

## EXERCISE 22.8

### Question. 1

#### Solution:

From the question it is given that,  $dy/dx = (x + y + 1)^2$  ... [equation (i)]

Let us assume that,  $x + y + 1 = v$

Differentiating with respect to  $x$  on both side, we get,

$$1 + (dy/dx) = dv/dx$$

Transposing,

$$dy/dx = (dv/dx) - 1 \quad \dots \text{[equation (ii)]}$$

Substituting equation (ii) in equation (i),

$$\text{Then, } (dv/dx) - 1 = v^2$$

$$dv/dx = v^2 + 1$$

Now, taking like variables on same side,

$$dv/(v^2 + 1) = dx$$

Integrating on both side we get,

$$\int (dv/(v^2 + 1)) = \int dx$$

$$\text{We know that, } \int (dx/(a^2 + x^2)) = (1/a) \tan^{-1} (x/a) + c$$

$$\tan^{-1} (v) = x + c$$

Substitute value of  $v$  we get,

$$\tan^{-1} (x + y + 1) = x + c$$

### Question. 2

#### Solution:

From the question it is given that,  $(dy/dx) \cos (x - y) = 1$  ... [equation (i)]

Let us assume that,  $x - y = v$

Differentiating with respect to  $x$  on both side, we get,

$$1 - (dy/dx) = dv/dx$$

Transposing,

$$dy/dx = 1 - (dv/dx) \quad \dots \text{[equation (ii)]}$$

Substituting equation (ii) in equation (i),

$$\text{Then, } 1 - (dv/dx) \cos v = 1$$

$$1 - (dv/dx) = \sec v$$

$$1 - \sec v = dv/dx$$

Now, taking like variables on same side,

$$dx = dv/(1 - \sec v)$$

$$dx = (\cos/(1 - \cos v)) dv$$

$$dx = \left[ \frac{(\cos^2 (v/2)) - (\sin^2 (v/2))}{2 \sin^2 (v/2)} \right] dv$$

Integrating on both side we get,

$$\int dx = \int \left[ \frac{(\cos^2 (v/2)) - (\sin^2 (v/2))}{2 \sin^2 (v/2)} \right] dv$$

We know that,  $\int \operatorname{cosec}^2 x = -\cot x + c$

$$\int dx = \int \left( \frac{1}{2} \cot (v/2) \right) dv - \int \frac{1}{2} dv$$

$$2 \int dx = \int (\operatorname{cosec}^2 (v/2) - 1) dv - \int dv$$

$$2x = -2 \cot (v/2) dv - v - v + c_1$$

$$2(x + v) = -2 \cot (v/2) + c_1$$

$$x + x - y = -\cot ((x - y)/2) + c$$

$$c + y = \cot ((x - y)/2)$$

### Question. 3

#### Solution:

From the question it is given that,  $(dy/dx) = ((x - y) + 3)/(2(x - y) + 5) \dots$  [equation (i)]

Let us assume that,  $x - y = v$

Differentiating with respect to  $x$  on both side, we get,

$$1 - (dy/dx) = dv/dx$$

Transposing,

$$dy/dx = 1 - (dv/dx) \dots$$
 [equation (ii)]

Substituting equation (ii) in equation (i),

$$\text{Then, } 1 - (dv/dx) = (v + 3)/(2v + 3)$$

Transposing we get,

$$dv/dx = 1 - ((v + 3)/(2v + 5))$$

$$dv/dx = (2v + 5 - v - 3)/(2v + 5)$$

$$dv/dx = (v + 2)/(2v + 5)$$

Now, taking like variables on same side,

$$((2v + 5)/(v + 2))dv = dx$$

$$(((2v + 4) + 1)/(v + 2)) dv = dx$$

On dividing we get,

$$(2 + (1/(v + 2))) dv = dx$$

Integrating on both side we get,

$$\int (2 + (1/(v + 2))) dv = \int dx$$

We know that,  $\int dx/x = \log x + c$  and  $\int adx = ax + c$

$$2v + \log [v + 2] = x + c$$

$$2(x - y) + \log [x - y - 2] = x + c$$

### Question. 4

**Solution:**

From the question it is given that,  $(dy/dx) = (x + y)^2$  ... [equation (i)]

Let us assume that,  $x + y = v$

Differentiating with respect to  $x$  on both side, we get,

$$1 + (dy/dx) = dv/dx$$

Transposing,

$$dy/dx = (dv/dx) - 1 \quad \dots \text{[equation (ii)]}$$

Substituting equation (ii) in equation (i),

$$\text{Then, } (dv/dx) - 1 = v^2$$

Transposing we get,

$$dv/dx = v^2 + 1$$

Now, taking like variables on same side,

$$dv/(v^2 + 1) = dx$$

Integrating on both side we get,

$$\int (1/(v^2 + 1)) dv = \int dx$$

$$\tan^{-1} v = x + c$$

$$\tan^{-1} (x + y) = x + c$$

$$x + y = \tan (x + c)$$

**Question. 5**

**Solution:**

From the question it is given that,  $(x + y)^2 (dy/dx) = 1$  ... [equation (i)]

Let us assume that,  $x + y = v$

Differentiating with respect to  $x$  on both side, we get,

$$1 + (dy/dx) = dv/dx$$

Transposing,

$$dy/dx = (dv/dx) - 1 \quad \dots \text{[equation (ii)]}$$

Substituting equation (ii) in equation (i),

$$\text{Then, } v^2 ((dv/dx) - 1) = 1$$

$$dv/dx = (1/v^2) + 1$$

$$dv/dx = (v^2 + 1)/v^2$$

Now, taking like variables on same side,

$$(v^2/(v^2 + 1)) dv = dx$$

$$((v^2 + 1 - 1)/(v^2 + 1)) dv = dx$$

Integrating on both side we get,

$$\int ((v^2 + 1 - 1)/(v^2 + 1)) dv = \int dx$$

$$\int (1 - (1/(v^2 + 1))) dv = \int dx$$

$$v - \tan^{-1} v = x + c$$

$$x + y - \tan^{-1} (x + y) = x + c$$

$$y - \tan^{-1} (x + c) = c$$



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