta - 1)} = \frac{2 \cos^2 \theta - 1}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{1}{\cos \theta} = \sec \theta = \text{R.H.S.}$$

$$(ii) (\operatorname{cosec} \theta - \sin \theta) (\sec \theta - \cos \theta) (\tan \theta + \cot \theta) = 1.$$

$$\text{L.H.S.} = (\operatorname{cosec} \theta - \sin \theta) (\sec \theta - \cos \theta) (\tan \theta + \cot \theta)$$

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$$\begin{aligned} &= \left(\frac{1}{\sin \theta} - \sin \theta \right) \left(\frac{1}{\cos \theta} - \cos \theta \right) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) \\ &= \left(\frac{1 - \sin^2 \theta}{\sin \theta} \right) \left(\frac{1 - \cos^2 \theta}{\cos \theta} \right) \left(\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \right) \\ &= \frac{(\cos^2 \theta \times \sin^2 \theta)}{\sin \theta \cos \theta} \times \frac{1}{\sin \theta \cos \theta} \\ &= \frac{\sin^2 \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} = 1 = \text{R.H.S.} \end{aligned}$$

6. (i) $\sin^2 \theta + \cos^4 \theta = \cos^2 \theta + \sin^4 \theta$

(ii) $\frac{\cot \theta}{\operatorname{cosec} \theta + 1} + \frac{\operatorname{cosec} \theta + 1}{\cot \theta} = 2 \sec \theta$

Solution:

Given,

(i) $\sin^2 \theta + \cos^4 \theta = \cos^2 \theta + \sin^4 \theta$

L.H.S. = $\sin^2 \theta + \cos^4 \theta$

= $(1 - \cos^2 \theta) + \cos^4 \theta$

= $\cos^4 \theta - \cos^2 \theta + 1$

= $\cos^2 \theta (\cos^2 \theta - 1) + 1$

= $\cos^2 \theta (-\sin^2 \theta) + 1$

= $1 - \sin^2 \theta \cos^2 \theta$

Now,

R.H.S. = $\cos^2 \theta + \sin^4 \theta$

= $(1 - \sin^2 \theta) + \sin^4 \theta$

= $\sin^4 \theta - \sin^2 \theta + 1$

= $\sin^2 \theta (\sin^2 \theta - 1) + 1$

= $\sin^2 \theta (-\cos^2 \theta) + 1$

= $1 - \sin^2 \theta \cos^2 \theta$

Hence, L.H.S. = R.H.S.

(ii) $\frac{\cot \theta}{\operatorname{cosec} \theta + 1} + \frac{\operatorname{cosec} \theta + 1}{\cot \theta} = 2 \sec \theta$

L.H.S. = $\frac{\cot \theta}{\operatorname{cosec} \theta + 1} + \frac{\operatorname{cosec} \theta + 1}{\cot \theta}$



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$$\begin{aligned}
 &= \frac{\frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta} + 1} + \frac{1}{\frac{\cos \theta}{\sin \theta}} \\
 &= \frac{\frac{\cos \theta}{\sin \theta}}{\frac{1 + \sin \theta}{\sin \theta}} + \frac{1 + \sin \theta}{\frac{\cos \theta}{\sin \theta}} \\
 &= \frac{\cos \theta}{\sin \theta} \times \frac{\sin \theta}{1 + \sin \theta} + \frac{1 + \sin \theta}{\sin \theta} \times \frac{\sin \theta}{\cos \theta} \\
 &= \frac{\cos \theta}{1 + \sin \theta} + \frac{1 + \sin \theta}{\cos \theta} \\
 &= \frac{\cos^2 \theta + 1 + \sin^2 \theta + 2 \sin \theta}{(1 + \sin \theta) \cos \theta} \\
 &= \frac{1 + 1 + 2 \sin \theta}{(1 + \sin \theta) \cos \theta} = \frac{2 + 2 \sin \theta}{(1 + \sin \theta) \cos \theta} \\
 &= \frac{2(1 + \sin \theta)}{\cos \theta (1 + \sin \theta)} = \frac{2}{\cos \theta} \\
 &= 2 \sec \theta = \text{R.H.S.}
 \end{aligned}$$

7. (i) $\sec^4 A (1 - \sin^4 A) - 2 \tan^2 A = 1$

(ii) $\frac{1}{\sin A + \cos A + 1} + \frac{1}{\sin A + \cos A - 1} = \sec A + \operatorname{cosec} A$

Solution:

