NCERT Solutions for Class-XII Maths

Chapter-7.11

NCERT Maths Class 12

1. $\int_0^1 \frac{x}{x^2+1}dx$

1. $I = \int^{\frac{\pi}{2}} \cos^2 xdx$ **....(1)**

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right)dx$ $\left(\int_0^0 f(x)dx = \int_0^0 f(a-x)dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 xdx$ **....(2)**

Adding (1) and (2), we obtain

$2I = \int_0^{\frac{x}{2}} \left(\sin^2 x + \cos^2 x\right)dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1dx$

$\Rightarrow 2I = [x]_0^{\frac{x}{2}}$

$\Rightarrow 2I = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

2. $\int_0^{\frac{x}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

2. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

let, $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ .....(1)

as, $\left\{\int_0^a f(x)dx = \int_0^a f(a-x)dx\right\}$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \qquad (2)$$

Adding (1) and (2), we get

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}[1]dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}-0$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

3. $\int_0^{\frac{x}{2}}\frac{\sin^{\frac{3}{2}}x\,dx}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}$

3. $\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(1)$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots.(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1.dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

4. $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$

4. $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$

let, $I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$ …..(1)

as, $\left\{ \int_0^a f(x)dx = \int_0^a f(a-x)dx \right\}$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx$$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\cos^5 x + \sin^5 x} dx$ …..(2)

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} [1] dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2} - 0$$

$$\Rightarrow 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

5. $\int_{-5}^{5}|x+2|\,dx$

5. $I=\int_{-5}^{5}|x+2|\,dx$

It can be seen that (x + 2) ≤ 0 on [−5, −2] and (x + 2) ≥ 0 on [−2, 5].

$\therefore I=\int_{-5}^{-2}-(x+1)dx+\int_{-2}^{5}(x+2)dx \qquad \left(\int_a^b f(x)=\int_a^c f(x)+\int_c^b f(x)\right)$

$I=-\left[\frac{x^2}{2}+2x\right]_{-5}^{-2}+\left[\frac{x^2}{2}+2x\right]_{-2}^{5}$

$=-\left[\frac{(-2)^2}{2}+2(-2)-\frac{(-5)^2}{2}-2(-5)\right]+\left[\frac{(5)^2}{2}+2(5)-\frac{(-2)^2}{2}-2(-2)\right]$

$=-\left[2-4-\frac{25}{2}+10\right]+\left[\frac{25}{2}+10-2+4\right]$

$=-2+4+\frac{25}{2}-10+\frac{25}{2}+10-2+4$

$=29$

6. $\int_{2}^{8}|x-5|\,dx$

6. $\int_{2}^{8}|x-5|\,dx$

As we can see that $(x-5)\le 0$ on $[2,5]$ and $(x+2)\ge 0$ on $[5,8]$

as, $\left\{\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx\right\}$

$\Rightarrow I=\int_{2}^{5}-(x-5)dx+\int_{5}^{8}(x-5)dx$

$\Rightarrow I=-\left[\frac{x^2}{2}-5x\right]_{2}^{5}+\left[\frac{x^2}{2}-5x\right]_{5}^{8}$

$\Rightarrow I=-\left[\frac{(5)^2}{2}-5(5)-\frac{(2)^2}{2}+5(2)\right]+\left[\frac{(8)^2}{2}-5(8)-\frac{(5)^2}{2}+5(5)\right]$

$\Rightarrow I=-\left[\frac{25}{2}-25-2+10\right]+\left[\frac{64}{2}-40-\frac{25}{2}+25\right]$

$\Rightarrow I=-\frac{25}{2}+17+32-15-\frac{25}{2}$

$\Rightarrow I=34-25$

$\Rightarrow I = 9$

7. $\int_{0}^{1}x(1-x)^n dx$

7. $\int_0^1 x(1-x)^n\ dx$

let, $I = \int_0^1 x(1-x)^n\ dx$

as, $\left\{ \int_0^a f(x)dx = \int_0^a f(a-x)dx \right\}$

$\Rightarrow I = \int_0^1 (1-x)(1-(1-x))^n\ dx$

$\Rightarrow I = \int_0^1 (1-x)(x)^n\ dx$

$\Rightarrow I = \int_0^1 (x)^n - (x)^{n+1} dx$

$$\Rightarrow I = \left[\frac{(x)^{n+1}}{n+1} - \frac{(x)^{2+2}}{n+2} \right]_0^1$$

