

EXERCISE 12B

Question 1.

Expand:

(i) $(2a + b)^2$

Solution:-

$$(2a + b)^2 = (2a)^2 + (b)^2 + 2 \times 2a \times b [(a + b)^2 = a^2 + b^2 + 2ab] = 4a^2 + b^2 + 4ab$$

(ii) $(a - 2b)^2$

Solution:-

$$(a - 2b)^2 = (a)^2 + (2b)^2 - 2 \times a \times 2b [(a - b)^2 = a^2 + b^2 - 2ab] = a^2 + 4b^2 - 4ab$$

(iii) $(a + \frac{1}{2a})^2$

Solution:-

$$= (a)^2 + (\frac{1}{2a})^2 + 2 \times a \times \frac{1}{2a} = a^2 + \frac{1}{4a^2} + \frac{2a}{2a} = a^2 + \frac{1}{4a^2} + 1$$

(iv) $(2a - \frac{1}{a})^2$

Solution:-

$$= (2a)^2 + (\frac{1}{a})^2 - 2 \times 2a \times \frac{1}{a} = 4a^2 + \frac{1}{a^2} - 4$$

(v) $(a + b - c)^2$

Solution:-

$$= (a)^2 + (b)^2 + (-c)^2 (+2 \times a \times b + 2 \times b \times (-c) + 2 \times (-c) \times (a))$$

$$= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

(vi) $(a - b + c)^2$

Solution:-

$$= (a)^2 + (-b)^2 + (c)^2 + 2 \times a \times -b + 2(-b)(c) + 2 \times c \times a = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$$

(vii) $(3x + \frac{1}{3x})^2$

Solution:-

$$= (3x)^2 + (\frac{1}{3x})^2 + 2 \times 3x \times \frac{1}{3x} = 9x^2 + \frac{1}{9x^2} + 2$$

(viii) $(2x - \frac{1}{2x})^2$

Solution:-

$$= (2x)^2 + \left(\frac{1}{2x}\right)^2 - 2 \times 2x \times \frac{1}{2x} = 4x^2 + \frac{1}{4x^2} - 2$$

Question 2.

Find the square of:

(i) $x+3y$

Solution:-

$$(x + 3y)^2 = (x)^2 + (3y)^2 + 2 \times x \times 3y = x^2 + 9y^2 + 6xy$$

(ii) $2x-5y$

Solution:-

$$(2x-5y)^2 = (2x)^2 + (5y)^2 - 2 \times 2x \times 5y = 4x^2 + 25y^2 - 20xy$$

(iii) $\left(a + \frac{1}{5a}\right)$

Solution:-

$$\left(a + \frac{1}{5a}\right)^2 = (a)^2 + \left(\frac{1}{5a}\right)^2 + 2 \times a \times \frac{1}{5a} \left(a^2 + \frac{1}{25a^2} + \frac{2}{5}\right)$$

(iv) $\left(2a - \frac{1}{a}\right)$

Solution:-

$$\left(2a - \frac{1}{a}\right)^2 = (2a)^2 + \left(\frac{1}{a}\right)^2 - 2 \times 2a \times \frac{1}{a} = 4a^2 + \frac{1}{a^2} - 4$$

(v) $x-2y+1$

Solution:-

$$(x - 2y + 1)^2 = (x)^2 + (-2y)^2 + (1)^2 + 2 \times x (x - 2y + 2 \times (-2y) \times 1 + 2 \times 1 \times x) = x^2 + 4y^2 + 1 - 4xy - 4y + 2x$$

(vi) $3a-2b-5c$

Solution:-

$$(3a - 2b - 5c)^2 = (3a)^2 + (-2b)^2 + (-5c)^2 (+2 \times 3a \times -2b + 2 \times (-2b)(-5c) (+2 \times 3a \times -5c))$$

(vii) $\left(2x + \frac{1}{x} + 1\right)$

Solution:-

$$\left(2x + \frac{1}{x} + 1\right)^2 = (2x)^2 + \left(\frac{1}{x}\right)^2 + (1)^2 + 2x = (2x \times \frac{1}{x} + 2 \times \frac{1}{x} \times 1 + 2 \times 1 \times 2x) = 4x^2 + \frac{1}{x^2} + 1 + 4 + \frac{2}{x} + 4x = 4x^2 + \frac{1}{x^2} + 5 + \frac{2}{x} + 4x$$

(viii) $\left(5 - x + \frac{2}{x}\right)$

Solution:-

$$(5 - x + \frac{2}{x})^2 = (5)^2 + (-x)^2 + (\frac{2}{x})^2 + 2 \times 5 \times (-x) + 2(-x) \times \frac{2}{x} + 2 \times \frac{2}{x} \times 5$$

$$= 25 + x^2 + \frac{4}{x^2} - 10x - 4 + \frac{20}{x} = 21 + x^2 + \frac{4}{x^2} - 10x + \frac{20}{x}$$

