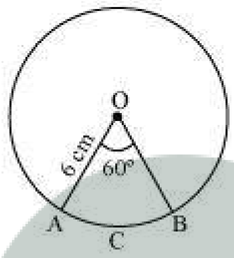
NCERT Solutions for Class 10 Maths Chapter 12 Area Related to Circles Ex. 12.1

Unless stated otherwise, use $\pi \frac{22}{7}$

1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°

Solution: Let OACB be a sector of the circle making 60° angle at centre O of the circle



Area of sector of angle θ = $\frac{\theta}{360}$ * 4πr^2

Area of sector OACB =$\frac{60}{360} * \frac{22}{7}$ * (6)^2

= $\frac{1}{6} * \frac{22}{7} * 6 * 6$

= $\frac{132}{7}$ cm^2

Hence,

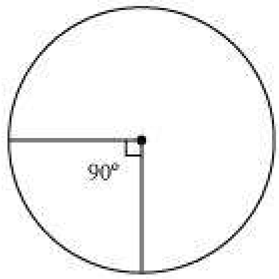
The area of the sector of the circle making 60° at the centre of the circle is$\frac{132}{7}$ cm^2

2. Find the area of a quadrant of a circle whose circumference is 22 cm

Solution: Let the radius of the circle be *r*

Circumference = 22 cm

$2\pi r = 22$



$r = \frac{22}{2\pi} = \frac{11}{\pi}$

Quadrant of circle will subtend 90° angle at the centre of the circle

Area of such quadrant of the circle = $\frac{90}{360} * \pi * r^2$

$= \frac{1}{4\pi} * \pi * (\frac{11}{\pi})^2$

$= \frac{121}{4\pi}$

$= \frac{121*7}{4*22}$

$= \frac{77}{8}$ cm^2

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes

Solution: As we know that in 1 hour (i.e., 60 minutes), the minute hand rotates 360°

In 5 minutes, minute hand will rotate = $\frac{360}{60} * 5$

$= 30^o$

Therefore, the area swept by the minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius

Area of sector of angle θ $= \frac{\theta}{360} * \pi r^2$

Area of sector of 30° = $\frac{30}{360} * \frac{22}{7}$ * 14 * 14

= $\frac{22}{12}$ * 2 * 14

= $\frac{11*14}{3}$

= $\frac{154}{3}$ cm^2

Hence,

The area swept by the minute hand in 5 minutes is $\frac{154}{3}$ cm^2

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major sector (Use π = 3.14)

Solution: Let AB be the chord of the circle subtending 90° angle at centre O of the circle.

Area of major sector OADB = $(\frac{360-90}{360})$ * πr^2

= $(\frac{270}{360})$ πr^2

= $\frac{3}{4}$ * 3.14 * 10 * 10

= 235.5 cm^2

Area of minor sector OACB = $\frac{90}{360}$ * πr^2

= $\frac{1}{4}$ * 3.14 * 10 * 10

= 78.5 cm^2

Area of ΔOAB = $\frac{1}{2}$ * OA * OB

$= \frac{1}{2} * 10 * 10$

$= 50\ cm^2$

Area of minor segment ACB = Area of minor sector OACB -Area of ΔOAB

= 78.5 - 50

$= 28.5\ cm^2$

5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) The length of the arc

(ii) Area of the sector formed by the arc

(iii) Area of the segment formed by the corresponding chord

Solution: Radius (*r*) of circle = 21 cm

Angle subtended by the given arc = 60°

Length of an arc of a sector of angle θ $= \frac{\theta}{360} * 2\pi r$

(i) Length of arc ACB $= \frac{60}{360} * 2 * \frac{22}{7} * 21$

$= \frac{1}{6} * 2 * 22 * 3$

= 22 cm

(ii) Area of sector OACB $= \frac{60}{360} * \pi r^2$

$= \frac{1}{6} * \frac{22}{7} * 21 * 21$

$= 231\ cm^2$

(iii) In ΔOAB,

∠OAB = ∠OBA (As OA = OB)

∠OAB + ∠AOB + ∠OBA = 180°

2∠OAB + 60° = 180°

∠OAB = 60°

Hence,

ΔOAB is an equilateral triangle

Area of ΔOAB $=\frac{\sqrt{3}}{4}$ (Side)^2

$= \frac{\sqrt{3}}{4} * (21)^2$

$= \frac{441\sqrt{3}}{4}$ cm^2

Area of segment ACB = Area of sector OACB - Area of ΔOAB

$= (231 - \frac{441\sqrt{3}}{4})$ cm^2

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle

(Use π = 3.14 and $\sqrt{3}$ = 1.73)

