$\frac{x}{1}=\frac{z}{-1}, y=0$ and $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$

& $\overrightarrow{L_2} = \frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

**To find –** Angle between the two pair of lines

Direction ratios of $L_1$ = (1,0,-1)

Direction ratios of $L_2$ = (3,4,5)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3 + 0\times 4 + (-1)\times 5}{\sqrt{1^2+0^2+1^2}\sqrt{3^2+4^2+5^2}}\right)$$

$$= \cos^{-1}\left(\frac{3-5}{5\sqrt{2}\times\sqrt{2}}\right)$$

$$= \cos^{-1}\left(\frac{1}{5}\right)$$

**8. Question**

Find the angle between each of the following pairs of lines:

$\frac{5-x}{3}=\frac{y+3}{-2}, z=5$ and $\frac{x-1}{1}=\frac{1-y}{3}=\frac{z-5}{2}$

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x-5}{-3}=\frac{y+3}{-2}=\frac{z-5}{0}$

& $\overrightarrow{L_2} = \frac{x-1}{1}=\frac{y-1}{-3}=\frac{z-5}{2}$

**To find –** Angle between the two pair of lines

Direction ratios of $L_1$ = (-3,-2,0)

Direction ratios of $L_2$ = (1,-3,2)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{(-3)\times 1+(-2)\times(-3)+0\times 2}{\sqrt{3^2+2^2+0^2}\sqrt{1^2+3^2+2^2}}\right)$$

$$= \cos^{-1}\left(\frac{-3+6}{\sqrt{13}\times\sqrt{14}}\right)$$

$$= \cos^{-1}\left(\frac{3}{\sqrt{182}}\right)$$

**9. Question**

Show that the lines $\frac{x-3}{2}=\frac{y+1}{-3}=\frac{z-2}{4}$ and $\frac{x+2}{2}=\frac{y-4}{4}=\frac{z+5}{2}$ are perpendicular to each other.

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x-3}{2}=\frac{y+1}{-3}=\frac{z-2}{4}$

& $\overrightarrow{L_2} = \frac{x+2}{2}=\frac{y-4}{4}=\frac{z+5}{2}$



**To prove –** The lines are perpendicular to each other

Direction ratios of $L_1$ = (2,-3,4)

Direction ratios of $L_2$ = (2,4,2)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times 2+(-3)\times 4+4\times 2}{\sqrt{2^2+3^2+4^2}\sqrt{2^2+4^2+2^2}}\right)$$

$$= \cos^{-1}\left(\frac{4-12+8}{\sqrt{29}\times\sqrt{24}}\right)$$

$$= \cos^{-1}(0)$$

$= \frac{\pi}{2}$

Hence, **the lines are perpendicular to each other.**

**10. Question**

If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{5}$ are perpendicular to each other then find the value of λ.

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$

& $\overrightarrow{L_2} = \frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-5}$

**To find –** The value of λ

Direction ratios of $L_1$ = (-3,2λ,2)

Direction ratios of $L_2$ = (3λ,1,-5)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

Since the lines are perpendicular to each other,

The angle between the lines

$$\Rightarrow \cos^{-1}\left(\frac{(-3)\times 3\lambda + 2\lambda\times 1 + 2\times(-5)}{\sqrt{3^2+(2\lambda)^2+2^2}\sqrt{(3\lambda)^2+1^2+5^2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}\left(\frac{-9\lambda + 2\lambda - 10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}\left(\frac{-7\lambda - 10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \left(\frac{-7\lambda - 10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right) = \cos\frac{\pi}{2} = 0$$

$$\Rightarrow -7\lambda - 10 = 0$$

$$\Rightarrow \boldsymbol{\lambda = -\frac{10}{7}}$$

**11. Question**

Show that the lines x = - y = 2z and x + 2 = 2y – 1 = - z + 1 are perpendicular to each other.