$$\Rightarrow I = \left[\frac{(n+2)-(n+1)}{(n+1)(n+2)} \right]$$

$$\Rightarrow I = \left[\frac{(n+2)-(n+1)}{(n+1)(n+2)} \right]$$

$$\Rightarrow I = \left[\frac{1}{(n+1)(n+2)} \right]$$

8. $\int_0^{\frac{x}{4}} \log(1+\tan x)dx$

8. Let I $I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

$$\therefore I = \int_0^{\frac{x}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\frac{2}{(1+\tan x)}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log 2dx-\int_0^{\frac{\pi}{4}}\log\left(1+\tan x\right)dx$$

$$\Rightarrow I=\int_e^{\frac{\pi}{4}}\log 2dx-1 \qquad \text{[From (1)]}$$

$$\Rightarrow 2I=\left[x\log 2\right]_0^{\frac{\pi}{4}}$$

$$\Rightarrow 2I=\frac{\pi}{4}\log 2$$

$$\Rightarrow I=\frac{\pi}{8}\log 2$$

9. $\int_0^2 x\sqrt{2-x}dx$

9. $\int_0^2 x\sqrt{2-x}dx$

let, $I=\int_0^2 x\sqrt{2-x}dx$ .....(1)

as, $\left\{\int_0^a f\left(x\right)dx=\int_0^a f\left(a-x\right)dx\right\}$

$$\Rightarrow I=\int_0^2 (2-x)\sqrt{2-(2-x)}dx$$

$$\Rightarrow I=\int_0^2 (2-x)\sqrt{(x)}dx$$

$$\Rightarrow I=\int_0^2 (2x^{\frac{1}{2}}-x^{\frac{3}{2}})dx$$

$$\Rightarrow I=\left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$$

$$\Rightarrow I=\left[\frac{4}{3}\left((2)^{\frac{3}{2}}\right)-\frac{2}{5}\left((2)^{\frac{5}{2}}\right)\right]$$

$$\Rightarrow I=\left[\frac{4}{3}\left(x^{\frac{3}{2}}\right)-\frac{2}{5}\left(x^{\frac{5}{2}}\right)\right]_0^2$$

$$\Rightarrow I=\frac{4}{3}\times 2\sqrt{2}-\frac{2}{5}\times 4\sqrt{2}$$

$$\Rightarrow I=\frac{8\sqrt{2}}{3}-\frac{8\sqrt{2}}{5}$$

$$\Rightarrow I = \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$\Rightarrow I = \frac{16\sqrt{2}}{15}$$

10. $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$

$\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$

let, $I = \int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{2\log\sin x - \log(2\sin x\cos x)\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{2\log\sin x - \log(2) - \log(\sin x) - \log(\cos x)\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{\log\sin x - \log 2 - \log\cos x\}dx \quad \text{….(1)}$$

as, $\left\{\int_0^a f(x)dx = \int_0^a f(a-x)dx\right\}$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left\{\log\sin\left(\frac{\pi}{2} - x\right) - \log 2 - \log\cos\left(\frac{\pi}{2} - x\right)\right\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{\log\cos x - \log 2 - \log\sin x\}dx \quad \text{….(2)}$$

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} (-\log 2 - \log 2)\, dx$$

$$2I = -2\log 2\int_0^{\frac{\pi}{2}} (1)\, dx$$

$$\Rightarrow 2I = -2\log 2\int_0^{\frac{\pi}{2}} [1]dx$$

$$\Rightarrow 2I = -2\log 2[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = -2\log 2\left(\frac{\pi}{2}\right)$$

$$\Rightarrow 2I = -2\log 2\left[\frac{\pi}{2} - 0\right]$$

$\Rightarrow I = \frac{\pi}{2}(-\log 2)$

$\Rightarrow I = \frac{\pi}{2}\left(\log \frac{1}{2}\right)$

11. $\int_{\frac{-\pi}{2}}^{\frac{x}{2}} \sin^2 x dx$

11. Let $I = \int_{\frac{-x}{2}}^{\frac{\pi}{2}} \sin^2 x dx$

As $\sin^2 (-x) = (\sin (-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if f(x) is an even function, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

