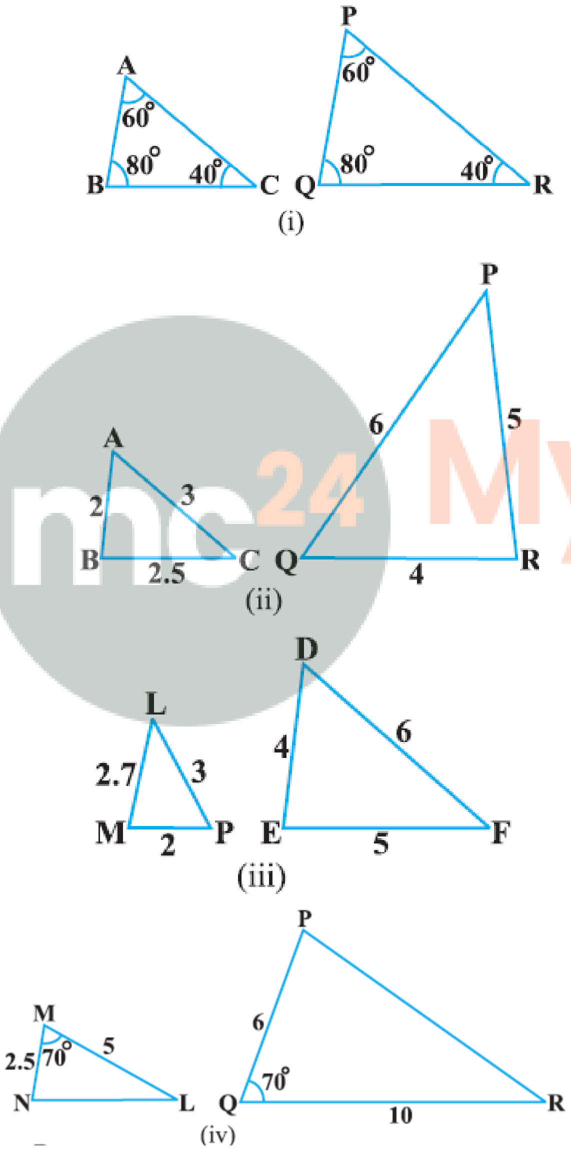
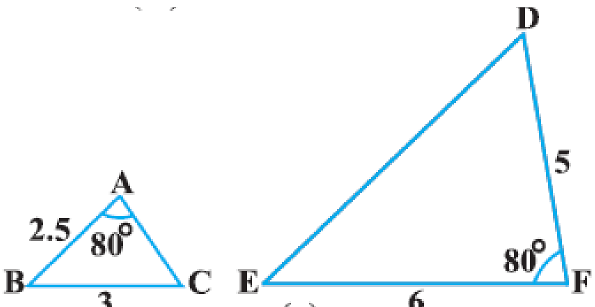
Chapter 6

Chapter 6: Triangles

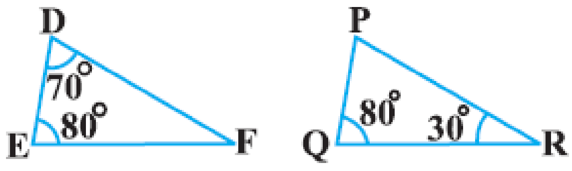
EXERCISE 6.3:

1. State which pairs of triangles in Fig. are similar.Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:





(v)



(vi)

Solution: (i) From the figure:

∠A = ∠P = 60°

∠B = ∠Q = 80°

∠C = ∠R = 40°

Therefore, ΔABC ~ΔPQR [By AAA similarity]

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

(ii) The triangles ABC and QRP are similar to each other by SSS similarity

(iii) The given triangles are not similar because the corresponding sides are not proportional

(iv) In triangle MNL and QPR, we have

$$\frac{MN}{QP} = \frac{ML}{QR} = \frac{1}{2}$$

∠M = ∠Q = 70°

ΔMNL ~ ΔQPR (SAS similarity)

