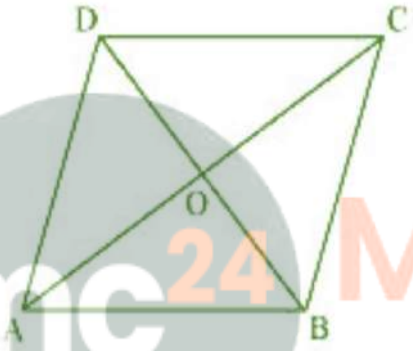
NCERT Solutions for Class-XI Maths

Chapter-12 Exercise-Miscellaneous

NCERT Math Class 11

1. Three vertices of a parallelogram $ABCD$ are $A(3,-1,2), B(1,2,-4)$ and $C(-1,1,2)$. Find the coordinates of the fourth vertex.

1. The three vertices of a parallelogram $ABCD$ are given as $A(3,-1,2), B(1,2,-4)$, and $C(-1,1,2)$. Let the coordinates of the fourth vertex be $D(x,y,z)$.



We know that the diagonals of a parallelogram bisect each other.
Therefore, in parallelogram $ABCD$, AC and BD bisect each other.

$\therefore$ Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow (1,0,2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2} = 1, \frac{y+2}{2} = 0, \text{ and } \frac{z-4}{2} = 2$$

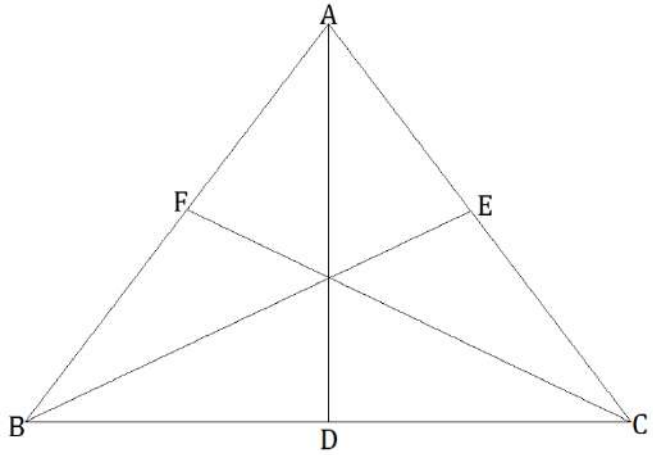
$$\Rightarrow x = 1, y = -2, \text{ and } z = 8$$

Thus, the coordinates of the fourth vertex are $(1,-2,8)$.

2. Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

2. Given: The vertices of the triangle are A (0, 0, 6), B (0, 4, 0) and C (6, 0, 0).

$\Rightarrow x_1 = 0, y_1 = 0, z_1 = 6; x_2 = 0, y_2 = 4, z_2 = 0; x_3 = 6, y_3 = 0, z_3 = 0$



We know that the median is a line segment through a vertex of a triangle to the midpoint of the side opposite to the vertex.

So, let the medians of this triangle be AD, BE and CF corresponding to the vertices A, B and C respectively.

$\Rightarrow$ D, E and F are the midpoints of the sides BC, AC and AB respectively.

By Midpoint Formula, we know that the coordinates of the mid-point of the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

So, we have

The coordinates of D $= \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right) = \left(\frac{6}{2}, \frac{4}{2}, \frac{0}{2}\right) = (3, 2, 0)$

The coordinates of E $= \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2}\right) = \left(\frac{6}{2}, \frac{0}{2}, \frac{6}{2}\right) = (3, 0, 3)$

And the coordinates of F $= \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) = \left(\frac{0}{2}, \frac{4}{2}, \frac{6}{2}\right) = (0, 2, 3)$

By Distance Formula, we know that the distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by $PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$.

The lengths of the medians are:

$$AD = \sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2} = \sqrt{3^2 + 2^2 + (-6)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$$

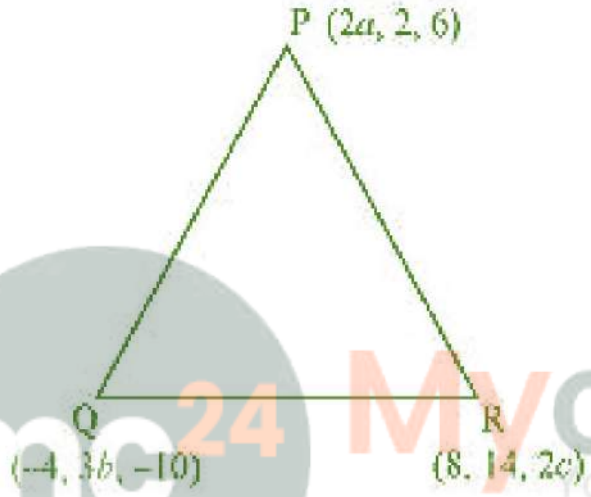
$$BE = \sqrt{(3-0)^2+(0-4)^2+(3-0)^2} = \sqrt{3^2+(-4)^2+3^2} = \sqrt{9+16+9} = \sqrt{34}$$

$$CF = \sqrt{(0-6)^2+(2-0)^2+(3-0)^2} = \sqrt{(-6)^2+2^2+3^2} = \sqrt{36+4+9} = \sqrt{49} = 7$$

So, the lengths of the medians of the given triangle are 7, $\sqrt{34}$ and 7.