$I = 2\int_0^{\frac{x}{2}} \sin^2 x dx$

$= 2\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx$

$= \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx$

$= \int_0^{\frac{\pi}{2}} (1-\cos 2x) dx$

$= \left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2}$

12. $\int_0^{\pi} \frac{x dx}{1+\sin x}$

12. Let $I = \int_0^{\pi} \frac{x dx}{1+\sin x}$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)} dx$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin x} dx$

Adding (1) and (2), we obtain

$2I = \int_0^{\pi} \frac{\pi}{1+\sin x} dx$

$\Rightarrow 2I = \pi\int_0^{\pi} \left\{\sec^2 x - \tan x \sec x\right\} dx$

$\Rightarrow 2I = \pi[\tan x - \sec x]_0^x$

$\Rightarrow 2I = \pi[2]$

$\Rightarrow I = \pi$

13. $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$

13. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^7 x) dx$

let, $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^7 x) dx$

As we can see $f(x) = \sin^7 x$ and $f(-x) = \sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$.

i.e. $f(x) = -f(-x)$

so, sin2x is an odd function.

It is also known that if f(x) is an odd function then,

$\left\{\int_{-a}^{a} f(x) dx = 0\right\}$

$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^7 x) dx = 0$

14. $\int_0^{2\pi} \cos^5 x dx$

14. Let $I = \int_0^{2\pi} \cos^5 x dx$

$\cos^5(2\pi - x) = \cos^5 x$

It is known that,

$\int_0^{2d} f(x) dx = 2\int_0^{a} f(x) dx, \text{if } f(2a - x) = f(x)$

$= 0 \text{ if } f(2a - x) = -f(x)$

$\therefore I = 2\int_0^{\pi} \cos^5 x dx$

$\Rightarrow I = 2(0) = 0 \qquad \left[\cos^5(\pi - x) = -\cos^5 x\right]$

15. $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

15. $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

let, $I=\int_0^{\frac{\pi}{2}}\frac{\sin x-\cos x}{1+\sin x\cos x}dx$ .....(1)

as, $\left\{\int_0^a f(x)dx=\int_0^a f(a-x)dx\right\}$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}dx$$

Adding (1) and (2), we get

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin x-\cos x+\cos x-\sin x}{1+\sin x\cos x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\frac{0}{1+\sin x\cos x}dx$$

16. $\int_{20}^{\pi}\log(1+\cos x)dx$

16. Let $I=\int_0^{\pi}\log(1+\cos x)dx$ ....(1)

$\Rightarrow I=\int_1^{\pi}\log(1+\cos(\pi-x))dx$ $\left(\int_0^{\pi}f(x)dx=\int_0^{\pi}f(a-x)dx\right)$

Adding (1) and (2), we obtain

$$2I=\int_0^{\pi}\{\log(1\cos x)+\log(1-\cos x)\}dx$$

$$2I=\int_0^{\pi}\log(1-\cos^2 x)\,dx$$

$$2I=\int_0^{\pi}\log(\sin^2 x)\,dx$$

$$2I=\int_0^{\pi}2.\log(\sin x)\,dx$$

$$2I=2.\int_0^{\pi}\log(\sin x)\,dx$$

$I=\int_0^{\pi}\log(\sin x)\,dx$ .....(3)

because, $\int_0^{2a}f(x)dx=2.\int_0^{a}f(x)dx, \text{if } f(2a-x)=f(x)$

Here, if $f(x) = \log(\sin x)$ and $f(\pi - x)=\log(\sin(\pi - x)) = \log(\sin x) = f(x)$

$$\Rightarrow I = 2.\int_0^{\frac{\pi}{2}} \log\sin x dx \quad \ldots(4)$$

$$\Rightarrow I = 2.\int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2} - x\right)dx$$

$$\Rightarrow I = 2.\int_0^{\frac{\pi}{2}} \log\cos x dx \quad \ldots..(5)$$

Adding (1) and (2), we get

$$\Rightarrow 2I = 2.\int_0^{\frac{\pi}{2}} (\log\sin x + \log\cos x)dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log\sin x + \log\cos x + \log 2 - \log 2)dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log(2\sin x\cos x) - \log 2)dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log(\sin 2x)dx - \int_0^{\frac{\pi}{2}} \log 2dx$$