Solution: Radius (*r*) of circle = 15 cm

Area of sector OPRQ $=\frac{60}{360} * \pi r^2$

$= \frac{1}{6} * 3.14 * (15)^2$

= 117.75 cm^2

In ΔOPQ,

∠OPQ = ∠OQP (As OP = OQ)

∠OPQ + ∠OQP + ∠POQ = 180°

2 ∠OPQ = 120°

$\angle OPQ = 60°$

ΔOPQ is an equilateral triangle.

Area of $\Delta OPQ = \frac{\sqrt{3}}{4} * (\text{Side})^2$

$= \frac{\sqrt{3}}{4} * (15)^2$

$= \frac{225\sqrt{3}}{4}$

$= 56.25 * \sqrt{3}$

$= 97.3125\ cm^2$

Area of segment PRQ = Area of sector OPRQ - Area of ΔOPQ

= 117.75 - 97.3125

$= 20.4375\ cm^2$

Area of major segment PSQ = Area of circle - Area of segment PRQ

$= \pi (15)^2 – 20.4375$

= 3.14 * 225 – 20.4375

= 706.5 – 20.4375

$= 686.0625\ cm^2$

7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Solution: Let us draw a perpendicular OV on chord ST. It will bisect the chord ST.

SV = VT

In ΔOVS,

$\frac{OV}{OS}$ = Cos 60°

$\frac{OV}{12}$ = $\frac{1}{2}$

OV = 6 cm

$\frac{SV}{SO}$ = Sin 60°

$\frac{SV}{12}$ = $\frac{\sqrt{3}}{2}$

SV = $6\sqrt{3}$ cm

ST = 2 * SV

= 2 * $6\sqrt{3}$

= 12 $\sqrt{3}$ cm

Area of ΔOST =$\frac{1}{2}$ * 12 $\sqrt{3}$ * 6

= $36\sqrt{3}$

= 36 * 1.73

= 62.28 cm^2

Area of sector OSUT =$\frac{120}{360}$ * π (12)^2

= $\frac{1}{3}$ * 3.14 * 144

= 150.72 cm^2

Area of segment SUT = Area of sector OSUT - Area of ΔOST

= 150.72 - 62.28

= 88.44 cm^2

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 12.11). Find

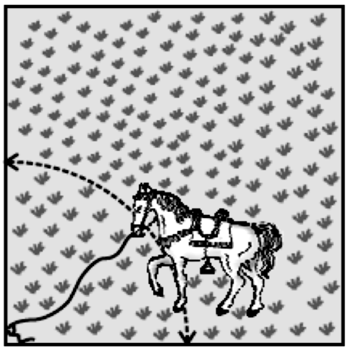


Fig. 12.11

(i) The area of that part of the field in which the horse can graze

(ii) The increase in the grazing area if the rope were10 m long instead of 5 m. (Use π = 3.14)

Solution: From the figure, it can be observed that the horse can graze a sector of 90° in a circle of 5 m radius

(i) Area that can be grazed by horse = Area of sector OACB

$= \frac{90}{360} * \pi r^2$

$= \frac{1}{4} * 3.14 * 5 * 5$

$= 19.625\ m^2$

Area that can be grazed by the horse when length of rope is 10 m long = $\frac{90}{360} * \pi\ (10)^2$

$= \frac{1}{4} * 3.14 * 100$

$= 78.5\ m^2$

(ii) Increase in grazing area = (78.5 - 19.625)

$= 58.875\ m^2$

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10equal sectors as shown in Fig. 12.12. Find:

(i) The total length of the silver wire required

(ii) The area of each sector of the brooch

Fig. 12.12

Solution: (i) Total length of wire required will be the length of 5 diameters and the circumference of the brooch

Radius of circle =$\frac{35}{2}$ mm

Circumference of brooch = $2\pi r$

$= 2 * \frac{22}{7} * \frac{35}{2}$

= 110 mm

Length of wire required = 110 + 5 × 35

= 110 + 175 = 285 mm

According to the question,

Each of 10 sectors of the circle is subtend36° at the centre of the circle

(ii) Area of each sector =$\frac{36}{360} * \pi r^2$

$= \frac{1}{10} * \frac{22}{7} * (\frac{35}{2}) * (\frac{35}{2})$

$= \frac{385}{4}$ mm^2

10. An umbrella has 8 ribs which are equally spaced (see Fig. 12.13). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella

