NCERT Solutions for Class 10 Maths Chapter 9 Some applications of Trigonometry

EXERCISE 9.1

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see Fig. 9.11)

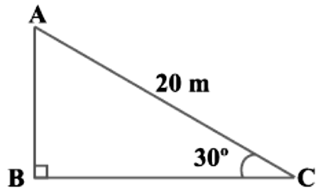

Fig. 9.11

Solution:

In this figure; AC is the rope which is 20 m long and we have to find AB.

In Δ ABC;

$$Sin\ \theta = \frac{p}{h}$$

$$Sin 30° = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{20}$$

Hence, AB = 10cm

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2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree

Solution:

Let BD be the tree.

Now, due to storm AD is broken and leans as AC.

AC is the broken part of the tree and AB is the part which is still upright.

Hence,

BC = 8 m

And,

∠ BCA=30°

Sum of AB and AC, which is equal to BD will give the height of the tree.

In Δ ABC:

$$Cos\ \theta = \frac{b}{h}$$

$$Cos 30° = \frac{BC}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{8}{AC}$$

AC = $\frac{16}{\sqrt{3}}$

Hence,

$$tan\ \theta = \frac{p}{b}$$

$$tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{8}$$

AB = $\frac{8}{\sqrt{3}}$

Now,

Height of tree= AB + AC

$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$

$= \frac{24}{\sqrt{3}}$

$= 8\sqrt{3}\, m$

Hence, height of the tree is $8\sqrt{3}\, m$

3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and Fig. 9.10 is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Solution:

In the first case:

p = 1.5 m and angle of elevation = 30°

Now,

$$Sin\ \theta = \frac{p}{h}$$

$$Sin 30° = \frac{1.5}{h}$$

$$\frac{1}{2} = \frac{1.5}{h}$$

Hence, h = 3 m

In the second case:

p = 3 m, angle of elevation = 60°

$$Sin\ \theta = \frac{p}{h}$$

$$Sin60° = \frac{3}{h}$$

$$\frac{\sqrt{3}}{2} = \frac{3}{h}$$

Hence, h = $2\sqrt{3}$ m

Therefore, the length of first case and second case are 3m and $2\sqrt{3}m$ respectively.

4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower

Solution: In this question:

b = 30 m,

Angle of elevation = 30°

And

p = ?

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$$tan\ \theta = \frac{p}{h}$$

$$tan30° = \frac{p}{30}$$

$$\frac{1}{\sqrt{3}} = \frac{p}{30}$$

p = $\frac{30}{\sqrt{3}}$

Hence, p = $10\sqrt{3}\ m$

5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string

Solution: In this question:

p = 60 m, angle of elevation = 60° and h = ?

Now,

$$Sin\ \theta = \frac{p}{h}$$

$$Sin60° = \frac{60}{h}$$

$$\frac{\sqrt{3}}{2} = \frac{60}{h}$$

h = $\frac{120}{\sqrt{3}}$

Hence, h= $40\sqrt{3}\ m$

6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building

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Solution:

According to the question:

p = 30 – 1.5 = 28.5 m,

Angle of elevation in first case = 30°

And

Angle of elevation, in second case = 60°.

Difference between bases in the first case and second case will give the distance covered by the boy.

1st case:

$$tan\ \theta = \frac{p}{b}$$

$$tan30° = \frac{28.5}{b}$$

$$\frac{1}{\sqrt{3}} = \frac{28.5}{b}$$

b = $28.5\sqrt{3}$

2nd case:

$$tan\ \theta = \frac{p}{b}$$

$$tan60° = \frac{28.5}{b}$$

$$\sqrt{3} = \frac{28.5}{b}$$

$$b = \frac{28.5}{\sqrt{3}}$$

Hence, the distance covered by boy:

= $28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$

$$= \frac{28.5 * 3 - 28.5}{\sqrt{3}}$$

$$= \frac{57}{\sqrt{3}}$$

= $19\sqrt{3}$

7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower

Solution:

As per the question:

Let the height of building = p = 20 m

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Therefore, total height of building and transmission tower = p’

The angle of elevation to the top of the building = 45° and that to the top of transmission tower = 60°

Height of transmission tower = ?

1st case:

$$tan\ \theta = \frac{p}{b}$$

$$tan45° = \frac{20}{b}$$

$$1 = \frac{20}{b}$$

$$b = 20\ m$$

2nd case:

$$tan\ \theta = \frac{p'}{b}$$

$$tan60° = \frac{p'}{20}$$

$$\sqrt{3} = \frac{p'}{20}$$

$$p' = 20\sqrt{3}$$

Now, the height of the tower can be calculated as:

p'- p = 20√3 - 20

= 20(√3 - 1)

= 20(1.732 - 1)

= 20 × 0.732

= 14.64 m

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8. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal

Solution:Let AD be the height of the statue which is 1.6 m,

And DB be the height of the pedestal

And C be the point where observer is present.