**HINT:** The given lines are $\frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$ and $\frac{x+2}{1}=\frac{y-\frac{1}{2}}{1}=\frac{z-1}{-2}$.

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$

& $\overrightarrow{L_2} = \frac{x+2}{2}=\frac{y-1/2}{1}=\frac{z-1}{-2}$

**To prove –** The lines are perpendicular to each other

Direction ratios of $L_1$ = (2,-2,1)

Direction ratios of $L_2$ = (2,1,-2)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$=\cos^{-1}\left(\frac{2\times 2+(-2)\times 1+1\times(-2)}{\sqrt{2^2+2^2+1^2}\sqrt{1^2+1^2+2^2}}\right)$$

$$=\cos^{-1}\left(\frac{4-2-2}{\sqrt{29}\times\sqrt{24}}\right)$$

$$=\cos^{-1}(0)$$

$$=\frac{\pi}{2}$$

Hence, **the lines are perpendicular to each other.**

**12. Question**

Find the angle between two lines whose direction ratios are

i. 2, 1, 2 and 4, 8, 1

ii. 5, -12, 13 and -3, 4, 5

iii. 1, 1, 2 and $(\sqrt{3}-1),(-\sqrt{3}-1),4$

iv. a, b, c and (b – c), (c – a), (a – b)

**Answer**

**(i): Given –** Direction ratios of $L_1$ = (2,1,2) & Direction ratios of $L_2$ = (4,8,1)

**To find –** Angle between the two pair of lines

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times 4+1\times 8+2\times 1}{\sqrt{2^2+1^2+2^2}\sqrt{4^2+8^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{8+8+2}{3\times 9}\right)$$

$$= \cos^{-1}\left(\frac{18}{27}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{2}{3}\right)}$$

**(ii): Given –** Direction ratios of $L_1$ = (5,-12,13) & Direction ratios of $L_2$ = (-3,4,5)

**To find –** Angle between the two pair of lines

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$



The angle between the lines

$$= \cos^{-1}\left(\frac{5\times(-3)+(-12)\times 4+13\times 5}{\sqrt{5^2+12^2+13^2}\sqrt{3^2+4^2+5^2}}\right)$$

$$= \cos^{-1}\left(\frac{-15-48+65}{13\sqrt{2}\times 5\sqrt{2}}\right)$$

$$= \cos^{-1}\left(\frac{2}{130}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{1}{65}\right)}$$

**(iii) Given –** Direction ratios of $L_1$ = (1,1,2) & Direction ratios of $L_2$ = (√3-1,-√3-1,4)

**To find –** Angle between the two pair of lines

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times(\sqrt{3}-1)+1\times(-\sqrt{3}-1)+2\times 4}{\sqrt{1^2+1^2+2^2}\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{\sqrt{3}-1-\sqrt{3}-1+8}{\sqrt{6}\sqrt{24}}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3}$$

**(iv) Given –** Direction ratios of $L_1$ = (a,b,c) & Direction ratios of $L_2$ = ((b-c),(c-a),(a-b))

**To find –** Angle between the two pair of lines

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{a\times(b-c)+b\times(c-a)+c\times(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right)$$

$$= \cos^{-1}\left(\frac{0}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right)$$

$$= \cos^{-1}(0)$$

$$= \frac{\pi}{2}$$

**13. Question**

If A(1, 2, 3), B(4, 5, 7), C(-4, 3, -6) and D(2, 9, 2) are four given points then find the angle between the lines AB and CD.

