## EXERCISE 10.2

**1. If f is defined by f (x) = $x^2$, find f' (2).**

**Solution:**

We have a polynomial function f(x) = $x^2$, and we have to find whether it is derivable at x = 2 or not, so by using the formula, $f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$,

We get, $f'(2) = \lim_{x \to 2} \frac{f(x)-f(2)}{x-2}$

$f'(2) = \lim_{x \to 2} \frac{x^2-2^2}{x-2}$

$f'(2) = \lim_{x \to 2} \frac{(x+2)(x-2)}{x-2}$

[Using $a^2 - b^2 = (a + b)(a - b)$]

$f'(2) = \lim_{x \to 2} x + 2 = 4$

Hence, the function is differentiable at x = 2 and its derivative equals to 4.



**2. If f is defined by f (x) = $x^2 - 4x + 7$, show that f' (5) = 2 f' (7/2)**

**Solution:**

We have a polynomial function f(x) = $x^2 - 4x + 7$, and we have to f '(x) its value at x = 5 and x = 7/2, so by using the formula, $f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$,

We get, $f'(5) = \lim_{x \to 5} \frac{f(x)-f(5)}{x-5}$

$f'(5) = \lim_{x \to 5} \frac{x^2-4x+7-(5^2-4\times5+7)}{x-5}$

$f'(5) = \lim_{x \to 5} \frac{x^2-4x-5}{x-5}$

f' (5) = $\lim_{x \to 5} \frac{x(x-5) + 1(x-5)}{x-5}$

f' (5) = $\lim_{x \to 5} (x + 1) = 6$

Hence to function is differentiable at x = 5 and has value 6.

f' (7/2) = $\lim_{x \to \frac{7}{2}} \frac{f(x) - f(\frac{7}{2})}{x - \frac{7}{2}}$

f' (7/2) = $\lim_{x \to \frac{7}{2}} \frac{x^2 - 4x + 7 - [(\frac{7}{2})^2 - 4 \times \frac{7}{2} + 7]}{x - \frac{7}{2}}$

f' (7/2) = $\lim_{x \to \frac{7}{2}} \frac{x^2 - 4x + 7 - [(\frac{7}{2})^2 - 4 \times \frac{7}{2} + 7]}{x - \frac{7}{2}}$

f' (7/2) = $\lim_{x \to \frac{7}{2}} \frac{x^2 - 4x + \frac{7}{4}}{x - \frac{7}{2}}$

f' (7/2) = $\lim_{x \to \frac{7}{2}} \frac{x^2 - 4x + \frac{7}{4}}{x - \frac{7}{2}}$

f' (7/2) = $\lim_{x \to \frac{7}{2}} \frac{(2x-1)(2x-7)}{2(2x-7)}$

f' (7/2) = $\lim_{x \to \frac{7}{2}} \frac{(2x-1)}{2}$ = 3

Therefore f' (5) = 2 f' (7/2) = 6,
Hence the proof.



**3. Show that the derivative of the function f is given by f (x) = $2x^3 - 9x^2 + 12x + 9$, at x = 1 and x = 2 are equal.**

**Solution:**
We are given with a polynomial function f(x) = $2x^3 - 9x^2 + 12x + 9$, and we have

to find f '(x) at x = 1 and x = 2, so by using the formula, f '(c) = $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ , we get,

f' (1) = $\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$

f' (1) = $\lim_{x\to 1}\frac{2x^3-9x^2+12x+9-[2(1)^3-9(1)^2+12(1)+9]}{x-1}$

f' (1) = $\lim_{x\to 1}\frac{2x^3-9x^2+12x-5}{x-1}$

f' (1) = $\lim_{x\to 1}\frac{(x-1)(2x^2-7x+5)}{x-1}$

f' (1) = $\lim_{x\to 1} 2x^2 - 7x + 5 = 0$

For x = 2, we get,

f' (2) = $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$

f' (2) = $\lim_{x\to 2}\frac{2x^3-9x^2+12x+9-[2(2)^3-9(2)^2+12(2)+9]}{x-2}$

f' (2) = $\lim_{x\to 2}\frac{2x^3-9x^2+12x-4}{x-2}$

f' (2) = $\lim_{x\to 2}\frac{(x-2)(2x^2-5x+2)}{x-2}$

f' (2) = $\lim_{x\to 2} 2x^2 - 5x + 2 = 0$

Hence they are equal at x = 1 and x = 2.

**4. If for the function Ø (x) = λ $x^2$ + 7x – 4, Ø' (5) = 97, find λ.**

**Solution:**

We have to find the value of λ given in the real function and we are given with the differentiability of the function f(x) = λ$x^2$ + 7x - 4 at x = 5 which is f '(5) = 97, so we will adopt the same process but with a little variation.

So by using the formula, f '(c) $= \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$, we get,

f' (5) = $\lim_{x \to 5} \frac{f(x)-f(5)}{x-5}$

f' (5) = $\lim_{x \to 5} \frac{\lambda x^2 + 7x - 4 - [\lambda(5)^2 + 7(5) - 4]}{x-5}$

f' (5) = $\lim_{x \to 5} \frac{\lambda x^2 + 7x - 4 - [\lambda(5)^2 + 7(5) - 4]}{x-5}$

f' (5) = $\lim_{x \to 5} \frac{\lambda x^2 + 7x - 35 - 25\lambda}{x-5}$

As the limit has some finite value, then there must be the formation of some indeterminate form like $\frac{0}{0}, \frac{\infty}{\infty}$, so if we put the limit value, then the numerator will also be zero as the denominator, but there must be a factor (x - 5) in the numerator, so that this form disappears.

f' (5) = $\lim_{x \to 5} \frac{(x-5)(\lambda x + 5\lambda + 7)}{x-5}$

f' (5) = $\lim_{x \to 5} \lambda x + 5\lambda + 7 = 97$

f' (5) = $10\lambda + 7 = 97$

$10\lambda = 90$

$\lambda = 9$

**5. If f (x) = $x^3 + 7x^2 + 8x - 9$, find f' (4).**

**Solution:**

We are given with a polynomial function f(x) = $x^3 + 7x^2 + 8x - 9$, and we have to find whether it is derivable at x = 4 or not,

So by using the formula, f '(c) $= \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$,

We get, f '(4) = $\lim_{x \to 4} \frac{f(x)-f(4)}{x-4}$

f' (4) = $\lim_{x\to4}\frac{x^3+7x^2+8x-9-[4^3+7(4)^2+8(4)-9]}{x-4}$

f' (4) = $\lim_{x\to4}\frac{(x-4)(x^2+11x+52)}{x-4}$

f' (4) = $\lim_{x\to4} x^2 + 11x + 52$

f' (4) = 112.

**6. Find the derivative of the function f defined by f (x) = mx + c at x = 0.**

**Solution:**
We are given with a polynomial function f(x) = mx + c, and we have to find whether it is derivable at x = 0 or not,

So by using the formula, f' (c) = $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ ,

We get, f' (0) = $\lim_{x\to0}\frac{f(x)-f(0)}{x-0}$

f' (0) = $\lim_{x\to0}\frac{mx+c-[m(0)+c]}{x-0}$

f' (0) = $\lim_{x\to0}\frac{mx+c-c}{x-0}$

f' (0) = $\lim_{x\to0} m = m$
This is the derivative of a function at x = 0, and also this is the derivative of this function at every value of x.