Now,

In Δ ABC,

$$tan\ \theta = \frac{p}{b}$$

$$tan60° = \frac{p}{b}$$

$$\sqrt{3} = \frac{DB + 1.6}{b}$$

$$b = \frac{DB + 1.6}{\sqrt{3}} \quad (i)$$

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In Δ DBC,

$$tan\ \theta = \frac{p}{b}$$

$$tan45° = \frac{p}{b}$$

$$1 = \frac{DB}{b}$$

$$b = DB \quad (ii)$$

On comparing (i) and (ii),

$$DB = \frac{DB + 1.6}{\sqrt{3}}$$

$$DB\sqrt{3} = DB + 1.6$$

$$DB\left(\sqrt{3} - 1\right) = 1.6$$

$$DB = \frac{1.6}{\sqrt{3} - 1}$$

$$DB = 0.8(\sqrt{3} - 1)m$$

9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building

Solution: Let us take DC = 50 m,

∠ACB = 30°

And,

∠ DBC = 60°;

AB= ?

In Δ DCB;

$$\tan\theta = \frac{p}{b}$$

$$\tan 60° = \frac{50}{b}$$

$$\sqrt{3} = \frac{50}{b}$$

$$b = \frac{50}{\sqrt{3}}$$

In Δ ACB

$$\tan\theta = \frac{p}{b}$$

$$\tan 30° = \frac{AB}{b}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{\frac{50}{\sqrt{3}}}$$

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$$AB = \frac{50}{\sqrt{3} \times \sqrt{3}}$$

AB= $16\frac{2}{3}$ m

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles

Solution:

Let us take BD = 80 m,

∠ACB = 30°

And,

∠ ECD = 60°;

AB= ED = ?

In Δ ACB

$$\tan\theta = \frac{p}{b}$$

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC}$$

$$AB = \frac{BC}{\sqrt{3}} \qquad (i)$$

In Δ EDC:

$$\tan\theta = \frac{p}{b}$$

$$\tan 60° = \frac{ED}{CD}$$

$$\sqrt{3} = \frac{ED}{80 - BC}$$

$$ED = \sqrt{3}(80 - BC) \qquad (ii)$$

We know that AB = ED

Hence, from (i) and (ii),

$$\frac{BC}{\sqrt{3}} = \sqrt{3}(80 - BC)$$

$$BC = 3(80 - BC)$$

$$BC = 240 - 3BC$$

$$4BC = 240$$

$$BC = 60m$$

Now, using the value of BC in (i),

AB = $20\sqrt{3}\ m$

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11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see Fig. 9.12). Find the height of the tower and the width of the canal

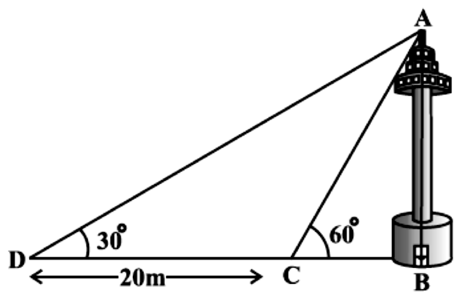

Fig. 9.12

Solution:

According to this figure:

CD = 20 m,

∠ACB = 60°,

∠ ADB = 30°,

AB = ? and BD = ?

In Δ ABC,

$$tan\ \theta = \frac{p}{b}$$

$$tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{BC}$$

$$AB = \sqrt{3}(BC) \quad (i)$$

In Δ ABD,

$$tan\ \theta = \frac{p}{b}$$

$$tan 30° = \frac{AB}{BC + 20}$$

$$\frac{BC + 20}{\sqrt{3}} = AB \quad (ii)$$

From (i) and (ii),

$$\sqrt{3}(BC) = \frac{BC + 20}{\sqrt{3}}$$

$$3BC = BC + 20$$

$$BC = 10\ m$$

Using the value of BC in (i) we get;

AB = 10√3 m which is the height of the tower

Width of canal = 10 m

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12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is60° and the angle of depression of its foot is 45°. Determine the height of the tower

Solution: Let us take AB = 7m,

∠ACB = 45°,

∠EAD = 60°

AB = DC and EC = ED + DC ?

