EXERCISE 2.4

For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorisation.

(i) (–8/3), 4/3
(ii) 21/8, 5/16
(iii) -2√3, -9
(iv) (-3/(2√5)), -½

Solution:

(i) Sum of the zeroes = - 8/3
Product of the zeroes = 4/3
$P(x) = x^2$ - (sum of the zeroes) + (product of the zeroes)
Then, $P(x)= x^2 - 8x/3 + 4/3$
$P(x)= 3x^2 - 8x + 4$
Using splitting the middle term method,
$3x^2 - 8x + 4 = 0$
$3x^2 - (6x + 2x) + 4 = 0$
$3x^2 - 6x - 2x + 4 = 0$
$3x(x - 2) - 2(x - 2) = 0$
$(x - 2)(3x - 2) = 0$
$\Rightarrow x = 2, 2/3$

(ii) Sum of the zeroes = 21/8
Product of the zeroes = 5/16
$P(x) = x^2$ - (sum of the zeroes) + (product of the zeroes)
Then, $P(x)= x^2 - 21x/8 + 5/16$
$P(x)= 16x^2 - 42x + 5$
Using splitting the middle term method,
$16x^2 - 42x + 5 = 0$
$16x^2 - (2x + 40x) + 5 = 0$
$16x^2 - 2x - 40x + 5 = 0$
$2x (8x - 1) - 5(8x - 1) = 0$
$(8x - 1)(2x - 5) = 0$
$\Rightarrow x = 1/8, 5/2$

(iii) Sum of the zeroes = $- 2\sqrt{3}$
Product of the zeroes = - 9
$P(x) = x^2$ - (sum of the zeroes) + (product of the zeroes)
Then, $P(x) = x^2 - 2\sqrt{3}x - 9$
Using splitting the middle term method,
$x^2 - 2\sqrt{3}x - 9 = 0$
$x^2 - (-\sqrt{3}x + 3\sqrt{3}x) - 9 = 0$
$x^2 + \sqrt{3}x - 3\sqrt{3}x - 9 = 0$
$x(x + \sqrt{3}) - 3\sqrt{3}(x + \sqrt{3}) = 0$
$(x + \sqrt{3})(x - 3\sqrt{3}) = 0$

$\Rightarrow x = -\sqrt{3}, 3\sqrt{3}$

(iv) Sum of the zeroes = $-3/2\sqrt{5}x$
Product of the zeroes = - ½
$P(x) = x^2$ - (sum of the zeroes) + (product of the zeroes)
Then, $P(x) = x^2 - 3/2\sqrt{5}x$ - ½
$P(x) = 2\sqrt{5}x^2 - 3x - \sqrt{5}$
Using splitting the middle term method,
$2\sqrt{5}x^2 - 3x - \sqrt{5} = 0$
$2\sqrt{5}x^2 - (5x - 2x) - \sqrt{5} = 0$
$2\sqrt{5}x^2 - 5x + 2x - \sqrt{5} = 0$
$\sqrt{5}x(2x - \sqrt{5}) - (2x - \sqrt{5}) = 0$
$(2x - \sqrt{5})(\sqrt{5} - 1) = 0$
$\Rightarrow x = -1/\sqrt{5}, \sqrt{5}/2$

2. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, $a + b$, $a + 2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

Solution:

Given that a, a+b, a+2b are roots of given polynomial $x^3-6x^2+3x+10$
Sum of the roots $\Rightarrow$ a+2b+a+a+b = -coefficient of x^2/ coefficient of x^3
$\Rightarrow 3a+3b = -(-6)/1 = 6$
$\Rightarrow 3(a+b) = 6$
$\Rightarrow a+b = 2$ ----------(1) b = 2-a

Product of roots $\Rightarrow$ (a+2b)(a+b)a = -constant/coefficient of x^3
$\Rightarrow (a+b+b)(a+b)a = -10/1$

Substituting the value of a+b=2 in it
$\Rightarrow (2+b)(2)a = -10$
$\Rightarrow (2+b)2a = -10$
$\Rightarrow (2+2-a)2a = -10$
$\Rightarrow (4-a)2a = -10$
$\Rightarrow 4a-a^2 = -5$
$\Rightarrow a^2-4a-5 = 0$
$\Rightarrow a^2-5a+a-5 = 0$
$\Rightarrow (a-5)(a+1) = 0$
a-5 = 0 or a+1 = 0
a = 5 a = -1

a = 5, -1 in (1) a+b = 2

When a = 5, 5+b=2 $\Rightarrow$ b=-3
a = -1, -1+b=2 $\Rightarrow$ b= 3

$\therefore$ If a=5 then b= -3

or
If a= -1 then b=3

3. Given that √2 is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.
Solution:

Given, √2 is one of the zero of the cubic polynomial.
Then, (x-√2) is one of the factor of the given polynomial p(x) = $6x^3+\sqrt{2}x^2-10x- 4\sqrt{2}$.
So, by dividing p(x) by x-√2

$$
\begin{array}{r}
6x^2 + 7\sqrt{2}\,x + 4 \\
(x - \sqrt{2}) \overline{)\,6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}} \\
6x^3 - 6\sqrt{2}\,x^2 \\
\hline
7\sqrt{2}\,x^2 - 10x - 4\sqrt{2} \\
7\sqrt{2}\,x^2 - 14x \\
\hline
4x - 4\sqrt{2} \\
4x - 4\sqrt{2} \\
\hline
0
\end{array}
$$

$6x^3+\sqrt{2}x^2-10x-4\sqrt{2}$= (x-√2) ($6x^2$ +7√2x + 4)
By splitting the middle term,
We get,
(x-√2) ($6x^2$ + 4√2x + 3√2x + 4)
= (x-√2) [2x(3x+2√2) + √2(3x+2√2)]
= (x-√2) (2x+√2) (3x+2√2)

To get the zeroes of p(x),
Substitute p(x)= 0
(x-√2) (2x+√2) (3x+2√2)= 0
x= √2 , x= -√2/2 ,x= -2√2/3
which is equal to,
x= √2 , x= -1 /√2 ,x= -2√2/3 [Rationalising second zero]
Hence, the other two zeroes of p(x) are -1/√2 and -2√2/3