**Answer**

**Given -**

A = (1,2,3)

B = (4,5,7)

C = (-4,3,-6)

D = (2,9,2)

**Formula to be used –** If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((4-1),(5-2),(7-3))

=(3,3,4)

Similarly, the direction ratios of the line CD can be given by

((2+4),(9-3),(2+6))

=(6,6,8)

**To find –** Angle between the two pair of lines AB and CD

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{3\times 6+3\times 6+4\times 8}{\sqrt{3^2+3^2+4^2}\sqrt{6^2+6^2+8^2}}\right)$$

$$= \cos^{-1}\left(\frac{18+18+32}{\sqrt{34}\times 2\sqrt{34}}\right)$$

$$= \cos^{-1}\left(\frac{68}{2\times 34}\right)$$

$$= \cos^{-1} 1$$

$$= \mathbf{0}$$



## Exercise 27D

### 1. Question

Find the shortest distance between the given lines.

$$\vec{r} = (\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k}),$$

$$\vec{r} = (2\hat{i}+\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k}).$$

**Answer**

**Given equations :**

$$\bar{r} = (\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$$

$\bar{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and $\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

**Answer :**

For given lines,

$\bar{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\bar{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

Here,

$\overline{a_1} = \hat{i} + \hat{j}$

$\overline{b_1} = 2\hat{i} - \hat{j} + \hat{k}$

$\overline{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

$\overline{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$= \hat{i}(-2 + 5) - \hat{j}(4 - 3) + \hat{k}(-10 + 3)$

$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{i} - \hat{j} - 7\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{3^2 + (-1)^2 + (-7)^2}$

$= \sqrt{9 + 1 + 49}$

$= \sqrt{59}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{i} + (1 - 1)\hat{j} + (-1 - 0)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} + 0\hat{j} - \hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (3\hat{i} - \hat{j} - 7\hat{k}).(\hat{i} + 0\hat{j} - \hat{k})$

$= (3 \times 1) + ((-1) \times 0) + ((-7) \times (-1))$

= 3 + 0 + 7

= 10

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{10}{\sqrt{59}} \right|$$

**2. Question**

Find the shortest distance between the given lines.

$$\vec{r} = \left(-4\hat{i} + 4\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right),$$

$$\vec{r} = \left(-3\hat{i} - 8\hat{j} - 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 3\hat{k}\right)$$

**Answer**

**Given equations :**

$$\bar{r} = \left(-4\hat{\imath} + 4\hat{\jmath} + \hat{k}\right) + \lambda\left(\hat{\imath} + \hat{\jmath} - \hat{k}\right)$$

$$\bar{r} = \left(-3\hat{\imath} - 8\hat{\jmath} - 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + 3\hat{k}\right)$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**Answer :**

For given lines,

$$\bar{r} = \left(-4\hat{\imath} + 4\hat{\jmath} + \hat{k}\right) + \lambda\left(\hat{\imath} + \hat{\jmath} - \hat{k}\right)$$

$$\bar{r} = \left(-3\hat{\imath} - 8\hat{\jmath} - 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + 3\hat{k}\right)$$

Here,

$\overline{a_1} = -4\hat{i} + 4\hat{j} + \hat{k}$

$\overline{b_1} = \hat{i} + \hat{j} - \hat{k}$

$\overline{a_2} = -3\hat{i} - 8\hat{j} - 3\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 3 & 3 \end{vmatrix}$$

$= \hat{i}(3+3) - \hat{j}(3+2) + \hat{k}(3-2)$

$\therefore \overline{b_1} \times \overline{b_2} = 6\hat{i} - 5\hat{j} + \hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{6^2 + (-5)^2 + 1^2}$

$= \sqrt{36 + 25 + 1}$

$= \sqrt{62}$

$\overline{a_2} - \overline{a_1} = (-3+4)\hat{i} + (-8-4)\hat{j} + (-3-1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} - 12\hat{j} - 4\hat{k}$



Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(6\hat{i} - 5\hat{j} + \hat{k}\right).\left(\hat{i} - 12\hat{j} - 4\hat{k}\right)$

$= (6 \times 1) + ((-5) \times (-12)) + (1 \times (-4))$

= 6 + 60 – 4

= 62

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{62}{\sqrt{62}}\right|$$

$d = \sqrt{62}\ units$

**3. Question**

Find the shortest distance between the given lines.