3. If the origin is the centroid of the triangle PQR with vertices $P(2a,2,6), Q(-4,3b,10)$ and $R(8,14,2c)$, then find the values of a, b and c.

3.



It is known that the coordinates of the centroid of the triangle, whose vertices are $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3), are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

Therefore, coordinates of the centroid of $\triangle PQR$

$$= \left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

It is given that origin is the centroid of $\triangle PQR$.

$$\therefore (0,0,0) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

$$\Rightarrow \frac{2a+4}{3} = 0, \frac{3b+16}{3} = 0 \text{ and } \frac{2c-4}{3} = 0$$

$$\Rightarrow a = -2, b = -\frac{16}{3} \text{ and } c = 2$$

Thus, the respective values of a, b, and c are $-2, -\frac{16}{3}$, and 2.

4. Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P (3, –2, 5).

4. Let the point on y-axis be A (0, y, 0).

Then, it is given that the distance between the points A (0, y, 0) and P (3, -2, 5) is $5\sqrt{2}$.

Now, by Distance Formula, we know that the distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

So, the distance between the points A (0, y, 0) and P (3, -2, 5) is given by

$$AP = \sqrt{(3-0)^2 + (-2-y)^2 + (5-0)^2} = \sqrt{3^2 + (-2-y)^2 + 5^2}$$

$$= \sqrt{(-2-y)^2 + 9 + 25} = \sqrt{(-2-y)^2 + 34}$$

$$\Rightarrow \sqrt{(-2-y)^2 + 34} = 5\sqrt{2}$$

Squaring both sides, we get

$(-2 -y)^2 + 34 = 25 \times 2$

$\Rightarrow (-2 -y)^2 = 50 - 34$

$\Rightarrow 4 + y^2 + (2 \times -2 \times -y) = 16$

$\Rightarrow y^2 + 4y + 4 -16 = 0$

$\Rightarrow y^2 + 4y - 12 = 0$

$\Rightarrow y^2 + 6y - 2y - 12 = 0$

$\Rightarrow y (y + 6) - 2 (y + 6) = 0$

$\Rightarrow (y + 6) (y - 2) = 0$

$\Rightarrow y = -6, y = 2$

So, the points (0, 2, 0) and (0, -6, 0) are the required points on the y-axis.

5. A point R with x-coordinate 4 lies on the line segment joining the points $P(2,-3,4)$ and $Q(8,0,10)$. Find the coordinates of the point R.

[Hint suppose R divides PQ in the ratio $k:1$. The coordinates of the point R are given by $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$]

5. The coordinates of points P and Q are given as $P(2,-3,4)$ and $Q(8,0,10)$. Let R divide line segment PQ in the ratio $k:1$.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1}\right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$$

It is given that the x-coordinate of point R is 4 .

$$\therefore \frac{8k+2}{k+1}=4$$

$$\Rightarrow 8k+2=4k+4$$

$$\Rightarrow 4k=2$$

$$\Rightarrow k=\frac{1}{2}$$

Therefore, the coordinates of point R are

$$\left(4,\frac{-3}{\frac{1}{2}+1},\frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right)=(4,-2,6)$$

6. If A and B be the points (3, 4, 5) and (−1, 3, −7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

6. Given: The points A (3, 4, 5) and B (-1, 3, -7)

$\Rightarrow x_1 = 3, y_1 = 4, z_1 = 5; x_2 = -1, y_2 = 3, z_2 = -7;$

$PA^2 + PB^2 = k^2$ .....(i)

Let the point be P (x, y, z).

Now, by Distance Formula, we know that the distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

So, $PA=\sqrt{(3-x)^2+(4-y)^2+(5-z)^2}$

And $PB=\sqrt{(-1-x)^2+(3-y)^2+(-7-z)^2}$

Now, substituting these values in (i), we have

$[(3 - x)^2 + (4 - y)^2 + (5 - z)^2] + [(-1 - x)^2 + (3 - y)^2 + (-7 - z)^2] = k^2$

$\Rightarrow [(9 + x^2 - 6x) + (16 + y^2 - 8y) + (25 + z^2 - 10z)] + [(1 + x^2 + 2x) + (9 + y^2 - 6y) + (49 + z^2 + 14z)] = k^2$

$\Rightarrow 9 + x^2 - 6x + 16 + y^2 - 8y + 25 + z^2 - 10z + 1 + x^2 + 2x + 9 + y^2 - 6y + 49 + z^2 + 14z = k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = k^2 - 109$

$\Rightarrow 2 (x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$

$$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$$