(i) $\sec^4 A (1 - \sin^4 A) - 2 \tan^2 A = 1$

L.H.S. = $\sec^4 A (1 - \sin^4 A) - 2 \tan^2 A$

$$= \frac{1}{\cos^4 A} (1 + \sin^2 A) (1 - \sin^2 A) - 2 \tan^2 A$$

$$[\because a^2 - b^2 = (a + b)(a - b)]$$

$$= \frac{(1 + \sin^2 A) \cos^2 A}{\cos^4 A} - 2 \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1 + \sin^2 A}{\cos^2 A} - \frac{2 \sin^2 A}{\cos^2 A}$$

$$= \frac{1 + \sin^2 A - 2 \sin^2 A}{\cos^2 A}$$

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$$= \frac{1 - \sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1$$

($\because 1 - \sin^2 A = \cos^2 A$)

= R.H.S.

$$(ii) \frac{1}{\sin A + \cos A + 1} + \frac{1}{\sin A + \cos A - 1} = \sec A + \operatorname{cosec} A$$

$$\begin{aligned} \text{L.H.S.} &= \frac{1}{\sin A + \cos A + 1} + \frac{1}{\sin A + \cos A - 1} \\ &= \frac{\sin A + \cos A - 1 + \sin A + \cos A + 1}{(\sin A + \cos A + 1)(\sin A + \cos A - 1)} \end{aligned}$$

$$\begin{aligned} &= \frac{2(\sin A + \cos A)}{(\sin A + \cos A)^2 - (1)^2} \\ &= \frac{2(\sin A + \cos A)}{\sin^2 A + \cos^2 A + 2 \sin A \cos A - 1} \end{aligned}$$

$$\begin{aligned} &= \frac{2(\sin A + \cos A)}{1 + 2 \sin A \cos A - 1} \\ &= \frac{2(\sin A + \cos A)}{2 \sin A \cos A} \\ &= \frac{\sin A + \cos A}{\sin A \cos A} = \frac{\sin A}{\sin A \cos A} + \frac{\cos A}{\sin A \cos A} \end{aligned}$$

$$= \frac{1}{\cos A} + \frac{1}{\sin A}$$

$$= \sec A + \operatorname{cosec} A = \text{R.H.S.}$$

$$8. (i) \frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta \cos \theta} + \sin \theta \cos \theta = 1$$

$$(ii) (\sec A - \tan A)^2 (1 + \sin A) = 1 - \sin A.$$

Solution:

$$(i) \frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta \cos \theta} + \sin \theta \cos \theta = 1$$

$$\begin{aligned} \text{LHS} &= \frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta \cos \theta} + \sin \theta \cos \theta \\ &= \frac{(\sin \theta + \cos \theta)(\sin^2 \theta - \sin \theta \cos \theta + \cos^2 \theta)}{(\sin \theta \cos \theta)} + \sin \theta \cos \theta \end{aligned}$$

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$$= \sin^2 \theta + \cos^2 \theta - \sin \theta \cos \theta + \sin \theta \cos \theta$$

$$= 1 = \text{R.H.S.}$$

(ii) $(\sec A - \tan A)^2 (1 + \sin A) = 1 - \sin A$

$$\text{L.H.S.} = \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A} \right)^2 (1 + \sin A)$$

$$= \left(\frac{1 - \sin A}{\cos A} \right)^2 (1 + \sin A)$$

$$= \frac{(1 - \sin A)^2}{\cos^2 A} (1 + \sin A)$$

$$= \frac{(1 - \sin A)^2 (1 + \sin A)}{1 - \sin^2 A}$$

$$= \frac{(1 - \sin A)^2 (1 + \sin A)}{(1 - \sin A)(1 + \sin A)}$$

$$= 1 - \sin A = \text{R.H.S.}$$

9. (i) $\frac{\cos A}{1 - \tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$

(ii) $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \operatorname{cosec} A$

(iii) $\frac{\tan^2 \theta}{\tan^2 \theta - 1} - \frac{\operatorname{cosec}^2 \theta}{\sec^2 \theta - \operatorname{cosec}^2 \theta} = \frac{1}{\sin^2 \theta - \cos^2 \theta}$

Solution:

(i) $\frac{\cos A}{1 - \tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$

$$\text{L.H.S.} = \frac{\cos A}{1 - \tan A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= 1 - \frac{\sin A}{\cos A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos A \times \cos A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{(\cos A - \sin A)} = \cos A + \sin A = \text{R.H.S.}$$