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k}),$$

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k}).$$

**Answer**

**Given equations :**

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\bar{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$



**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) \,.\, (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

**Answer :**

For given lines,

$$\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$$

$$\bar{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$$

Here,

$$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\overline{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} + \hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(-3 - 6) - \hat{j}(1 - 4) + \hat{k}(3 + 6)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-9)^2 + 3^2 + 9^2}$$

$$= \sqrt{81 + 9 + 81}$$

$$= \sqrt{171}$$

$$\overline{a_2} - \overline{a_1} = (4 - 1)\hat{i} + (5 - 2)\hat{j} + (6 - 3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(-9\hat{i} + 3\hat{j} + 9\hat{k}\right).\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$$

$$= ((-9) \times 3) + (3 \times 3) + (9 \times 3)$$

= - 27 + 9 + 27

= 9

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{9}{\sqrt{171}}\right|$$

$$\therefore d = \frac{9}{\sqrt{19}.\sqrt{9}}$$

$$\therefore d = \frac{3}{\sqrt{19}}$$

$$\therefore d = \frac{3\sqrt{19}}{19}$$

**4. Question**

Find the shortest distance between the given lines.

$$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right),$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right).$$

**Answer**

**Given equations :**

$$\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

**To Find** : d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

**Answer :**

For given lines,

$\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} + \hat{k}$

$\overline{b_1} = \hat{i} - \hat{j} + \hat{k}$

$\overline{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

$\overline{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(-2 - 1) - \hat{j}(2 - 2) + \hat{k}(1 + 2)$

$\therefore \overline{b_1} \times \overline{b_2} = -3\hat{i} + 0\hat{j} + 3\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-3)^2 + 0^2 + 3^2}$

$= \sqrt{9 + 0 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} - 3\hat{j} - 2\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-3\hat{i} + 0\hat{j} + 3\hat{k}).(\hat{i} - 3\hat{j} - 2\hat{k})$

$= ((-3) \times 1) + (0 \times (-3)) + (3 \times (-2))$

= - 3 + 0 – 6

= - 9

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{-9}{3\sqrt{2}}\right|$$

$$\therefore d = \frac{3}{\sqrt{2}}$$

$$\therefore d = \frac{3\sqrt{2}}{2}$$

**5. Question**

Find the shortest distance between the given lines.



$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}),$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k}).$$

**Answer**

**<u>Given equations</u> :**

$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$

**<u>To Find</u> :** d

**<u>Formula</u> :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

then,

$$\bar{a}.\bar{b}=(a_1\times b_1)+(a_2\times b_2)+(a_3\times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r}=\overline{a_1}+\lambda\overline{b_1}$ and

$\bar{r}=\overline{a_2}+\lambda\overline{b_2}$ is given by,

$$d=\left|\frac{(\overline{b_1}\times\overline{b_2}).(\overline{a_2}-\overline{a_1})}{|\overline{b_1}\times\overline{b_2}|}\right|$$



__Answer__ :

For given lines,

$$\bar{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$$

$$\bar{r}=(3\hat{i}+3\hat{j}-5\hat{k})+\mu(-2\hat{i}+3\hat{j}+8\hat{k})$$

Here,

$$\overline{a_1}=\hat{i}+2\hat{j}-4\hat{k}$$

$$\overline{b_1}=2\hat{i}+3\hat{j}+6\hat{k}$$

$$\overline{a_2}=3\hat{i}+3\hat{j}-5\hat{k}$$

$$\overline{b_2}=-2\hat{i}+3\hat{j}+8\hat{k}$$

Therefore,

$$\overline{b_1}\times\overline{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 6\\ -2 & 3 & 8\end{vmatrix}$$

$$=\hat{i}(24-18)-\hat{j}(16+12)+\hat{k}(6-6)$$

$$\therefore\overline{b_1}\times\overline{b_2}=6\hat{i}-28\hat{j}+0\hat{k}$$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{6^2 + (-28)^2 + 0^2}$