(ix) $2x-3y+z$

Solution:-

$$(2x - 3y + z)^2 = (2x)^2 + (-3y)^2 + (z)^2 + 2 \times 2xx (-3y) + 2(-3y) \times z + 2 \times z \times 2x$$

$$= 4x^2 + 9y^2 + z^2 - 12xy - 6yz + 4zx$$

(x) $(x + \frac{1}{x} - 1)$

Solution:-

$$(x + \frac{1}{x} - 1)^2 = (x)^2 + (\frac{1}{x})^2 + (-1)^2 + 2 \times x \times \frac{1}{x} + 2 \times \frac{1}{x} \times (-1) + 2(-1) \times x$$

$$= x^2 + \frac{1}{x^2} + 1 + 2 - \frac{2}{x} - 2x = x^2 + \frac{1}{x^2} + 3 - \frac{2}{x} - 2x$$

Question 3.

Evaluate:

Using expansion of $(a + b)^2$ or $(a - b)^2$

(i) $(208)^2$

Solution:-

$$= (200 + 8)^2 = (200)^2 + (8)^2 + 2(200)(8) = 40000 + 64 + 3200 = 43264$$

(ii) $(92)^2$

Solution:-

$$= (100 - 8)^2 = (100)^2 + (8)^2 - 2(100)(8)$$

$$= 10000 + 64 - 1600 = 8464$$

(iii) $(415)^2$

Solution:-

$$(400 + 15)^2 = (400)^2 + (15)^2 + 2(400)(15) = 160000 + 225 + 12000 = 172225$$

(iv) $(188)^2$

Solution:-

$$(200 - 12)^2 = (200)^2 + (12)^2 - 2(200)(12) = 40000 + 144 - 4800$$

$$= 35344$$

(v) $(9.4)^2$

Solution:-

$$= (10 - .6)^2 = (10)^2 + (.6)^2 - 2(10)(.6) = 100 + .36 - 12$$

$$= 88 + .36 = 88.36$$

(vi) $(20.7)^2$

Solution:-

$$= (20 + .7)^2 = (20)^2 + (.7)^2 + 2(20)(.7)$$

$$= 400 + .49 + 28 = 428 + .49 = 428.49$$

Question 4.

Expand:

(i) $(2a + b)^3$

Solution:-

$$= (2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b) [(a + b)^3 = a^3 + b^3 + 3ab(a + b)] = 8a^3 + b^3 + 6ab(2a + b)$$

$$= 8a^3 + b^3 + 12a^2b + 6ab^2$$

(ii) $(a - 2b)^3$

Solution:-

$$= (a)^3 - (2b)^3 - 3 \times a \times 2b(a - 2b) [(a - b)^3 = a^3 - b^3 - 3ab(a - b)] = a^3 - 8b^3 - 6ab(a - 2b)$$

$$= a^3 - 8b^3 - 6a^2b + 12ab^2$$

(iii) $(3x - 2y)^3$

Solution:-

$$= (3x)^3 - (2y)^3 - 3 \times 3x \times 2y(3x - 2y) = 27x^3 - 8y^3 - 18xy(3x - 2y)$$

$$= 27x^3 - 8y^3 - 54x^2y + 36xy^2$$

(iv) $(x + 5y)^3$

Solution:-

$$= (x)^3 + (5y)^3 + 3 \times x \times 5y(x + 5y) = x^3 + 125y^3 + 15xy(x + 5y)$$

$$= x^3 + 125y^3 + 15x^2y + 75y^2$$

(v) $(a + \frac{1}{a})^3$

Solution:-

$$= a^3 + (\frac{1}{a})^3 + 3 \times a \times \frac{1}{a} \times (a + \frac{1}{a}) = a^3 + \frac{1}{a^3} + 3(a + \frac{1}{a}) = a^3 + \frac{1}{a^3} + 3a + \frac{3}{a}$$

(vi) $(2a - \frac{1}{2a})^3$

Solution:-

$$= (2a)^3 - \left(\frac{1}{2a}\right)^3 - 3 \times 2a \times \frac{1}{2a} \left(2a - \frac{1}{2a}\right) = 8a^3 - \frac{1}{8a^3} - 3 \left(2a - \frac{1}{2a}\right) = 8a^3 - \frac{1}{8a^3} - 6a + \frac{3}{2a}$$

Question 5.

Find the cube of:

(i) $a+2$

Solution:-

$$(a + 2)^3 = (a)^3 + (2)^3 + 3 \times a \times 2(a + 2) = a^3 + 8 + 6a(a + 2) = a^3 + 8 + 6a^2 + 12a$$

$$= a^3 + 6a^2 + 12a + 8$$

(ii) $2a-1$

Solution:-

$$(2a - 1)^3 = (2a)^3 - (1)^3 - 3 \times 2a \times 1(2a - 1) = 8a^3 - 1 - 6a(2a - 1) = 8a^3 - 1 - 12a^2 + 6a$$

$$= 8a^3 - 12a^2 + 6a - 1$$

(iii) $2a+3b$

Solution:-

$$(2a + 3b)^3 = (2a)^3 + (3b)^3 + 3 \times 2a \times 3b$$

$$(2a + 3b)^3$$

$$= 8a^3 + 27b^3 + 18ab(2a + 3b) = 8a^3 + 27b^3 + 36a^2b + 54ab^2 = 8a^3 + 36a^2b + 54ab^2 + 27b^3$$

