EXERCISE 8.1

Choose the correct answer from the given four options:

1. If cos A = 4/5 , then the value of tan A is

(A) 3/5 (B) ¾ (C) 4/3 (D) 5/3

Solution:

According to the question,

cos A = 4/5 …(1)

We know,

tan A = sinA/cosA

To find the value of sin A,

We have the equation,

$\sin^2\theta + \cos^2\theta = 1$

So, $\sin\theta = \sqrt{(1-\cos^2\theta)}$

Then,

$\sin A = \sqrt{(1-\cos^2 A)}$ …(2)

$\sin^2 A = 1-\cos^2 A$

$\sin A = \sqrt{(1-\cos^2 A)}$

Substituting equation (1) in (2),

We get,

Sin A $= \sqrt{(1-(4/5)^2)}$

$= \sqrt{(1-(16/25))}$

$= \sqrt{(9/25)}$

= ¾

Therefore,

$$\tan A = \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$$

2. If sin A = ½ , then the value of cot A is

(A) √3 (B) 1/√3 (C) √3/2 (D) 1

Solution:

According to the question,

Sin A = ½ … (1)

We know that,

$$\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} \text{ … (2)}$$

To find the value of cos A.

We have the equation,

$\sin^2\theta + \cos^2\theta = 1$

So, $\cos\theta = \sqrt{(1-\sin^2\theta)}$

Then,

$\cos A = \sqrt{(1-\sin^2 A)}$ … (3)

$\cos^2 A = 1-\sin^2 A$

$\cos A = \sqrt{(1-\sin^2 A)}$

Substituting equation 1 in 3, we get,

$\cos A = \sqrt{(1-1/4)} = \sqrt{(3/4)} = \sqrt{3}/2$

Substituting values of sin A and cos A in equation 2, we get

$\cot A = (\sqrt{3}/2) \times 2 = \sqrt{3}$

3. The value of the expression [cosec (75° + θ) – sec (15° – θ) – tan (55° + θ) + cot (35° – θ)] is

(A) – 1 (B) 0 (C) 1 (D) 3 2

Solution:

According to the question,

We have to find the value of the equation,

$\text{cosec}(75°+\theta) - \sec(15°-\theta) - \tan(55°+\theta) + \cot(35°-\theta)$

$= \text{cosec}[90°-(15°-\theta)] - \sec(15°-\theta) - \tan(55°+\theta) + \cot[90°-(55°+\theta)]$

Since, $\text{cosec}(90°-\theta) = \sec\theta$

And, $\cot(90°-\theta) = \tan\theta$

We get,

$= \sec(15°-\theta) - \sec(15°-\theta) - \tan(55°+\theta) + \tan(55°+\theta)$

$= 0$

4. Given that sinθ = a b , then cosθ is equal to

(A) $b/\sqrt{(b^2- a^2)}$ (B) b/a (C) $\sqrt{(b^2-a^2)}/b$ (D) $a/\sqrt{(b^2-a^2)}$

Solution:

According to the question,

$\sin\theta = a/b$

We know, $\sin^2\theta + \cos^2\theta = 1$

$\sin^2 A = 1-\cos^2 A$

$\sin A = \sqrt{(1-\cos^2 A}$

So, $\cos\theta = \sqrt{(1-a^2/b^2)} = \sqrt{((b^2-a^2)/b^2)} = \sqrt{(b^2-a^2)}/b$

Hence, $\cos\theta = \sqrt{(b^2 - a^2)}/b$

5. If cos (α + β) = 0, then sin (α – β) can be reduced to

(A) cos β (B) cos 2β (C) sin α (D) sin 2α

Solution:

According to the question,

$\cos(\alpha+\beta) = 0$

Since, $\cos 90° = 0$

We can write,

$\cos(\alpha+\beta)= \cos 90°$

By comparing cosine equation on L.H.S and R.H.S,

We get,

$(\alpha+\beta)= 90°$

$\alpha = 90°-\beta$

Now we need to reduce $\sin(\alpha-\beta)$,

So, we take,

$\sin(\alpha-\beta) = \sin(90°-\beta-\beta) = \sin(90°-2\beta)$

$\sin(90°-\theta) = \cos\theta$

So, $\sin(90°-2\beta) = \cos 2\beta$

Therefore, $\sin(\alpha-\beta) = \cos 2\beta$

6. The value of (tan1° tan2° tan3° ... tan89°) is

(A) 0 (B) 1 (C) 2 (D) ½

Solution:

tan 1°. tan 2°.tan 3° …… tan 89°

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.tan 45°.tan 46°.tan 47°…tan 87°.tan 88°.tan 89°

Since, tan 45° = 1,

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.1.tan 46°.tan 47°…tan 87°.tan 88°.tan 89°

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.1.tan(90°-44°).tan(90°-43°)…tan(90°-3°). tan(90°-2°).tan(90°-1°)

Since, tan(90°-θ) = cot θ,

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.1.cot 44°.cot 43°…cot 3°.cot 2°.cot 1°

Since, tan θ = (1/cot θ)

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.1. (1/tan 44°). (1/tan 43°)… (1/tan 3°). (1/tan 2°). (1/tan 1°)

$$= \left(\tan 1^\circ \times \frac{1}{\tan 1^\circ}\right).\left(\tan 2^\circ \times \frac{1}{\tan 2^\circ}\right)\ldots\left(\tan 44^\circ \times \frac{1}{\tan 44^\circ}\right)$$

= 1

Hence, tan 1°.tan 2°.tan 3° …… tan 89° = 1

7. If cos 9α = sinα and 9α < 90° , then the value of tan5α is

(A) 1/√3 (B) √3 (C) 1 (D) 0

Solution:

According to the question,

cos 9∝ = sin ∝ and 9∝<90°

i.e. 9α is an acute angle

We know that,

sin(90°-θ) = cos θ

So,

cos 9∝ = sin (90°-∝)

Since, cos 9∝ = sin(90°-9∝) and sin(90°-∝) = sin∝

Thus, sin (90°-9∝) = sin∝

90°-9∝ =∝

10∝ = 90°

∝ = 9°

Substituting ∝ = 9° in tan 5∝, we get,

tan 5∝ = tan (5×9) = tan 45° = 1

∴, tan 5∝ = 1