$= \sqrt{36 + 784 + 9}$

$= \sqrt{820}$

$\overline{a_2} - \overline{a_1} = (3-1)\hat{\imath} + (3-2)\hat{\jmath} + (-5+4)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(6\hat{\imath} - 28\hat{\jmath} + 0\hat{k}\right).\left(2\hat{\imath} + \hat{\jmath} - \hat{k}\right)$

$= (6 \times 2) + ((-28) \times 1) + (0 \times (-1))$

= 12 – 28 + 0

= - 16

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{-16}{\sqrt{820}}\right|$$

$$d = \frac{16}{\sqrt{820}}\ units$$



**6. Question**

Find the shortest distance between the given lines.

$$\vec{r} = \left(6\hat{i} + 3\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 4\hat{k}\right),$$

$$\vec{r} = \left(-9\hat{i} + \hat{j} - 10\hat{k}\right) + \mu\left(4\hat{i} + \hat{j} + 6\hat{k}\right).$$

**Answer**

**<u>Given equations</u> :**

$\bar{r} = \left(6\hat{\imath} + 3\hat{k}\right) + \lambda\left(2\hat{\imath} - \hat{\jmath} + 4\hat{k}\right)$

$\bar{r} = \left(-9\hat{\imath} + \hat{\jmath} - 10\hat{k}\right) + \mu\left(4\hat{\imath} + \hat{\jmath} + 6\hat{k}\right)$

**<u>To Find</u> :** d

**<u>Formula</u> :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

**Answer :**

For given lines,

$$\bar{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$$

$$\bar{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$$

Here,

$$\overline{a_1} = 6\hat{i} + 3\hat{k}$$

$$\overline{b_1} = 2\hat{i} - \hat{j} + 4\hat{k}$$

$$\overline{a_2} = -9\hat{i} + \hat{j} - 10\hat{k}$$

$$\overline{b_2} = 4\hat{i} + \hat{j} + 6\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 4 & 1 & 6 \end{vmatrix}$$

$$= \hat{i}(-6-4) - \hat{j}(12-16) + \hat{k}(2+4)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -10\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-10)^2 + 4^2 + 6^2}$$

$$= \sqrt{100 + 16 + 36}$$

$$= \sqrt{152}$$

$$\overline{a_2} - \overline{a_1} = (-9-6)\hat{i} + (1-0)\hat{j} + (6-3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = -15\hat{i} + \hat{j} + 3\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(-10\hat{i} + 4\hat{j} + 6\hat{k}\right).\left(-15\hat{i} + \hat{j} + 3\hat{k}\right)$$

$$= ((-10) \times (-15)) + (4 \times 1) + (6 \times 3)$$

= 150 + 4 + 18

= 172

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{172}{\sqrt{152}}\right|$$

$$\therefore d = \frac{172}{2\sqrt{38}}$$

$$\therefore d = \frac{86}{\sqrt{38}}$$

$$d = \frac{86}{\sqrt{38}}\ units$$

**7. Question**

Find the shortest distance between the given lines.

$$\vec{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k},$$

$\vec{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}.$

**Answer**

**Given equations :**

$\bar{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$

$\bar{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

**Answer :**

Given lines,

$\bar{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$

$\bar{r} = (1 + s)\hat{i} + (3s - 7)\hat{j} + (2s - 2)\hat{k}$

Above equations can be written as

$\bar{r} = (3\hat{i} + 4\hat{j} - 2\hat{k}) + t(-\hat{i} + 2\hat{j} + \hat{k})$

$\bar{r} = (\hat{i} - 7\hat{j} - 2\hat{k}) + s(\hat{i} + 3\hat{j} + 2\hat{k})$

Here,

$\overline{a_1} = 3\hat{i} + 4\hat{j} - 2\hat{k}$

$\overline{b_1} = -\hat{i} + 2\hat{j} + \hat{k}$

$\overline{a_2} = \hat{i} - 7\hat{j} - 2\hat{k}$

$\overline{b_2} = \hat{i} + 3\hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$= \hat{i}(4 - 3) - \hat{j}(-2 - 1) + \hat{k}(-3 - 2)$