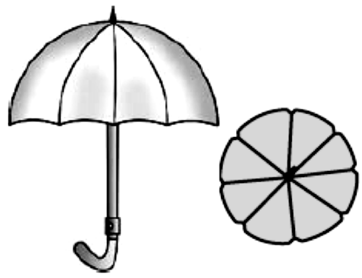


Fig. 12.13

Solution: There are 8 ribs in the given umbrella. The area between two consecutive ribs is subtending $\frac{360}{8} = 45^{o}$ at the centre of the assumed flat circle

Area between two consecutive ribs of circle $= \frac{45}{360} * \pi r^2$

$= \frac{1}{8} * \frac{22}{7} * (45)^2$

$= \frac{11}{28} * 2025$

$= \frac{22275}{28}$ cm^2

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades

Solution: It can be seen from the figure that each blade of wiper would sweep an area of a sector of 115° in a circle with 25 cm radius

Area of such sector $= \frac{115}{360} * \pi * (25)^2$

$= \frac{23}{72} * \frac{22}{7} * 25 * 25$

$= \frac{158125}{252}$ cm^2

Area swept by 2 blades = 2 * $\frac{158125}{252}$

= $\frac{158125}{156}$ cm^2

12. To warn ships for underwater rocks, a lighthouse spreads a red colored light over a sector of angle80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)

Solution: It can be seen from the figure that the lighthouse spreads light across a sector of 80° in a circle of 16.5 km radius

Area of sector OACB =$\frac{80}{360}$ * πr^2

= $\frac{2}{9}$ * 3.14 * 16.5 * 16.5

= 189.97 km^2

13. A round table cover has six equal designs as shown in Fig. 12.14. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs 0.35 per cm^2. (Use $\sqrt{3}$ = 1.7)

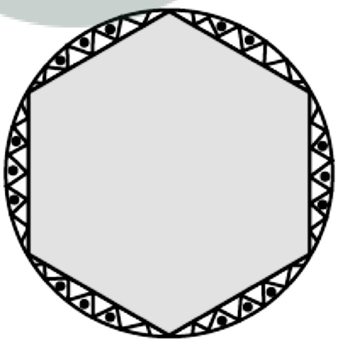


Fig. 12.14

Solution: It can be concluded that these designs are segments of the circle.

Let us take segment APB.

Chord AB is a side of the hexagon.

And,

Each chord will substitute$\frac{360}{6}$ = 60^{o} at the centre of the circle

In ΔOAB,

∠OAB = ∠OBA (As OA = OB)

∠AOB = 60°

∠OAB + ∠OBA + ∠AOB = 180°

2∠OAB = 180° - 60° = 120°

∠OAB = 60°

Hence,

ΔOAB is an equilateral triangle.

Area of ΔOAB =$\frac{\sqrt{3}}{4}$ (Side)^2

= $\frac{\sqrt{3}}{4}$ * (28)^2

= 196 $\sqrt{3}$

= 333.2 cm^2

Area of sector OAPB =$\frac{60}{360}$ * πr^2

= $\frac{1}{6}$ * $\frac{22}{7}$ * 28 * 28

= $\frac{1232}{3}$ cm^2

Now,

Area of segment APB = Area of sector OAPB - Area of ΔOAB

= ($\frac{1232}{3}$ - 333.2) cm^2

Area of design = 6 * ($\frac{1232}{3}$ - 333.2)

= 2464 – 1992.2

= 464.8 cm^2

Cost of making 1 cm^2 designs = Rs 0.35

Cost of making 464.76 cm^2 designs = 464.8 * 0.35

= Rs 162.68

Hence, the cost of making such designs would be Rs 162.68

14. Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

A. $\frac{p}{180} \times 2\pi R$

B. $\frac{p}{180} \times \pi R^2$

C. $\frac{p}{360} \times 2\pi R$

D. $\frac{p}{720} \times 2\pi R^2$

Solution: Area of sector of angle θ $= \frac{\theta}{360} * \pi R^2$

So,

Area of sector of angle p $= \frac{p}{360} (\pi R^2)$

$= (\frac{p}{720})(2\pi R^2)$

Hence, (D) is the correct answer