(v) In triangle ABC and DEF, we have

AB = 2.5, BC = 3

∠A = 80°

EF = 6

DF = 5

∠F = 80°

$$\frac{AB}{DF} = \frac{2.5}{5} = \frac{1}{2}$$

And, $\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$

∠B ≠ ∠F
Hence, triangle ABC and DEF are not similar

(vi) In triangle DEF, we have
∠D + ∠E + ∠F = 180° (Sum of angles of triangle)
70° + 80° + ∠F = 180°
∠F = 30°
In PQR, we have
∠P + ∠Q + ∠R = 180°
∠P + 80° + 30° = 180°
∠P = 70°
In triangle DEF and PQR, we have
∠D = ∠P = 70°
∠F = ∠Q = 80°
∠F = ∠R = 30°
Hence, ΔDEF ~ ΔPQR   (AAA similarity)

2. In Fig. 6.35, Δ ODC ~ Δ OBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB

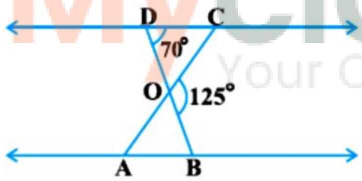


Fig. 6.35

Solution: From the figure,
DOB is a straight line
∠ DOC + ∠ COB = 180°
∠ DOC = 180° - 125°
= 55°
In ΔDOC,
∠ DCO + ∠ CDO + ∠ DOC = 180°
(Sum of the measures of the angles of a triangle is 180º)
∠ DCO + 70º + 55º = 180°
∠ DCO = 55°
It is given that ΔODC∼ ΔOBA
∠ OAB = ∠ OCD (Corresponding angles are equal in similar triangles)
∠ OAB = 55°

3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Solution: In ΔDOC and ΔBOA,

∠CDO = ∠ABO (Alternate interior angles as AB || CD)

∠DCO = ∠BAO (Alternate interior angles as AB || CD)

∠DOC = ∠BOA (Vertically opposite angles)

Therefore,

$\frac{DO}{BO} = \frac{OC}{OA}$ (Corresponding sides are proportional)

$Hence,$

$$\frac{OA}{OC} = \frac{OB}{OD}$$

4. In Fig. 6.36, $\frac{QR}{QS} = \frac{QT}{PR}$ and ∠1 = ∠2. Show that Δ PQS ~ Δ TQR.

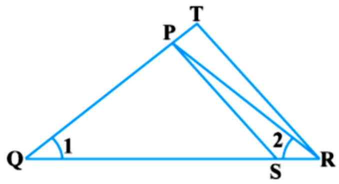


Fig. 6.36

Solution: Given: In ΔPQR,

∠PQR = ∠PRQ

PQ = PR (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i),

$\frac{QR}{QS} = \frac{QT}{QP}$ (ii)

In triangle PQS and TQR, we get

$\frac{QR}{QS} = \frac{QT}{QP}$ [Using (i)]

∠Q = ∠Q

Therefore,

Triangle PQS ~ TQR (SAS similarity rule)

5. S and T are points on sides PR and QR of Δ PQR such that ∠P = ∠RTS. Show that Δ RPQ ~ Δ RTS

Solution: In ΔRPQ and ΔRST,

∠ RTS = ∠ QPS (Given)

∠ R = ∠ R (Common)

∴ ΔRPQ ~ΔRTS (By AA similarity)

6. In Fig. 6.37, if Δ ABE ≅Δ ACD, show that Δ ADE ~ Δ ABC.

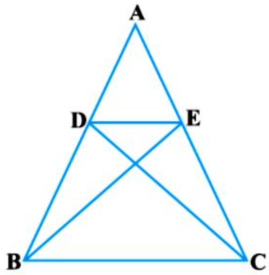


Fig. 6.37

Solution: It is given in the question that ΔABE ≅ ΔACD

∴ AB = AC (By CPCT) (i)