$\therefore \overline{b_1} \times \overline{b_2} = \hat{i} + 3\hat{j} - 5\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{1^2 + 3^2 + (-5)^2}$

$= \sqrt{1 + 9 + 25}$

$= \sqrt{35}$

$\overline{a_2} - \overline{a_1} = (1 - 3)\hat{i} + (-7 - 4)\hat{j} + (-2 + 2)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -2\hat{i} - 11\hat{j} + 0\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (\hat{i} + 3\hat{j} - 5\hat{k}).(-2\hat{i} - 11\hat{j} + 0\hat{k})$

$= (1 \times (-2)) + (3 \times (-11)) + ((-5) \times 0)$

= - 2 – 33 + 0

= - 35

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left|\frac{-35}{\sqrt{35}}\right|$$

$$\therefore d = \sqrt{35}$$

$$d = \sqrt{35}\ units$$

**8. Question**

Find the shortest distance between the given lines.

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k},$$

$$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}.$$

**Answer**

**Given equations :**

$$\bar{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k}$$

$$\bar{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

**Answer :**

Given lines,

$$\bar{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k}$$

$$\bar{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

Above equations can be written as

$$\bar{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

$$\bar{r} = (\hat{i} - \hat{j} + 2\hat{k}) + s(-\hat{i} + 2\hat{j} + \hat{k})$$

Here,

$$\bar{a_1} = -\hat{i} + \hat{j} - \hat{k}$$

$$\bar{b_1} = \hat{i} + \hat{j} - \hat{k}$$

$$\bar{a_2} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\bar{b_2} = -\hat{i} + 2\hat{j} + \hat{k}$$

Therefore,

$$\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1 + 2) - \hat{j}(1 - 1) + \hat{k}(2 + 1)$$

$$\therefore \bar{b_1} \times \bar{b_2} = 3\hat{i} - 0\hat{j} + 3\hat{k}$$

$$\therefore |\bar{b_1} \times \bar{b_2}| = \sqrt{3^2 + 0^2 + 3^2}$$

$$= \sqrt{9 + 0 + 9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

$$\bar{a_2} - \bar{a_1} = (1 + 1)\hat{i} + (-1 - 1)\hat{j} + (2 + 1)\hat{k}$$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} - 2\hat{j} + 3\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1}) = (3\hat{i} - 0\hat{j} + 3\hat{k}) . (2\hat{i} - 2\hat{j} + 3\hat{k})$

$= (3 \times 2) + (0 \times (-2)) + (3 \times 3)$

= 6 + 0 + 9

= 15

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{15}{3\sqrt{2}} \right|$$

$$\therefore d = \frac{5}{\sqrt{2}}$$

$$\therefore d = \frac{5\sqrt{2}}{2}$$

$$d = \frac{5\sqrt{2}}{2} \text{ units}$$



**9. Question**

Compute the shortest distance between the lines $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} - \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} - \hat{k})$. Determine whether these lines intersect or not.

**Answer**

**Given equations :**

$\bar{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} - \hat{k})$

$\bar{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} - \hat{k})$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$



**<u>Answer</u> :**

For given lines,

$\bar{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} - \hat{k})$

$\bar{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} - \hat{k})$

Here,

$\overline{a_1} = \hat{i} - \hat{j}$

$\overline{b_1} = 2\hat{i} - \hat{k}$

$\overline{a_2} = 2\hat{i} - \hat{j}$

$\overline{b_2} = \hat{i} - \hat{j} - \hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 1 & -1 & -1 \end{vmatrix}$$

$= \hat{i}(0 - 1) - \hat{j}(-2 + 1) + \hat{k}(-2 - 0)$

$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + \hat{j} - 2\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-1)^2 + 1^2 + (-2)^2}$