(iv) $3b-2a$

Solution:-

$$(3b - 2a)^3 = (3b)^3 - (2a)^3 - 3 \times 3b \times 2a(3b - 2a) = 27b^3 - 8a^3 - 18ab(3b - 2a)$$

$$= 27b^3 - 8a^3 - 54ab^2 + 36a^2b = 27b^3 - 8a^3 - 54ab^2 + 36a^2b = 27b^3 - 54ab^2 + 36a^2b - 8a^3$$

(v) $\left(2x + \frac{1}{x}\right)^3$

Solution:-

$$\left(2x + \frac{1}{x}\right)^3 = (2x)^3 + \left(\frac{1}{x}\right)^3 + 3 \times 2x \times \frac{1}{x} \left(2x + \frac{1}{x}\right) = 8x^3 + \frac{1}{x^3} + 6 \left(2x + \frac{1}{x}\right)$$

$$= 8x^3 + \frac{1}{x^3} + 12x + \frac{6}{x} = 8x^3 + 12x + \frac{6}{x} + \frac{1}{x^3}$$

(vi) $\left(x - \frac{1}{2}\right)^3$

Solution:-

$$\left(x - \frac{1}{2}\right)^3 = (x)^3 - \left(\frac{1}{2}\right)^3 - 3 \times x \times \frac{1}{2} \left(x - \frac{1}{2}\right) = x^3 - \frac{1}{8} - \frac{3x}{2} \left(x - \frac{1}{2}\right) = x^3 - \frac{1}{8} - \frac{3x^2}{2} + \frac{3x}{4}$$

$$= x^3 - \frac{3x^2}{2} + \frac{3x}{4} - \frac{1}{8}$$

EXERCISE 12C

Question:1

If $a+b=5$ and $ab=6$; find $(a^2 + b^2)$

Solution:-

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow (5)^2 = a^2 + b^2 + 2 \times 6$$

$$\Rightarrow 25 = a^2 + b^2 + 12$$

$$\Rightarrow 25 - 12 = a^2 + b^2$$

$$\Rightarrow 13 = a^2 + b^2$$

$$\therefore a^2 + b^2 = 13$$

Question:2

If $a-b=6$, and $ab=16$; find $(a^2 + b^2)$

Solution:

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\Rightarrow (6)^2 = a^2 + b^2 - 2 \times 16$$

$$\Rightarrow 36 = a^2 + b^2 - 32$$

$$\Rightarrow 36 + 32 = a^2 + b^2$$

$$\Rightarrow 68 = a^2 + b^2$$

$$\therefore a^2 + b^2 = 68$$

Question:3

If $(a^2 + b^2) = 29$ and $ab = 10$; find :

(i) $a+b$

Solution:-

$$(a + b)^2 = 29 + 2 \times 10$$

$$\Rightarrow (a + b)^2 = 29 + 20$$

$$\Rightarrow (a + b)^2 = 49$$

$$\Rightarrow a + b = \sqrt{49}$$

$$\Rightarrow a+b=7$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

(ii) a-b

$$\Rightarrow (a - b)^2 = 29 - 2 \times 10$$

$$\Rightarrow (a - b)^2 = 29 - 20$$

$$\Rightarrow (a - b)^2 = 9$$

$$\Rightarrow a - b = \sqrt{9}$$

$$\Rightarrow a-b=3$$

Question:4

If $(a^2 + b^2) = 10$ and $ab=3$; find

(i)a-b

Solution:-

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\Rightarrow (a - b)^2 = 10 - 2 \times 3$$

$$\Rightarrow (a - b)^2 = 10 - 6$$

$$\Rightarrow (a - b)^2 = 4$$

$$\Rightarrow (a - b) = \sqrt{4} \Rightarrow a-b=2$$

(ii) a+b

Solution:-

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow (a + b)^2 = 10 + 2 \times 3$$

$$\Rightarrow (a + b)^2 = 10 + 6$$

$$\Rightarrow (a + b)^2 = 16$$

$$m(a + b) = \sqrt{16}$$

$$\Rightarrow (a+b)=4$$

Question:5

If $(a + \frac{1}{a}) = 3$; find $a^2 + \frac{1}{a^2}$

Solution:-

$$(a + \frac{1}{a})^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow (3)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow 9 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow 7 = a^2 + \frac{1}{a^2}$$

$$\therefore a^2 + \frac{1}{a^2} = 7$$

Alternative Method:

$$a + \frac{1}{a} = 3$$

$$\Rightarrow (a + \frac{1}{a})^2 = (3)^2$$

$$\Rightarrow a^2 + \frac{1}{a^2} + 2 = 9$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 9 - 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 7$$