And,

AD = AE (By CPCT) (ii)

In ΔADE and ΔABC,

Dividing equation (ii) by (i)

∠A = ∠A (Common)

Hence,

ΔADE~ΔABC (By SAS similarity)

7. In Fig. 6.38, altitudes AD and CE of Δ ABCintersect each other at the point P. Showthat:

(i) Δ AEP ~ Δ CDP

(ii) Δ ABD ~ Δ CBE

(iii) Δ AEP ~ Δ ADB

(iv) Δ PDC ~ Δ BEC

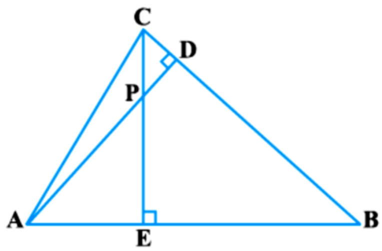


Fig. 6.38

Solution: (i) In ΔAEP and ΔCDP,

∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)

Hence, by using AA similarity,

ΔAEP ~ ΔCDP

(ii) In ΔABD and ΔCBE,

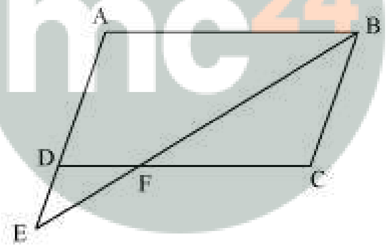
∠ADB = ∠CEB (Each 90°)
∠ABD = ∠CBE (Common)
Hence, by using AA similarity,
ΔABD ~ ΔCBE

(iii) In ΔAEP and ΔADB,
∠AEP = ∠ADB (Each 90°)
∠PAE = ∠DAB (Common)
Hence, by using AA similarity,
ΔAEP ~ ΔADB

(iv) In ΔPDC and ΔBEC,
∠PDC = ∠BEC (Each 90°)
∠PCD = ∠BCE (Common angle)
Hence, by using AA similarity,
ΔPDC ~ ΔBEC

8. E is a point on the side AD produced of aparallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB

Solution: In ΔABE and ΔCFB,



∠A = ∠C (Opposite angles of a parallelogram)
∠AEB = ∠CBF (Alternate interior angles because AE || BC)
Therefore,
ΔABE ~ ΔCFB (By AA similarity)

9. In Fig. 6.39, ABC and AMP are two righttriangles, right angled at B and Mrespectively. Prove that:

(i) Δ ABC ~ Δ AMP

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

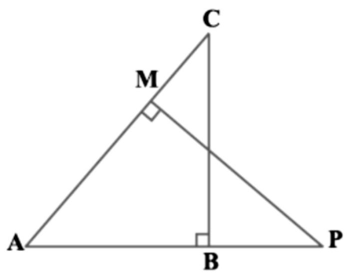


Fig. 6.39

Solution: (i) In ΔABC and ΔAMP,

∠ABC = ∠AMP (Each 90°)

∠A = ∠A (Common)

∴ΔABC ~ ΔAMP (By AA similarity)

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$ (Corresponding sides of similar triangles are proportional)

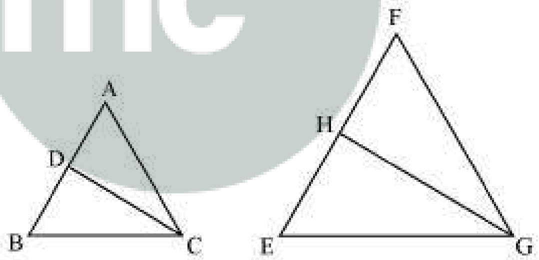
10. CD and GH are respectively the bisectorsof ∠ ACB and ∠ EGF in such a way that D and H lieon sides AB and FE of Δ ABC and Δ EFGrespectively. If Δ ABC ~ Δ FEG, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) Δ DCB ~ Δ HGE