$= \sqrt{1 + 1 + 4}$

$= \sqrt{6}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{i} + (-1 + 1)\hat{j} + (0 - 0)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} + 0\hat{j} + 0\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1}) = (-\hat{i} + \hat{j} - 2\hat{k}) . (\hat{i} + 0\hat{j} + 0\hat{k})$

$= ((-1) \times 1) + (1 \times 0) + ((-2) \times 0)$

= - 1 + 0 + 0

= - 1

Therefore, the shortest distance between the given lines is

$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$



$\therefore d = \left| \frac{-1}{\sqrt{6}} \right|$

$\therefore d = \frac{1}{\sqrt{6}}$

$\therefore d = \frac{\sqrt{6}}{6}$

$d = \frac{\sqrt{6}}{6}\ units$

As d ≠ 0

Hence, the given lines do not intersect.

**10. Question**

Show that the lines $\vec{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda\ (2\hat{i} - 7\hat{j} + 5\hat{k})$, and $\vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu$ $(2\hat{i} + \hat{j} - 3\hat{k})$ do not intersect.

**Answer**

**<u>Given equations</u> :**

$\bar{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$

$\bar{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) \, . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

**Answer :**

For given lines,

$\bar{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$

$\bar{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$

Here,

$\overline{a_1} = 3\hat{i} - 15\hat{j} + 9\hat{k}$

$\overline{b_1} = 2\hat{i} - 7\hat{j} + 5\hat{k}$

$\overline{a_2} = -\hat{i} + \hat{j} + 9\hat{k}$

$\overline{b_2} = 2\hat{i} + \hat{j} - 3\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$= \hat{i}(21 - 5) - \hat{j}(-6 - 10) + \hat{k}(2 + 14)$

$\therefore \overline{b_1} \times \overline{b_2} = 17\hat{i} + 16\hat{j} + 16\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{17^2 + 16^2 + 17^2}$

$= \sqrt{289 + 256 + 289}$

$= \sqrt{834}$

$\overline{a_2} - \overline{a_1} = (-1 - 3)\hat{i} + (1 + 15)\hat{j} + (9 - 9)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -4\hat{i} + 16\hat{j} + 0\hat{k}$



Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(17\hat{i} + 16\hat{j} + 16\hat{k}\right).\left(-4\hat{i} + 16\hat{j} + 0\hat{k}\right)$

$= (17 \times (-4)) + (16 \times 16) + (16 \times 0)$

= - 68 + 256 + 0

= 188

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{188}{\sqrt{834}}\right|$$

$$\therefore d = \frac{188}{\sqrt{834}} \text{units}$$

As d ≠ 0

Hence, the given lines do not intersect.

**11. Question**

Show that the lines $\vec{r} = \left(2\hat{i} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2j + 3\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu$ $\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ intersect.

Also, find their point of intersection.

**Answer**

**Given equations :**

$$\bar{r} = \left(2\hat{\imath} - 3\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right)$$

$$\bar{r} = \left(2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right)$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$



**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**Answer :**

For given lines,

$$\bar{r} = (2\hat{i} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\bar{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$$

Here,

$$\bar{a_1} = 2\hat{i} - 3\hat{k}$$

$$\overline{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(12 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{3^2 + 2^2 + (-1)^2}$$

$$= \sqrt{9 + 4 + 1}$$

$$= \sqrt{14}$$

$$\overline{a_2} - \bar{a_1} = (2 - 2)\hat{i} + (6 - 0)\hat{j} + (3 + 3)\hat{k}$$

$$\therefore \overline{a_2} - \bar{a_1} = 0\hat{i} + 6\hat{j} + 6\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \bar{a_1}) = \left(3\hat{i} + 2\hat{j} - \hat{k}\right).\left(0\hat{i} + 6\hat{j} + 6\hat{k}\right)$$

$$= (3 \times 0) + (2 \times 6) + ((-1) \times 6)$$

= 0 + 12 – 6

= 6

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \bar{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$\therefore d = \left|\frac{6}{\sqrt{14}}\right|$

$\therefore d = \frac{6}{\sqrt{14}}$ units

As d ≠ 0

Hence, the given lines do not intersect.