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$$(ii) (\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \operatorname{cosec} A$$

$$\text{L.H.S.} = (\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A)$$

$$\begin{aligned} &= \left(\frac{1}{\cos A} - \frac{1}{\sin A} \right) \left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right) \\ &= \frac{\sin A - \cos A}{\sin A \cos A} \times \frac{\sin A \cos A + \sin^2 A + \cos^2 A}{\sin A \cos A} \\ &= \frac{\sin A - \cos A}{\sin A \cos A} \times \frac{\sin A \cos A + 1}{\sin A \cos A} \\ &= \frac{(\sin A - \cos A)(\sin A \cos A + 1)}{\sin^2 A \cos^2 A} \end{aligned}$$

$$\text{R.H.S.} = \tan A \sec A - \cot A \operatorname{cosec} A$$

$$\begin{aligned} &= \frac{\sin A}{\cos A} \cdot \frac{1}{\cos A} - \frac{\cos A}{\sin A} \cdot \frac{1}{\sin A} \\ &= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin^2 A} = \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A} \\ &= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\sin^2 A \cos^2 A} \\ &= \frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\sin^2 A \cos^2 A} \\ &= \frac{(\sin A - \cos A)(\sin A \cos A + 1)}{\sin^2 A \cos^2 A} \end{aligned}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$(iii) \frac{\tan^2 \theta}{\tan^2 \theta - 1} + \frac{\operatorname{cosec}^2 \theta}{\sec^2 \theta - \operatorname{cosec}^2 \theta} = \frac{1}{\sin^2 \theta - \cos^2 \theta}$$

$$\begin{aligned} \text{L.H.S.} &= \frac{\tan^2 \theta}{\tan^2 \theta - 1} + \frac{\operatorname{cosec}^2 \theta}{\sec^2 \theta - \operatorname{cosec}^2 \theta} \\ &= \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta} - 1} + \frac{\frac{1}{\sin^2 \theta}}{\frac{1}{\cos^2 \theta} - \frac{1}{\sin^2 \theta}} = \frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{\cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} + \frac{\frac{1}{\sin^2 \theta}}{\frac{\sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}} \\ &= \frac{\sin^2 \theta}{\sin^2 \theta - \cos^2 \theta} + \frac{\sin^2 \theta \cos^2 \theta}{\sin^2 \theta (\sin^2 \theta - \cos^2 \theta)} \\ &= \frac{\sin^2 \theta}{\sin^2 \theta - \cos^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} = \frac{1}{\sin^2 \theta - \cos^2 \theta} = \text{R.H.S.} \end{aligned}$$

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$$10. \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{1 - 2\cos^2 A} = \frac{2\sec^2 A}{\tan^2 A - 1}$$

Solution:

$$\begin{aligned} \text{L.H.S} &= \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} \\ &= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{\sin^2 A - \cos^2 A} \\ &= \frac{2(\sin^2 A + \cos^2 A)}{\sin^2 A - \cos^2 A} \\ &= \frac{2 \times 1}{\sin^2 A - \cos^2 A} = \frac{2}{\sin^2 A - \cos^2 A} \\ &= \text{R.H.S.} \end{aligned}$$

Now, we have

$$\begin{aligned} &= \frac{2}{\sin^2 A - \cos^2 A} \\ &= \frac{2}{1 - \cos^2 A - \cos^2 A} = \frac{2}{1 - 2\cos^2 A} \\ &= \text{R.H.S.} \end{aligned}$$

Now, we have

$$\begin{aligned} &= \frac{2}{\sin^2 A - \cos^2 A} \\ &= \frac{2}{\sin^2 A - (1 - \sin^2 A)} \\ &= \frac{2}{\sin^2 A - 1 + \sin^2 A} \\ &= \frac{2}{\sin^2 A - \cos^2 A} = \frac{\frac{2}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - \frac{\cos^2 A}{\cos^2 A}} \\ &= \frac{2\sec^2 A}{\tan^2 A - 1} = \text{R.H.S.} \end{aligned}$$

Hence,

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{1 - 2\cos^2 A} = \frac{2\sec^2 A}{\tan^2 A - 1}$$

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11. $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$

Solution:

Given,

$$2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$$

$$\text{L.H.S.} = 2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1$$

$$= 2[(\sin^2 \theta)^3 + (\cos^2 \theta)^3] - 3(\sin^4 \theta + \cos^4 \theta) + 1$$

$$= 2[(\sin^2 \theta + \cos^2 \theta)(\sin^4 \theta + \cos^4 \theta - \sin^2 \theta \cos^2 \theta)] - 3(\sin^4 \theta + \cos^4 \theta) + 1$$

$$= 2(\sin^4 \theta + \cos^4 \theta - \sin^2 \theta \cos^2 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1$$

$$= 2\sin^4 \theta + 2\cos^4 \theta - 2\sin^2 \theta \cos^2 \theta - 3\sin^4 \theta - 3\cos^4 \theta + 1$$

$$= 1 - \sin^4 \theta - \cos^4 \theta - 2\sin^2 \theta \cos^2 \theta$$

$$= 1 - [\sin^4 \theta + \cos^4 \theta + 2\sin^2 \theta \cos^2 \theta]$$

$$= 1 - 1$$

$$= 0 = \text{R.H.S.}$$

12. If $\cot \theta + \cos \theta = m$, $\cot \theta - \cos \theta = n$, then prove that $(m^2 - n^2)^2 = 16$.

Solution:

Given,

$$\cot \theta + \cos \theta = m \dots (i)$$

$$\cot \theta - \cos \theta = n \dots (ii)$$

Adding (i) and (ii), we get

$$2 \cot \theta = m + n \Rightarrow \cot \theta = \frac{m+n}{2}$$

$$\therefore \tan \theta = \frac{2}{m+n} \dots (iii)$$

Subtracting (ii) from (i),

$$2 \cos \theta = m - n \Rightarrow \cos \theta = \frac{m-n}{2}$$

$$\therefore \sec \theta = \frac{2}{m-n} \dots (iv)$$

Now, squaring and subtracting (iii) from (iv), we have

$$\sec^2 \theta - \tan^2 \theta = \left(\frac{2}{m-n}\right)^2 - \left(\frac{2}{m+n}\right)^2$$

$$1 = \frac{4}{(m-n)^2} - \frac{4}{(m+n)^2}$$

$$\Rightarrow 4 \left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2} \right] = 1$$

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$$4 \left[\frac{(m+n)^2 - (m-n)^2}{(m+n)^2 (m-n)^2} \right] = 1$$

$$\frac{4(4mn)}{(m^2 - n^2)^2} = 1$$

$$\frac{16mn}{(m^2 - n^2)^2} = 1$$

$$\therefore (m^2 - n^2)^2 = 16mn.$$

13. If $\sec \theta + \tan \theta = p$, prove that $\sin \theta = (p^2 - 1)/(p^2 + 1)$

Solution:

Given, $\sec \theta + \tan \theta = p$

$$\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = p$$

$$\frac{1 + \sin \theta}{\cos \theta} = p$$

Squaring on both sides,

$$\frac{(1 + \sin \theta)^2}{\cos^2 \theta} = p^2 \Rightarrow \frac{(1 + \sin \theta)^2}{1 - \sin^2 \theta} = p^2$$

$$\frac{(1 + \sin \theta)^2}{(1 - \sin \theta)(1 + \sin \theta)} = p^2$$

$$\Rightarrow \frac{1 + \sin \theta}{1 - \sin \theta} = \frac{p^2}{1}$$

Applying componendo and dividendo,

$$\frac{1 + \sin \theta + 1 - \sin \theta}{1 + \sin \theta - 1 + \sin \theta} = \frac{p^2 + 1}{p^2 - 1}$$

$$\frac{2}{2\sin \theta} = \frac{p^2 + 1}{p^2 - 1} \Rightarrow \frac{1}{\sin \theta} = \frac{p^2 + 1}{p^2 - 1}$$

$$\therefore \sin \theta = \frac{p^2 - 1}{p^2 + 1}$$

14. If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that $\cos^2 A = (m^2 - 1)/(n^2 - 1)$

Solution:

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Given,

$$\tan A = n \tan B \text{ and } \sin A = m \sin B$$

$$n = \tan A / \tan B$$

$$m = \sin A / \sin B$$

$$\frac{1}{\sin B} = \frac{m}{\sin A} \Rightarrow \operatorname{cosec} B = \frac{m}{\sin A}$$

$$\frac{1}{\tan B} = \frac{n}{\tan A} \Rightarrow \cot B = \frac{n}{\tan A}$$

$$\text{Now, } \operatorname{cosec}^2 B - \cot^2 B = 1$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1$$

$$m^2 - n^2 \cos^2 A = \sin^2 A$$

$$m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$m^2 - 1 = \cos^2 A + n^2 \cos^2 A$$

$$m^2 - 1 = (n^2 + 1) \cos^2 A$$

$$\Rightarrow \cos^2 A = \frac{m^2 - 1}{n^2 + 1}$$

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15. If $\sec A = x + 1/4x$, then prove that $\sec A + \tan A = 2x$ or $1/2x$