Let 2x = t ⇒ 2dx = dt

When x = 0, t = 0 and when x = π /2, t = π

$$\Rightarrow I = \left[\frac{1}{2}\int_0^{\pi} (\log(\sin t)dt\right] - \left(\frac{\pi}{2}\log 2\right)$$

$$\Rightarrow I = \left[\frac{I}{2}\right] - \left(\frac{\pi}{2}\log 2\right)$$

$$\Rightarrow I = -\left(\frac{\pi}{2}\log 2\right)$$

$$\Rightarrow I = -(\pi\log 2)$$

17. $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}}dx$

17. Let I = $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}}dx \quad \ldots.(1)$

It is known that, $\left(\int_0^a f(x)dx = \int_0^a f(a - x)dx\right)$

$$I = \int_0^a \frac{\sqrt{a - x}}{\sqrt{a - x} + \sqrt{x}}dx \quad \ldots.(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a - x}}{\sqrt{x} + \sqrt{a - x}}dx$$

$\Rightarrow 2I = \int_0^a 1.dx$

$\Rightarrow 2I = [x]_0^a$

$\Rightarrow 2I = a$

$\Rightarrow I = \frac{a}{2}$

18. $\int_0^4 |x-1|\,dx$

18. $I = \int_0^4 |x-1|dx$

It can be seen that, $(x - 1) \le 0$ when $0 \le x \le 1$ and $(x - 1) \ge 0$ when $1 \le x \le 4$

$I = \int_0^1 |x-1|\,dx + \int_0^1 |x-1|dx$ $\quad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$

$= \int_0^1 -(x-1)dx + \int_0^4 (x-1)dx$

$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$

$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$

$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$

$= 5$

19. Show that $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx$, if f and g are defined as f(x) = f(x) = f(a – x) and g(x) + g(a – x) = 4

19. $\int_0^a f(x)g(x)\,dx$

let, $I = \int_0^a f(x)g(x)\,dx$ …..(1)

as, $\left\{\int_0^a f(x)dx = \int_0^a f(a-x)dx\right\}$

$\Rightarrow I = \int_0^a f(a-x)g(a-x)\,dx$

$\Rightarrow I = \int_0^a f(x)g(a-x)\,dx$ …..(2)

Adding (1) and (2), we get

$2I = \int_0^a f(x)g(x) + f(x)g(a-x)\,dx$

$\Rightarrow 2I = \int_0^a f(x)g(x) + g(a-x)\,dx$

$\Rightarrow 2I = \int_0^a f(x) 4\, dx$ as, $g(x) + g(a-x) = 4$

$\Rightarrow I = \frac{1}{2}\int_0^a f(x) \times 4\, dx$

$\Rightarrow I = 2.\int_0^a f(x) dx$

20. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^3 + x\cos x + \tan^5 x + 1\right) dx$ is

(a) 0 (b) 2

(c) π (d) 1

20. Let $I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^3 + x\cos x + \tan^5 x + 1\right) dx$

$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 x dx + \int_{\frac{\pi}{2}}^{\frac{x}{2}} 1.dx$

It is known that if f (x) is an even function, then $\int_{-a}^{a} f(x) dx = 2\int_0^a f(x) dx$

if f (x) is an odd function, then $\int_{-a}^{a} f(x) dx = 0$

$I = 0 + 0 + 02\int_0^{\frac{x}{2}} 1.dx$

$= 2[x]_0^{\frac{\pi}{2}}$

$= \frac{2\pi}{2}$

$= \pi$

Hence, the correct Answer is C.

21. The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$ is

(a) 2 (b) $\frac{3}{4}$

(c) 0 (d) –2

21. $\int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$

let, $I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$ …..(1)

as, $\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right)dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\cos x}{4+3\sin}\right)dx \qquad .....(2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} \left\{\log\left(\frac{4+3\sin x}{4+3\cos x}\right)+\left(\frac{4+3\cos x}{4+3\sin}\right)\right\}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\times\frac{4+3\cos x}{4+3\sin}\right)dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1 dx$$

$$\Rightarrow I = 0$$

Correct answer is (c)