**12. Question**

Show that the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\ \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and $\vec{r} = \left(4\hat{i} + \hat{j}\right) + \mu\ \left(5\hat{i} + 2\hat{j} + \hat{k}\right)$ intersect.

Also, find their point of intersection.

**Answer**

**Given equations :**

$\bar{r} = \left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right) + \lambda\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right)$

$\bar{r} = \left(4\hat{\imath} + \hat{\jmath}\right) + \mu\left(5\hat{\imath} + 2\hat{\jmath} + \hat{k}\right)$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}) . (\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

**Answer :**

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\bar{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

Here,

$$\bar{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\bar{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\bar{a_2} = 4\hat{i} + \hat{j}$$

$$\bar{b_2} = 5\hat{i} + 2\hat{j} + \hat{k}$$

Therefore,

$$\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(3 - 8) - \hat{j}(2 - 20) + \hat{k}(4 - 15)$$

$$\therefore \bar{b_1} \times \bar{b_2} = -5\hat{i} + 18\hat{j} - 11\hat{k}$$

$$\therefore |\bar{b_1} \times \bar{b_2}| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$$

$$= \sqrt{25 + 324 + 121}$$

$$= \sqrt{470}$$

$$\bar{a_2} - \bar{a_1} = (4 - 1)\hat{i} + (1 - 2)\hat{j} + (0 - 3)\hat{k}$$

$$\therefore \bar{a_2} - \bar{a_1} = 3\hat{i} - \hat{j} - 3\hat{k}$$

Now,

$$(\bar{b_1} \times \bar{b_2}) . (\bar{a_2} - \bar{a_1}) = (-5\hat{i} + 18\hat{j} - 11\hat{k}) . (3\hat{i} - \hat{j} - 3\hat{k})$$

$$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$$

= - 15 – 18 + 33

= 0

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{0}{\sqrt{470}}\right|$$

$$\therefore d = 0 \text{ units}$$

As d = 0

Hence, the given lines not intersect each other.

Now, to find point of intersection, let us convert given vector equations into Cartesian equations.

For that substituting $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$$\therefore L1 : x\hat{i} + y\hat{j} + z\hat{k} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

$$\therefore L2 : x\hat{i} + y\hat{j} + z\hat{k} = \left(4\hat{i} + \hat{j}\right) + \mu\left(5\hat{i} + 2\hat{j} + \hat{k}\right)$$

$$\therefore L1 : (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$$

$$\therefore L2 : (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$$

$$\therefore L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\therefore L2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$$

General point on L1 is

$x_1 = 2\lambda+1$ , $y_1 = 3\lambda+2$ , $z_1 = 4\lambda+3$

let, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Therefore, point P satisfies equation of line L2.

$$\therefore \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = \frac{4\lambda + 3 - 0}{1}$$

$$\therefore \frac{2\lambda - 3}{5} = \frac{3\lambda + 1}{2}$$

$\Rightarrow 4\lambda - 6 = 15\lambda + 5$

$\Rightarrow 11\lambda = -11$

$\Rightarrow \lambda = -1$

Therefore, $x_1 = 2(-1)+1$ , $y_1 = 3(-1)+2$ , $z_1 = 4(-1)+3$

$\Rightarrow x_1 = -1$ , $y_1 = -1$ , $z_1 = -1$

Hence point of intersection of given lines is (-1, -1, -1).

**13. Question**

Find the shortest distance between the lines $L_1$ and $L_2$ whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$.

**HINT:** The given lines are parallel.

**Answer**

**Given equations :**

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors



$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two parallel lines :**

The shortest distance between the parallel lines $\bar{r} = \bar{a_1} + \lambda\bar{b}$ and