Solution:

Given, $\sec A = x + 1/4x$

We know that,

$$\tan A = \pm \sqrt{\sec^2 A - 1}$$

$$= \pm \sqrt{\left(x + \frac{1}{4x}\right)^2 - 1}$$

$$= \pm \sqrt{x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1}$$

$$= \pm \sqrt{x^2 + \frac{1}{16x^2} - \frac{1}{2}}$$

$$= \pm \left(x - \frac{1}{4x}\right)$$

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$$\therefore \sec A + \tan A = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$$

$$\text{or } x + \frac{1}{4x} - x + \frac{1}{4x} = \frac{1}{2x}$$

16. When $0^\circ < \theta < 90^\circ$, solve the following equations:

(i) $2 \cos^2 \theta + \sin \theta - 2 = 0$

(ii) $3 \cos \theta = 2 \sin^2 \theta$

(iii) $\sec^2 \theta - 2 \tan \theta = 0$

(iv) $\tan^2 \theta = 3 (\sec \theta - 1)$.

Solution:

Given, $0^\circ < \theta < 90^\circ$

(i) $2 \cos^2 \theta + \sin \theta - 2 = 0$

$$2(1 - \sin^2 \theta) + \sin \theta - 2 = 0$$

$$2 - 2 \sin^2 \theta + \sin \theta - 2 = 0$$

$$-2 \sin^2 \theta + \sin \theta = 0$$

$$\sin \theta (1 - 2 \sin \theta) = 0$$

So, either $\sin \theta = 0$ or $1 - 2 \sin \theta = 0$

If $\sin \theta = 0$

$$\Rightarrow \theta = 0^\circ$$

And, if $1 - 2 \sin \theta = 0$

$$\sin \theta = \frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ$$

Thus, $\theta = 0^\circ$ or 30°

(ii) $3 \cos \theta = 2 \sin^2 \theta$

$$3 \cos \theta = 2(1 - \cos^2 \theta)$$

$$3 \cos \theta = 2 - 2 \cos^2 \theta$$

$$2 \cos^2 \theta + 3 \cos \theta - 2 = 0$$

$$2 \cos^2 \theta + 4 \cos \theta - \cos \theta - 2 = 0$$

$$2 \cos \theta (\cos \theta + 2) - 1(\cos \theta + 2) = 0$$

$$(2 \cos \theta - 1)(\cos \theta + 2) = 0$$

So, either $2 \cos \theta - 1 = 0$ or $\cos \theta + 2 = 0$

If $2 \cos \theta - 1 = 0$

$$\cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = 60^\circ$$

And, for $\cos \theta + 2 = 0$

$$\Rightarrow \cos \theta = -2 \text{ which is not possible being out of range.}$$

Thus, $\theta = 60^\circ$

(iii) $\sec^2 \theta - 2 \tan \theta = 0$

$$(1 + \tan^2 \theta) - 2 \tan \theta = 0$$

$$\tan^2 \theta - 2 \tan \theta + 1 = 0$$



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$$(\tan \theta - 1)^2 = 0$$

$$\tan \theta - 1 = 0$$

$$\Rightarrow \tan \theta = 1$$

Thus, $\theta = 45^\circ$

$$(iv) \tan^2 \theta = 3 (\sec \theta - 1)$$

$$(\sec^2 \theta - 1) = 3 \sec \theta - 3$$

$$\sec^2 \theta - 1 - 3 \sec \theta + 3 = 0$$

$$\sec^2 \theta - 3 \sec \theta + 2 = 0$$

$$\sec^2 \theta - 2 \sec \theta - \sec \theta + 2 = 0$$

$$\sec \theta (\sec \theta - 2) - 1 (\sec \theta - 2) = 0$$

$$(\sec \theta - 1) (\sec \theta - 2) = 0$$

So, either $\sec \theta - 1 = 0$ or $\sec \theta - 2 = 0$

If $\sec \theta - 1 = 0$

$$\sec \theta = 1$$

$$\Rightarrow \theta = 0^\circ$$

And, if $\sec \theta - 2 = 0$

$$\sec \theta = 2$$

$$\Rightarrow \theta = 60^\circ$$

Thus, $\theta = 0^\circ$ or 60°

