

EXERCISE 8.2

Solve the following systems of homogeneous linear equations by matrix method:

$$1. \quad 2x - y + z = 0$$

$$3x + 2y - z = 0$$

$$x + 4y + 3z = 0$$

Solution:

Given

$$2x - y + z = 0$$

$$3x + 2y - z = 0$$

$$x + 4y + 3z = 0$$

The system can be written as

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A X = 0$$

$$\text{Now, } |A| = 2(6 + 4) + 1(9 + 1) + 1(12 - 2)$$

$$|A| = 2(10) + 10 + 10$$

$$|A| = 40 \neq 0$$

Since, $|A| \neq 0$, hence $x = y = z = 0$ is the only solution of this homogeneous equation.

$$2. \quad 2x - y + 2z = 0$$

$$5x + 3y - z = 0$$

$$x + 5y - 5z = 0$$

Solution:

$$\text{Given } 2x - y + 2z = 0$$

$$5x + 3y - z = 0$$

$$x + 5y - 5z = 0$$

The system can be written as

$$\begin{bmatrix} 2 & -1 & 2 \\ 5 & 3 & -1 \\ 1 & 5 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A X = 0$$

$$\text{Now, } |A| = 2(-15 + 5) + 1(-25 + 1) + 2(25 - 3)$$

$$|A| = -20 - 24 + 44$$

$$|A| = 0$$

Hence, the system has infinite solutions

Let $z = k$

$$2x - y = -2k$$

$$5x + 3y = k$$

$$\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ k \end{bmatrix}$$

$$A X = B$$

$$|A| = 6 + 5 = 11 \neq 0 \text{ So, } A^{-1} \text{ exist}$$

$$\text{Now adj } A = \begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}$$

$$X = A^{-1} B = \frac{1}{|A|} (\text{adj } A) B = \frac{1}{11} \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} -2k \\ k \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{-5k}{11} \\ \frac{12k}{11} \end{bmatrix}$$

$$\text{Hence, } x = \frac{-5k}{11}, y = \frac{12k}{11} \text{ and } z = k$$

$$3. \quad 3x - y + 2z = 0$$

$$4x + 3y + 3z = 0$$

$$5x + 7y + 4z = 0$$

Solution:

$$\text{Given } 3x - y + 2z = 0$$

$$4x + 3y + 3z = 0$$

$$5x + 7y + 4z = 0$$

The system can be written as

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 3 \\ 5 & 7 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A X = 0$$

$$\text{Now, } |A| = 3(12 - 21) + 1(16 - 15) + 2(28 - 15)$$

$$|A| = -27 + 1 + 26$$

$$|A| = 0$$

Hence, the system has infinite solutions

$$\text{Let } z = k$$

$$3x - y = -2k$$

$$4x + 3y = -3k$$

$$\begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ -3k \end{bmatrix}$$

$$A X = B$$

$$|A| = 9 + 4 = 13 \neq 0 \text{ So, } A^{-1} \text{ exist}$$

$$\text{Now adj } A = \begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}^T = \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}$$

$$X = A^{-1} B = \frac{1}{|A|} (\text{adj } A) B = \frac{1}{13} \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} -2k \\ -3k \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{-9k}{13} \\ \frac{-k}{13} \\ \frac{-9k}{13} \end{bmatrix}$$

$$\text{Hence, } x = \frac{-9k}{13}, y = \frac{-k}{13} \text{ and } z = k$$

$$4. \quad x + y - 6z = 0$$

$$x - y + 2z = 0$$

$$-3x + y + 2z = 0$$

Solution:

$$\text{Given } x + y - 6z = 0$$

$$x - y + 2z = 0$$

$$-3x + y + 2z = 0$$

The system can be written as

$$\begin{bmatrix} 1 & 1 & -6 \\ 1 & -1 & 2 \\ -3 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A X = 0$$

$$\text{Now, } |A| = 1(-2 - 2) - 1(2 + 6) - 6(1 - 3)$$

$$|A| = -4 - 8 + 12$$

$$|A| = 0$$

Hence, the system has infinite solutions

$$\text{Let } z = k$$

$$x + y = 6k$$

$$x - y = -2k$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

$$A X = B$$

$$|A| = -1 - 1 = -2 \neq 0 \text{ So, } A^{-1} \text{ exist}$$

$$\text{Now } \text{adj } A = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$X = A^{-1} B = \frac{1}{|A|} (\text{adj } A) B = \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

$$X = \frac{1}{-2} \begin{bmatrix} -6k + 2k \\ -6k - 2k \end{bmatrix}$$

$$X = \begin{bmatrix} -4k \\ -8k \end{bmatrix}$$

Hence, $x = 2k$, $y = 4k$ and $z = k$



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