(iii) Δ DCA ~ Δ HGF

Solution: It is given that ΔABC ~ ΔFEG



∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE

∠ACB = ∠FGE

∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)

(i) In ΔACD and ΔFGH,

∠A = ∠F (Proved above)

∠ACD = ∠FGH (Proved above)

∴ΔACD ~ ΔFGH (By AA similarity criterion)

$\frac{CD}{GH} = \frac{AC}{FG}$ (Corresponding sides of similar triangles are proportional)

(ii) In triangle DCB and HGE

∠DCB = ∠HGE (Proved above)

∠B = ∠E   (Proved above)
Therefore,
ΔDCB ~ ΔHGE (By AA similarity criterion)

(iii) In ΔDCA and ΔHGF,
∠ACD = ∠FGH (Proved above)
∠A = ∠F (Proved above)
ΔDCA ~ ΔHGF (By AA similarity)

11. In Fig. 6.40, E is a point on side CBproduced of an isosceles triangle ABCwith AB = AC. If AD ⊥ BC and EF ⊥ AC,prove that Δ ABD ~ Δ ECF

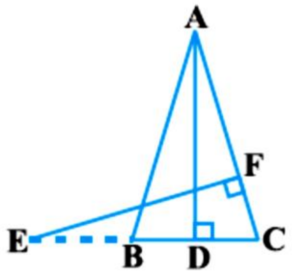


Fig. 6.40

Solution: Given that ABC is an isosceles triangle
AB = AC
⇒∠ABD = ∠ECF
In ΔABD and ΔECF,
∠ADB = ∠EFC (Each 90°)
∠BAD = ∠CEF (Proved above)
ΔABD ~ ΔECF (By using AA similarity criterion)

12. Sides AB and BC and median AD of atriangle ABC are respectively proportionalto sides PQ and QR and medianPM of
Δ PQR (see Fig. 6.41). Show that
Δ ABC ~ Δ PQR.

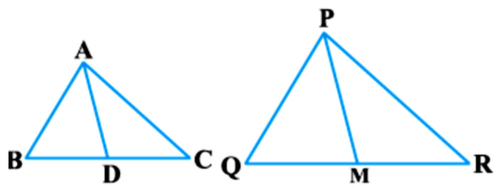


Fig. 6.41

Solution: Median divides the opposite side
BD = $\frac{BC}{2}$ and,
QM = $\frac{QR}{2}$

Given:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$\frac{AB}{PQ} = \frac{\frac{1}{2BC}}{\frac{1}{2QR}} = \frac{AD}{PM}$

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In ΔABD and ΔPQM,

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

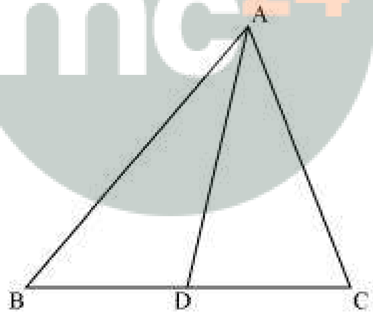
ΔABD ~ ΔPQM (By SSS similarity)
∠ABD = ∠PQM (Corresponding angles of similar triangles)
In ΔABC and ΔPQR,
∠ABD = ∠PQM (Proved above)

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

ΔABC ~ ΔPQR (By SAS similarity)

13. D is a point on the side BC of a triangleABC such that ∠ ADC = ∠ BAC. Show that CA2 = CB.CD

Solution: In ΔADC and ΔBAC,



∠ADC = ∠BAC (Given)
∠ACD = ∠BCA (Common angle)
ΔADC ~ ΔBAC (By AA similarity)
We know that corresponding sides of similar triangles are in proportion
Hence,

$$\frac{CA}{CB} = \frac{CD}{CA}$$

CA^2 = CB * CD

14. Sides AB and AC and median AD of atriangle ABC are respectivelyproportional to sides PQ and PR andmedian PM of another triangle PQR.
Show that Δ ABC ~ Δ PQR

Solution: Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML.