In Δ ABC:

$$tan\ \theta = \frac{p}{b}$$

$$tan45° = \frac{AB}{BC}$$

$$AB = BC = 7\ m$$

In Δ EDA,

$$tan\ \theta = \frac{p}{b}$$

$$tan60° = \frac{ED}{7}$$

$$\sqrt{3} = \frac{ED}{7}$$

$$ED = 7\sqrt{3}\ m$$

However,

Height of tower can be calculated as:

EC = ED + DC

= 7√3 + 7

= 7(√3 +1) m

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13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles ofdepression of two ships are 30° and 45°. If one ship is exactly behind the other on thesame side of the lighthouse, find the distance between the two ships

Solution: Let us take AB = 75m,

∠ACB = 45°,

∠EAD = 30°

BC =?, DC= ?

In Δ ABC,

$$tan\ \theta = \frac{p}{b}$$

$$tan 45° = \frac{75}{BC}$$

$$BC = 75\ m$$

In Δ ABD,

$$tan\ \theta = \frac{p}{b}$$

$$tan 30° = \frac{75}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{BD}$$

$$BD = 75\sqrt{3}\ m$$

$$BC + CD = 75\sqrt{3}\ m$$

$$75 + CD = 75\sqrt{3}\ m$$

$$CD = 75(\sqrt{3} - 1)\ m$$

14. A 1.2 m tall girl spots a balloon movingwith the wind in a horizontal line at aheight of 88.2 m from the ground. Theangle of elevation of the balloon fromthe eyes of the girl at any instant is60°. After some time, the angle

ofelevation reduces to 30° (see Fig. 9.13).Find the distance travelled by theballoon during the interval.

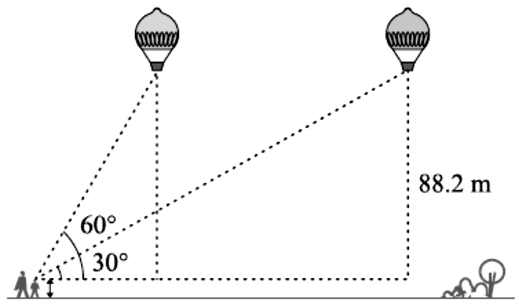

Fig. 9.13

Solution: From this figure:

BF = 1.2 m,

AG = 88.2 m,

AC = AG – GC

= 88.2 – 1.2

= 87 m

∠EBD = 30° and ∠ABC = 60°,

CD = ?

In Δ ABC;

$$tan\ \theta = \frac{p}{b}$$

$$tan60° = \frac{87}{BC}$$

$$\sqrt{3} = \frac{87}{BC}$$

$$BC = \frac{87}{\sqrt{3}}$$

In Δ ABC,

$$tan\ \theta = \frac{p}{b}$$

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$$tan30° = \frac{87}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{87}{BD}$$

$$BD = 87\sqrt{3}\, m$$

Now, the distance covered by the balloon will be:

CD = BD – BC

CD = $87\sqrt{3} - \frac{87}{\sqrt{3}}$

CD = $58\sqrt{3}\, m$

15. A straight highway leads to the foot of a tower. A man standing at the top of the towerobserves a car at an angle of depression of 30°, which is approaching the foot of thetower with a uniform speed. Six seconds later, the angle of depression of the car is foundto be 60°. Find the time taken by the car to reach the foot of the tower from this point

Solution: Let us assume that ∠ADB = 30°, ∠ACB = 60° and time taken to reach from D to C = 6 seconds.

Time taken to reach from C to B = ?

In Δ ABD

$$tan\ \theta = \frac{p}{b}$$

$$tan30° = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BD}$$

$$AB = \frac{BD}{\sqrt{3}} \qquad (i)$$

In Δ ABC;

$$tan\ \theta = \frac{p}{b}$$

$$tan60° = \frac{AB}{BC}$$

$$BC\sqrt{3} = AB \quad (ii)$$

Now, on comparing (i) and (ii),

$$BC\sqrt{3} = \frac{BD}{\sqrt{3}}$$

$$3BC = BC + 6$$

$$BC = 3$$

Hence, the time taken to reach from C to B = 6 second

16. The angles of elevation of the top of a tower from two points at a distance of 4 m and9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m

Solution: In this figure; ∠ADB = θ and ∠ACB = 90°-θ

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In Δ ABD;

$$tan\ \theta = \frac{AB}{DB}$$

$$tan\ \theta = \frac{AB}{9} \quad (i)$$

In Δ ABC,

$$\tan(90 - \theta) = \frac{AB}{BC}$$

$$cot\theta = \frac{AB}{4} \quad (ii)$$

But we know that,

$$cot\theta = \frac{1}{tan\theta}$$

Therefore,

$$\frac{AB}{4} = \frac{9}{AB}$$

AB = 3*2

AB = 6 m

Hence proved.