Then, join B to E, C to

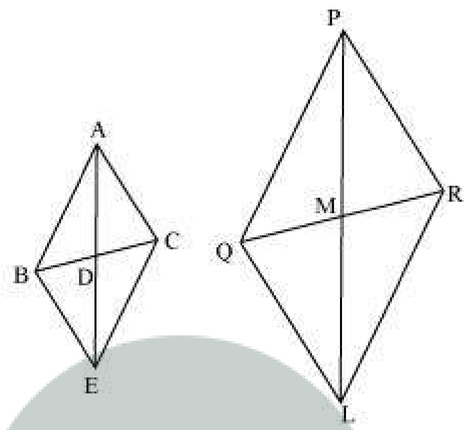
E, Q to L, and R to L

We know that medians divide opposite sides.

Hence, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)



In quadrilateral ABEC,

Diagonals AE and BC bisect each other at point D.

Therefore,

Quadrilateral ABEC is a parallelogram.

AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given in the question that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

ΔABE ∼ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal

∠BAE = ∠QPL (i)

Similarly, it can be proved that

ΔAEC ∼ ΔPLR and

∠CAE = ∠RPL (ii)

Adding equation (i) and (ii), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒∠CAB = ∠RPQ (iii)

In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

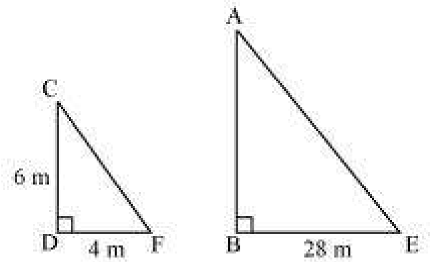
∠CAB = ∠RPQ [Using equation (iii)]

ΔABC ~ ΔPQR (By SAS similarity criterion)

15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same timea tower casts a shadow 28 m long. Find the height of the tower

Solution: Let AB and CD be a tower and a pole respectively

And, the shadow of BE and DF be the shadow of AB and CD respectively



At the same time, the light rays from the sun will fall on the tower and the pole at the same angle

Therefore,

∠DCF = ∠BAE

And,

∠DFC = ∠BEA

∠CDF = ∠ABE (Tower and pole are vertical to the ground)

ΔABE ~ ΔCDF (AAA similarity)

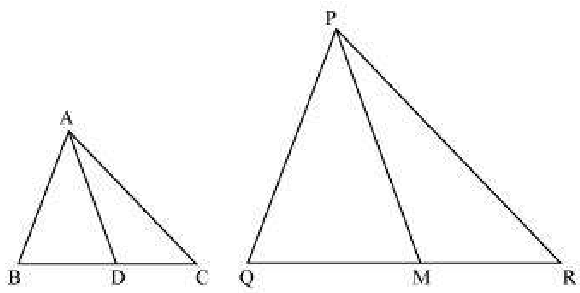
$$\frac{AB}{CD} = \frac{BE}{DF}$$
$$\frac{AB}{6} = \frac{28}{4}$$

AB = 42 m

16. If AD and PM are medians of triangles ABC and PQR, respectively whereΔ ABC ~ Δ PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Solution: It is given that ΔABC is similar to ΔPQR

We know that the corresponding sides of similar triangles are in proportion



$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ (i)

Also, ∠A = ∠P

∠B = ∠Q

∠C = ∠R (ii)

Since AD and PM are medians, they divide their opposite sides

BD = $\frac{BC}{2}$ and,

QM = $\frac{QR}{2}$ (iii)

From (i) and (iii), we get

$\frac{AB}{PQ} = \frac{BD}{QM}$(iv)

In ΔABD and ΔPQM,

∠B = ∠Q [Using (ii)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Using (iv)]

ΔABD ~ ΔPQM (By SAS similarity)

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$